A STUDY OF OPTION PRICING MODELS WITH DISTINCT INTEREST RATE

Neha Sisodia 1, Ravi Gor 2

1 Research Scholar, Department of Mathematics, Gujarat University, Navrangpura, Ahmedabad-380009 (Gujarat), India
2 Associate Professor, Department of Mathematics, Gujarat University, Navrangpura, Ahmedabad-380009, (Gujarat), India

ABSTRACT

This paper analyses the effect of different interest rates on two Option Pricing Models, Black-Scholes', and Heston. Here, the parameter interest rate is focused, and a comparison is done amongst the two models. An error estimator, UMBRAE (Unscaled Mean Bounded Relative Absolute Error) is calculated for pricing various European call options. The real market data is collected from NSE (National Stock Exchange). Moneyness (percentage difference of stock price and strike price) and Time-To-Maturity are used as the base for comparison. All the mathematical calculation is done in MATLAB software. We observe that Black-Scholes' model is preferred for lower interest rates than Heston options pricing model and vice-versa. This study is helpful in derivatives market.

Keywords: European Call Option, Black-Scholes’ Model, Heston Model, Moneyness, Time-To-Maturity, Interest Rate

1. INTRODUCTION

Financial markets are one of the most demanding and considered to be quick source of income nowadays. It is the place where various securities are traded like – stock market, bond market, derivatives market etc. The derivative market is the market for financial instruments like – futures contracts or options. The value of financial contract depends on an underlying asset. Prices for derivatives derive from fluctuations in the underlying asset. Options are a type of derivative product that allows investors to contemplate against the volatility of an underlying stock. Options are of two types, Call and Put option. Call option - the option holder purchases an asset at a specified price on or before a specified time. Put option is vice versa of call option. In this, option holders sell an asset at a specified price on or before a specified time. It is again divided among two styles European and American. European options are traded in both NSE and BSE in Indian stock market. They can be exercised only at the time of expiration. Black-Scholes’ model is a well-known basic option pricing model for determining the theoretical premium value for a call or a put option with six parameters such as, volatility, underlying stock price, type of option, strike price, time, and risk-free interest rate.


Interest rate is one of the important parameters in market fluctuations. It is the amount a lender charges a borrower and is a percentage of the principal - the amount loaned. The risk-free rate is used in both mathematical models Black-Scholes’ and Heston option pricing model. It is the zero risk investments, theoretical rate of return. However, a truly risk-free rate does not exist because even the safest investments carry a very small risk.


In this paper, initially we discuss the basic terminologies, Black-Scholes’ model, Heston Model and then Methodology. At last, the result is discussed regarding comparison of the two models for real market Indian stock data.

### 1.1. LITERATURE REVIEW

A Study of Option Pricing Models with Distinct Interest Rate


It has been seen that modified B-S model gives better output than the original B-S model. In continuation to this, a stochastic volatility model was developed by Heston in 1993. He proposed a stochastic volatility model which gives a closed form solution for calculating theoretical premium value of European Call options. It assumes underlying stock price and volatility as stochastic quantity. The stochastic volatility models help in removing excess of skewness and kurtosis and asset variance follow a mean reverting CIR process. Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Finance (IOSR-JEF), 12(1), 19-26..

and Heston option pricing formula on the basis of different parameters of moneyness and Time-To-Maturity.

In the paper of Santra A. and Chakrabarti, B. (2017). Comparison of Black-Scholes and Heston Models for Pricing Index Options, Indian Institute of Management, 796,2-6., Matlab software is introduced for all kind of mathematical calculation. An accuracy measure UMBRAE is used as provided in the paper of Chao C, Jamie T. and Jonathan M. (2017). A new accuracy measure based on bounded relative error for time series forecasting, Tianjin University, China, 12(3), 6-7. The two models are compared for real market data with this error estimator.


1. **Option:** An option is defined as the right, but not the obligation, to buy (call option) or sell (put option) a specific asset by paying a strike price on or before a specific date.
   - *Call option:* An option which allows its holder the right to buy the underlying asset at a strike price at some particular time in the future.
   - *Put option:* An option which allows its holder the right to sell the underlying asset at a strike price at some particular time in the future.

2. **Stochastic Process:** Any variable whose value changes over time in an uncertain way is said to follow a stochastic process.

3. **Strike Price:** The predetermined price of an underlying asset is called strike price.

