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VALUATION OF EUROPEAN PUT OPTION BY USING THE QUADRATURE METHOD UNDER THE VARIANCE GAMMA PROCESS

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ABSTRACT

Dynamic asset pricing model uses the Geometric Brownian Motion process. The Black-Scholes model known as standard model to price European option based on the assumption that underlying asset prices dynamic follows that log returns of asset is normally distributed. In this paper, we introduce a new stochastic process called levy process for pricing options. In this paper, we use the quadrature method to solve a numerical example for pricing options in the Indian context. The illustrations used in this paper for pricing the European style option. We also try to develop the pricing formula for European put option by using put-call parity and check its relevancy on actual market data and observe some underlying phenomenon.

1. INTRODUCTION

In option pricing theory the main problem is to find the fair value of an option. To find the value of European option a well-known model named Black-Scholes model which is based on certain assumptions. Black-Scholes model is based on the assumption that the underlying asset price observes the Geometric Brownian motion where the log returns of the asset price is normally distributed. Some research papers conclude that GBM fails to represent the characteristic features like excessive kurtosis and skewness.

Some study on stock indices shows that class of Variance Gamma process can capture those characteristic features. To solve a levy process there are four different types of method. We used quadrature method to solve a numerical example for pricing European call option.

Sullivan (2000) [3] derived the pricing of American options under the Geometric Brownian Motion successfully by applying Quadrature routines. Andricopoulos et. al (2003) [1] recognized that by using a discounted integration of the payoff the options could be priced accurately and it is also provided that the payoff is segmented which implies that the integral is only depends over the continuous segments of the payoff.

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Black Fischer, Myron Scholes (1973) [2] derived a closed form formulae for a European option on an underlying asset with returns following Geometric Brownian motion, i.e. log returns of asset follows normal distribution. Geske & Torous (1991) [4] showed that the distribution of stock returns exhibits deviations from normality; in particular skewness and kurtosis. Madan and Seneta (1990) [5] in their theses derived The Variance Gamma (VG) model for share market returns which is based on levy process. Con and Tankov (2004) [6] in their book represent the properties and construction of levy processes and discussed multivariate modelling via Brownian subordination using levy process.

Sato (2011) [7] represent processes with the properties of independent and stationary increments and made the link between such processes and their infinite divisible laws.

Permana et. al (2014) [8] concluded that the Variance Gamma Model performed better as compared to the GBM model in Indonesian market. The Variance Gamma model fitted with the first four moments along with skewness and excessive kurtosis.

Ivanovski et. al (2015) [9] conclude that the Geometric Brownian motion breaks to catch the characteristics feature of asset price dynamics that reveal heavy tails and excessive kurtosis.

In this paper, we studies variance gamma model which is based on levy process. Three parameter variance gamma models are used to price European option. We developed European put option formula by applying a put call parity to call option formula. The contribution of this article is to check the relevancy of Indian context and observe some underlying phenomenon.

2. QUADRATURE METHOD

Consider that the underlying asset price follows Geometric Brownian motion. The notations used in this method are as follows:

E be the Exercise price, S_t be the Stock price, r be the constant risk-free interest rate, q be the dividend yield, σ be the Volatility and $T = t + \Delta t$ be the time where the payoff be known.

Define log price $x = \log\left(\frac{S_t}{E}\right)$ and $y = \log\left(\frac{S_{t+\Delta t}}{E}\right)$

Option price at time t can be defined as

$$v(x,t) = e^{-r\Delta t}E[V(y, t + \Delta t)]$$

where expectation is the risk neutral (RN) and where $V(y, t + \Delta t)$ is time $t + \Delta t$ known payoff at the scaled log price y.

Option price at time t is given by

$$V(x,t) = A(X). \int_{-\infty}^{\infty} F(x,y)dy$$

Where,

$$A(X) = \frac{1}{\sqrt{2\pi\sigma^2\Delta t}} \exp\left(-\frac{1}{2}kx - \frac{1}{8}k^2\sigma^2\Delta t - r\Delta t\right)$$

$$F(x,y) = B(x,y). V(y, t + \Delta t)$$

$$B(x,y) = \exp\left(\frac{(y-x)^2}{2\sigma^2\Delta t} + \frac{1}{2}ky\right)$$

$$k = 2\left(\frac{r-q}{\sigma^2}\right) - 1$$

Price of European Call option taken on Tata Consultancy Services Limited based on Quadrature method.

