



# STABILITY ANALYSIS OF THE MAIN SPINDLE VIBRATIONS IN TRANSVERSAL DIRECTION AT CNC LATHE



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## ABSTRACT

The paper presents an original method for stability analysis of the main spindle variations in transversal direction at CNC lathe, which by means of the Mihailov criterion and adequate software packages, establishes the dynamic instability character due to the specific processing conditions and to the grinding regime parameters. This creates the premises for rational design of the turning process, as well as for the structural design of the turning tool.

## 1. INTRODUCTION

Increased demands on the quality of finished industrial products due to increasing market competition has led to design and usage of high productivity modern machines and equipment like CNC machines, processing centres but also integrated systems of CNC machines such as flexible processing cells and flexible processing systems [1], [2], [3].

For this reason, studying the vibrations and analyzing ways of reducing them plays an important role in designing high performance equipment [4].

Ensuring a low level of vibrations creates the premises for achieving a sufficient stability degree for high precision processing, with superior surface quality and increased productivity [5], [6].

All these eventually add up into complex products and parts with a high degree of automation and mechanization.

**2. ESTABLISHING THE METHOD FOR ANALYZING STABILITY OF THE MAIN SPINDLE IN CASE OF TRANSVERSAL VIBRATIONS AL CNC LATHE**

For this purpose, it is necessary to determine the equations that describe the vibration movement of the main spindle at CNC lathe. These are in fact a system of 1<sup>st</sup> degree differential equations.

For analysis of the vibrations in transversal direction we take into account the solutions of the system of first order differential equations that generate the vibration movement of the main spindle [7]:

$$\begin{aligned}
 \bar{v}_3^i &= e_{11} a_1^i + e_{12} a_2^i + e_{13} a_3^i + e_{14} a_4^i \\
 \bar{v}_{3,1}^i &= e_{21} a_1^i + e_{22} a_2^i + e_{23} a_3^i + e_{24} a_4^i \\
 \bar{v}_{3,11}^i &= e_{31} a_1^i + e_{32} a_2^i + e_{33} a_3^i + e_{34} a_4^i \\
 \bar{v}_{3,111}^i &= e_{41} a_1^i + e_{42} a_2^i + e_{43} a_3^i + e_{44} a_4^i
 \end{aligned}
 \tag{1}$$

The integration constants are given by:

$$\begin{aligned}
 a_1^0 &= \frac{n_1}{n}; a_2^0 = \frac{n_2}{n}; a_3^0 = \frac{n_3}{n}; a_4^0 = \frac{n_4}{n} \\
 a_1^1 &= \frac{n_5}{n}; a_2^1 = \frac{n_6}{n}; a_3^1 = \frac{n_7}{n}; a_4^1 = \frac{n_8}{n} \\
 a_1^2 &= \frac{n_9}{n}; a_2^2 = \frac{n_{10}}{n}; a_3^2 = \frac{n_{11}}{n}; a_4^2 = \frac{n_{12}}{n}
 \end{aligned}
 \tag{2}$$

In this case, we can study the system stability by analyzing the expression at the denominator of integration constants  $a_1^i, a_2^i, a_3^i, a_4^i, i \in \{0,1,2\}$ . The determinant n is given by:

|   |   |   |   |  |  |  |  |                           |                             |                             |                             |
|---|---|---|---|--|--|--|--|---------------------------|-----------------------------|-----------------------------|-----------------------------|
| $Eie_{41}(0)+$<br>$+\rho I\Omega^2 e_{21}(0)$ | $Eie_{42}(0)+$<br>$+\rho I\Omega^2 e_{22}(0)$ | $Eie_{43}(0)+$<br>$+\rho I\Omega^2 e_{23}(0)$ | $Eie_{44}(0)+$<br>$+\rho I\Omega^2 e_{24}(0)$ | 0  | 0  | 0  | 0  | 0                         | 0                           | 0                           | 0                           |
| $e_{31}(0)$                                   | $e_{32}(0)$                                   | $e_{33}(0)$                                   | $e_{34}(0)$                                   | 0  | 0  | 0  | 0  | 0                         | 0                           | 0                           | 0                           |
| $k_{33}e_{11}(l_1)-$<br>$-Eie_{41}(l_1)$      | $k_{33}e_{12}(l_1)-$<br>$-Eie_{42}(l_1)$      | $k_{33}e_{13}(l_1)-$<br>$-Eie_{43}(l_1)$      | $k_{33}e_{14}(l_1)-$<br>$-Eie_{44}(l_1)$      | $Eie_{41}(l_1)$                          | $Eie_{42}(l_1)$                          | $Eie_{43}(l_1)$                          | $Eie_{44}(l_1)$                          | 0                         | 0                           | 0                           | 0                           |
| $k_{54}e_{21}(l_1)-$<br>$-Eie_{31}(l_1)$      | $k_{54}e_{22}(l_1)-$<br>$-Eie_{32}(l_1)$      | $k_{54}e_{23}(l_1)-$<br>$-Eie_{33}(l_1)$      | $k_{54}e_{24}(l_1)-$<br>$-Eie_{34}(l_1)$      | $Eie_{31}(l_1)$                          | $Eie_{32}(l_1)$                          | $Eie_{33}(l_1)$                          | $Eie_{34}(l_1)$                          | 0                         | 0                           | 0                           | 0                           |
| $-e_{11}(l_1)$                                | $-e_{12}(l_1)$                                | $-e_{13}(l_1)$                                | $-e_{14}(l_1)$                                | $e_{11}(l_1)$                            | $e_{12}(l_1)$                            | $e_{13}(l_1)$                            | $e_{14}(l_1)$                            | 0                         | 0                           | 0                           | 0                           |
| $-e_{21}(l_1)$                                | $-e_{22}(l_1)$                                | $-e_{23}(l_1)$                                | $-e_{24}(l_1)$                                | $e_{21}(l_1)$                            | $e_{22}(l_1)$                            | $e_{23}(l_1)$                            | $e_{24}(l_1)$                            | 0                         | 0                           | 0                           | 0                           |
| 0   | 0   | 0   | 0   | $k_{33}e_{11}(l_2)-$<br>$-Eie_{41}(l_2)$ | $k_{33}e_{12}(l_2)-$<br>$-Eie_{42}(l_2)$ | $k_{33}e_{13}(l_2)-$<br>$-Eie_{43}(l_2)$ | $k_{33}e_{14}(l_2)-$<br>$-Eie_{44}(l_2)$ | $Eie_{41}(l_2)$           | $Eie_{42}(l_2)$             | $Eie_{43}(l_2)$             | $Eie_{44}(l_2)$             |
| 0   | 0   | 0   | 0   | $k_{54}e_{21}(l_2)-$<br>$-Eie_{31}(l_2)$ | $k_{54}e_{22}(l_2)-$<br>$-Eie_{32}(l_2)$ | $k_{54}e_{23}(l_2)-$<br>$-Eie_{33}(l_2)$ | $k_{54}e_{24}(l_2)-$<br>$-Eie_{34}(l_2)$ | $Eie_{31}(l_2)$           | $Eie_{32}(l_2)$             | $Eie_{33}(l_2)$             | $Eie_{34}(l_2)$             |
| 0   | 0   | 0   | 0   | $-e_{11}(l_2)$                           | $-e_{12}(l_2)$                           | $-e_{13}(l_2)$                           | $-e_{14}(l_2)$                           | $e_{11}(l_2)$             | $e_{12}(l_2)$               | $e_{13}(l_2)$               | $e_{14}(l_2)$               |
| 0   | 0   | 0   | 0   | $-e_{21}(l_2)$                           | $-e_{22}(l_2)$                           | $-e_{23}(l_2)$                           | $-e_{24}(l_2)$                           | $e_{21}(l_2)$             | $e_{22}(l_2)$               | $e_{23}(l_2)$               | $e_{24}(l_2)$               |
| 0   | 0   | 0   | 0   | 0  | $\rho I\Omega^2 e_{21}(l)$               | 0  | 0  | $+m_p \Omega^2 e_{11}(l)$ | $+\rho I\Omega^2 e_{22}(l)$ | $+\rho I\Omega^2 e_{23}(l)$ | $+\rho I\Omega^2 e_{24}(l)$ |
| 0   | 0   | 0   | 0   | 0  | 0  | 0  | 0  | $+m_p \Omega^2 e_{12}(l)$ | $+m_p \Omega^2 e_{13}(l)$   | $+m_p \Omega^2 e_{14}(l)$   |                             |
| 0   | 0   | 0   | 0   | 0  | 0  | 0  | 0  | $Eie_{31}(l)-$            | $Eie_{32}(l)-$              | $Eie_{33}(l)-$              | $Eie_{34}(l)-$              |
| 0   | 0   | 0   | 0   | 0  | 0  | 0  | 0  | $-J_0 \Omega^2 e_{11}(l)$ | $-J_0 \Omega^2 e_{12}(l)$   | $-J_0 \Omega^2 e_{13}(l)$   | $-J_0 \Omega^2 e_{14}(l)$   |