4. **Stochastic Volatility:** Volatility is a measure for variation of price of a stock over time. Stochastic in this sense refers to successive values of a random variable that are not independent.

5. **Expiration Date/ Time-to-maturity:** The date on which an option right expires and becomes worthless if not exercised. In European options, an option cannot be exercised until the expiration date.

6. **Moneyness:** It is the relative position of the current price of an underlying asset with respect to the strike price of a derivative, most commonly a call/put option.

7. **Geometric Brownian Motion:** A continuous time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion.

8. **Black-Scholes Inputs:**
   Black-Scholes model uses following six parameters in option pricing model.
   - Underlying stock price
   - Interest rate
   - Time to expiration
   - Strike price
   - Volatility
Risk-Free Interest Rate is measured in percentage per year. In a particular trade it is the rate at which cash over the life of the option is deposited or borrowed. Call option value increases with the risk-free rate. Put option value decreases as the risk-free rate increases. Its sensitivity with option price is termed as RHO.

MIBOR (Mumbai interbank offer rate) is the rate at which a bank lends loan to another bank on short-term. For a continuous developed market of India, a reference rate is all required for its debt market, which evolved MIBOR. It is used in conjunction with forward rates and the Mumbai interbank bid (MIFOR and MIBID). NSE (2022).


This model has number of assumptions.
- Random walk.
- Interest rate remains constant.
- Stock pays no dividends.
- No transaction cost.
- Option can only be exercised upon expiration.
- Stock returns are normally distributed; thus, the volatility remain constant throughout.


\[ dS_t = \mu S_t dt + \sigma_t S_t dW_t \]

\[ S_t \] - asset price,
\[ \mu \] - drift (that is constant),
\[ \sigma_t \] - return volatility(constant) and
\[ W_t \] - Brownian motion.


The risk neutral dynamics on asset is given by.

\[ dS_t = rS_t dt + \sigma_t S_t dW_t \]

Here, \( r \) is the risk-free rate and Geometric Brownian Motion is the solution to the above stochastic differential equation.

\[ S_t = S_0 \exp \left[ \sigma W_t + \left( \mu - \frac{\sigma^2}{2} \right) t \right] \]
Geometric Brownian Motion (GBM) model is the lognormal of the above equation for stock prices.

\[
\ln\left(\frac{S_t}{S_0}\right) = \sigma W_t + \left(\mu - \frac{\sigma^2}{2}\right) t
\]

Here, R.H.S. equation is a normal random variable whose mean is \(\left(\mu - \frac{\sigma^2}{2}\right) t\) and variance is \(\sigma^2 t\).

The Black-Scholes Formula for European call price is,

\[
C = S_0 N(d_1) - Ke^{-rt} N(d_2)
\]

Where, \(d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)t}{\sigma \sqrt{t}}\) and \(d_2 = d_1 - \sigma \sqrt{t}\)

\(K\) – strike price, \(S_0\) – current stock price, \(t\) – time to expiration, \(r\) – riskless interest rate (constant), \(\sigma\) – volatility of stock (constant). Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS Management Review, 31-42.


In Heston S.L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, The Review of Financial Studies, 6(2), 327-343. developed a Stochastic Volatility option pricing Model. Consider at time \(t\) the underlying asset \(S_t\) which obeys a diffusion process with volatility being treated as a latent stochastic process of Feller as proposed by Cox, Ingersoll, and Ross (CIR): Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS Management Review, 31-42.

\[
dS_t = rS_t \, dt + \sqrt{V_t} S_t \, dW^1_t
\]

\[
dV_t = k[\theta - V_t] \, dt + \sigma \sqrt{V_t} \, dW^2_t
\]

Here, \(dW^1_t\) and \(dW^2_t\) are two separate Brownian motion which are correlated with a correlation coefficient \(\rho > 0\):

\[
dW^1_t \, dW^2_t = \rho \, dt\]
Here, $S_t$ - asset price, $r$ - risk free rate, $V_t$ - variance at time $t$, $\theta > 0$ is the long term mean variance, $k > 0$ is variance mean-reversion speed, $\sigma \geq 0$ is the volatility of the variance.