S_t = Rs. 2216, E = Rs. 2240, r = 10%, T = 36 Days, σ (I.V) =15.85%, C = Rs.**62.20**

Intervals	European call prices v (x, t)
[0,1]	8.69
[0,2]	17.39
[0,3]	26.09
[0,4]	34.79
[0,5]	43.48
[0,6]	52.18
[0,7]	60.88

3. VARIANCE GAMMA MODEL

Madan and Seneta [5] and Madan and Milne [10] improvised the Variance Gamma process from two to three parameters. The two parameters control the kurtosis and volatility and the third parameter to rule skewness is adjoining by generalizing the Variance Gamma model.

The VG process is procured by evaluating Brownian motion at arbitrary time change given by a Gamma process. The Gamma process $G(t; \mu, v)$ with mean rate μ and variance rate ν is the process of independent gamma increments over non-overlapping intervals of time $(t, t + h)$.

By using associate of Brownian motion process, we define the VG process in terms of Brownian motion $b(t; \theta, \sigma)$ and gamma process $G(t; 1, v)$ as follows

$$X(t, \sigma, v, \theta) = b(G(t; 1, v); \theta, \sigma)$$

The three parameters elaborate in the VG model are:

σ: volatility of Brownian motion which controls volatility
υ: variance rate of gamma time change which controls kurtosis
θ: drift rate in Brownian motion which controls skewness

The asset price dynamics followed by VG process under the risk-neutral process is given by

$$S(t) = S(0). \exp\,(rt + X(t; \sigma, v, \theta) + \omega t$$

Where $\omega = \frac{1}{v}(1 - \theta v - 0.5\sigma^2 v)$

The log returns of the asset can be modeled as below:

$$R(T) = \log S(t) - \log S(0) = rt + X(t; \sigma, v, \theta) + \omega t = ct + X(t; \sigma, v, \theta)$$

where $c = r + \omega$, σ is the volatility and r is the risk-free interest rate

The call option price with strike price K can be calculated by the integral as defined as follows:

$$C = \exp\,(-rt) \int_0^{\infty} \int_k^{\infty} (y - K)\, f_y(y; m(x), s(x)) g_x(x; \frac{T}{v}, V) dydx$$

Where $f_y(y; m(x), s(x))$ represents the probability density function of log normal distribution and $g_x(x; \frac{T}{v}, V)$ represents probability density function of Gamma distribution.

The pricing of put option can be obtained by using put-call parity as below:

$$P = C - \exp(-rt)\,.\,[K - E(S(T))$$

Where,

$$E\big(S(T)\big) = \int_0^{\infty} E(E(S(T)|G(T) = x).\,(g_x(x;\frac{T}{v}, V))$$

4. DATA ANALYSIS

We have computed fair values of option by using Black-Scholes model and Variance Gamma Model.

Company Parameters	TECHM	ACC	TCS
Current price (s) in Rs.	794.50	1535.10	2136.10
Stock price (K) in Rs.	790	1520	2120
Time to expiration (T) in year	0.1071	0.1071	0.1071
Interest rate (r) in %	10	10	10
Implied volatility (σ) in %	32.73	30.45	25.24
Call premium (c) in Rs.	28.10	65.45	66.55
Put premium (P) in Rs.	24	40	41.60
Put premium from BS model	27.59	46.02	51.44
Put premium from VG model	23.41	38.25	

5. CONCLUSION

Based on our analysis in solving a levy process by quadrature method we can conclude that in a different interval we get the different values of European call option price. In symmetric intervals we get the same values of the option. We derive the put option formula by using put-call parity. The derived put option price formula follows the Variance Gamma process which is relevant to the actual market. We numerically calculated stock options listed in NSE to check the performance of the model. We compare our proposed model with classical Black-Scholes model numerically.

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CONFLICT OF INTEREST

The author have declared that no competing interests exist.

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