European call option price is given by:

$$C = S_0 \Pi_1 - e^{-rT} K \Pi_2$$

Here, $\Pi_1$ - delta of the option and $\Pi_2$ - risk-neutral probability of exercise (i.e., when $S_t > K$)

Heston characteristic function for $j=1, 2$ is given as:

$$f_j(x,v,T;\emptyset) = e^{C(T;\emptyset) + D(T;\emptyset)v + i\emptyset x}$$

Were,

$$C(T;\emptyset) = r\emptyset iT + \frac{a}{\sigma^2} \left\{ (b_j - \rho \sigma \emptyset i + d)T - 2\ln \left[ \frac{1 - e^{dT}}{1 - g} \right] \right\}$$

$$D(T;\emptyset) = \frac{b_j - \rho \sigma \emptyset i + d}{\sigma^2} \left[ 1 - e^{dT} \right] \left[ 1 - g e^{dT} \right]$$

$$g = \frac{b_j - \rho \sigma \emptyset i + d}{b_j - \rho \sigma \emptyset i - d}$$

$$d = \sqrt{(\rho \sigma \emptyset i - b_j)^2 - \sigma^2 (2u_j \emptyset i - \emptyset^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = k\theta, b_1 = k - \rho \sigma, b_2 = k$$

The required probabilities can be obtained by inverting the characteristic functions:

$$\Pi_j(x,v,T;\ln[K]) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \frac{e^{-i\emptyset \ln[K]} f_j(x,v,T;\emptyset)}{i\emptyset} d\emptyset$$

2. MATERIALS AND METHODS

Data: The live market data has been gathered from NSE India website. Five different companies are considered randomly for calculation of European call option.

Aurobindo Pharma Limited, Biocon Limited, Cipla Limited, Zydus Cadila Healthcare Limited and Glenmark are considered for the period from January 01 to January 31, 2021. February 25, 2021 is considered as the maturity date.

Moneyness - Percentage difference between the current underlying price and the strike price:

\[ \text{Moneyness} (\%) = \frac{S}{K} + 1 \]

The result has been bifurcated in terms of moneyness option and maturity time.

- **ITM (In the money)** - In call options if the strike price is lower than the underlying stock price.
- **ATM (At the money)** - In call options when the strike price is similar to the underlying stock price.
- **OTM (Out of the money)** - In call options if the strike price is more than the underlying stock price.
- **An error estimator UMBRAE (Unscaled Mean Bounded Relative Absolute Error)**:

\[ \text{UMBRAE} = \frac{1}{1 - \text{UMBRAE}} \]

\[ \text{MBRAE} = \frac{1}{n} \sum_{t=1}^{n} (\text{BRAE}) \]

\[ \text{BRAE} = \frac{|r_t|}{|r_t| + |r^*_t|} \]

\[ r_t = y_t - f_t \]

\[ r^*_t = y_t - f^*_t \]

Where, \( y_t \) is observed price of the model, \( f_t \) is the actual forecasted value of the market and, \( f^*_t \) is the forecasted value of the market from Naive Method.

- We have used MATLAB software to run the Black-Scholes model for calculation of European call option value.
- Risk-Free Interest rate: During the life of an option, the amount of money lends or borrowed at the particular rate is called Risk-Free Interest rate. Value of call option increases with increase in rate.
- Volatility: It is defined as the standard deviation of the continuously compounded return of the stock. Value of call option is high for the higher volatility.
- We have used MATLAB software to run the Heston model for calculation of European call option value.
- Initial Variance bounds of 0 and 1 have been considered.
- Long-term Variance bounds of 0 and 1 have been considered.
- Correlation: Correlation takes values from -1 to 1 between the stochastic processes.
Volatility of Variance: It gives positive value. As the volatility of assets may increase in short term, a broad range of 0 to 5 has been considered.

Mean-Reversion Speed: This is dynamically set with the help of a non-negative constraint (Feller, 1951). The constraint \(2k\theta - \sigma^2 > 0\) guarantees that the variance in CIR process is always strictly positive.

Initial Variance = 0.28087

Long-term Variance = 0.001001

Volatility of Variance = 0.1

Correlation Coefficient = 0.5

Mean Reversion Speed = 2.931465

Interest rate is the supreme parameter in the overall study of the paper. To observe the effect of change of interest rate in option pricing models, two different rates have been considered, \(R_1 = 10\%\) (arbitrary) and \(R_2 = 3.47\%\) (MIBOR- Mumbai Interbank offer Rate)

3. RESULTS AND DISCUSSIONS

Randomly five different stocks have been chosen and its theoretical premium value of European call option is computed for B-S model and Heston model Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS Management Review, 31-42. Two different interest rates i.e., \(R_1\) and \(R_2\) have considered for different option of moneyness i.e. In-the-money (ITM), Out-of-the-money (OTM) and At-the-money (ATM). Error estimation UMBRAE is evaluated and compared for both the models.
A Study of Option Pricing Models with Distinct Interest Rate

International Journal of Engineering Science Technologies

100
A Study of Option Pricing Models with Distinct Interest Rate

Above graph’s shows that the model price’s at R₁ are higher than at R₂. This indicates that higher the interest rate so is the premium value. Also, it is seen that Black-Scholes’ model gives closer result at MIBOR i.e., R₂ as compared to Heston model.

Further, UMBRAE (Unscaled Mean Bounded Relative Absolute Error) is computed, and comparison is done for various option of moneyness. Interest Rate is considered as the principal parameter in both Black-Scholes’ and Heston Option pricing model. Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Finance (IOSR-JEF), 12(1), 19-26.

Table 1: Aurobindo Pharma Limited

<table>
<thead>
<tr>
<th>Model</th>
<th>Error estimation UMBRAE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Call Option at R₁</td>
</tr>
<tr>
<td></td>
<td>ITM K=800</td>
</tr>
<tr>
<td>Black-Scholes</td>
<td>0.7</td>
</tr>
<tr>
<td>Heston</td>
<td>0.41</td>
</tr>
</tbody>
</table>

According to the above table, at R₁ Heston model outperforms Black-Scholes’ model at ITM while other shows maximum error value 1. At R₂ Black-Schole model outcompete Heston model at ITM and ATM options while OTM options shows no difference.
According to the above table, at $R_1$ Heston model outcompete Black-Schole model at ITM and ATM options. At $R_2$ Black-Scholes model outcompete Heston for all the three moneyness option.

According to the above table, at $R_1$ Heston model outcompete Black-Schole model at ITM and ATM options. At $R_2$ Black-Scholes model is better than Heston model at ITM while vice-versa in ATM option.

According to the above table, at $R_1$ Heston model is better than Black-Scholes’ model in ITM and ATM options and vice-versa in OTM option. At $R_2$ Black-Scholes’ model outcompete Heston model in all moneyness option.

According to the above table, at $R_1$ Heston model outperforms Black-Scholes’ model in ITM option and vice-versa at $R_2$. Rest all other shows maximum error value 1.
4. CONCLUSIONS AND RECOMMENDATIONS

We conclude the following from the above study.

R₁ = 10% (Arbitrary):
- Heston model is far better than Black-Scholes’ model at In-the-money and At-the-money option for all cases.
- Black-Scholes’ model is better than Heston only in Cipla limited while; others have similar error value maximum 1 at Cut-off-the-money option.

R₂ = 3.47 (MIBOR):
- Black-Scholes’ model proves to be better than Heston model in all except Zydus Cedilla limited at At-the-money option.
- Black-Scholes’ model performs better than Heston model in all chosen companies’ data at In-the-money option.
- Black-Scholes’ outcompete Heston model in the case of Biocon and Cipla limited. Remaining all gives similar error value indicating no such difference at Out-of-the-money option.

We have considered one-month different stock data from five different companies for two different interest rates R₁ and R₂. We have computed the error estimation for both the Black-Scholes’ model and Heston model for various option of moneyness i.e. In-the-money (ITM), out-of-the-money (OTM) and At-the-money (ATM) Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Finance (IOSR-JEF), 12(1), 19-26.. The comparison is then done in both the models. At last, we could conclude that, the Black-Scholes’ model is more reliable for interest rate at MIBOR while, Heston model must be used for other higher rates. Because as the rate increases the higher the premium value we get. Thus, the Black-Scholes’ model fails with the increasing interest rates. This kind of study is always useful to derivative market investors in both short term and long-term options [18]. In future, we could work on the large number of data, compute many more results, and forecast much accurately.

ACKNOWLEDGEMENT

This paper and research are possible with the exceptional support of my guide Dr. Ravi Gor. His knowledge and keen attention in the subject have been a positive inspiration for my work.

REFERENCES


Executive Post Graduate Programme in Machine Learning & AI. (N.d.). Website -


Indian Institute of Management Calcutta (n.d.). Website -


NSE (2022).