(3)

Computing this determinant gives an expression of the form [8]:

$$\begin{aligned}
 P(s) = & (0.206 \cdot \Omega^4 + 264.9 \cdot \Omega^2 + 16.0214 \cdot 10^{-6}) \cdot s^6 + (-35.11 \cdot \Omega^6 - 3672 \cdot 10^9 \cdot \Omega^4 + 41.08 \cdot 10^{28} \cdot \Omega^2 + \\
 & + 2534.609) \cdot s^4 + 6.98 \cdot 10^{-6} \cdot \Omega \cdot s^2 + (-37.82 \cdot 10^{-14} \cdot \Omega^8 + 2.39 \cdot 10^9 \cdot \Omega^6 + 245.27 \cdot 10^{-7} \cdot \Omega^5 + 3.35 \cdot \\
 & \cdot 10^{15} \cdot \Omega^4 - 57.61 \cdot \Omega^3 + 456 \cdot 10^{15} \cdot \Omega^2 + 182.6 \cdot 10^{15} \cdot \Omega + 1.08 \cdot 10^{16}) \cdot s^2 + 107.24 \cdot 10^{-5} \cdot \Omega s + \\
 & + (6.86 \cdot 10^{-6} \cdot \Omega^8 + 3.19 \cdot 10^{13} \cdot \Omega^6 + 8.3 \cdot 10^8 \cdot \Omega^5 + 37.82 \cdot 10^{-14} \cdot \Omega^2 + 1.27 \cdot 10^{22} \cdot \Omega^4 + \\
 & + 1.76 \cdot 10^{24} \cdot \Omega^2 - 4.57 \cdot 10^{14})
 \end{aligned}
 \tag{4}$$

Where:

- computations were made under the assumptions  $\sin \alpha \approx \alpha$ ,  $\cos \alpha \approx 1$  for small domains of  $\Omega$ ;
- $\Omega$  represents the angular rotation speed of the spindle.

Using special MATLAB programs, there were established the roots of the polynomial  $P(s) = 0$  [3]. These have the form:

$s_{1,2} = \pm j\omega_1$ , which represents a continuous sinusoidal component;

$s_{3,4} = \sigma_2 \pm j\omega_2$ , which represents a (stable) damping component;

$s_{5,6} = \sigma_3 \pm j\omega_2$ , a component which amplifies the instability character of the movement.

The solutions obtained for  $P(s) = 0$  are:

$$\begin{aligned}
 s_{1,2} &= \pm 7.0596 \cdot 10^{11} j \\
 s_{3,4} &= -42 \cdot 10^{-2} \pm 42 \cdot 10^{-2} j \\
 s_{5,6} &= 42 \cdot 10^{-2} \pm 42 \cdot 10^{-2} j
 \end{aligned}
 \tag{5}$$

Based on Mihailov's stability criterion (Fig. 1), given the maximal degree of  $P(s)$ , in order for the system to be stable  $P(j\omega)$  must cover all  $u = 6$  planes, corresponding to the highest power exponent.

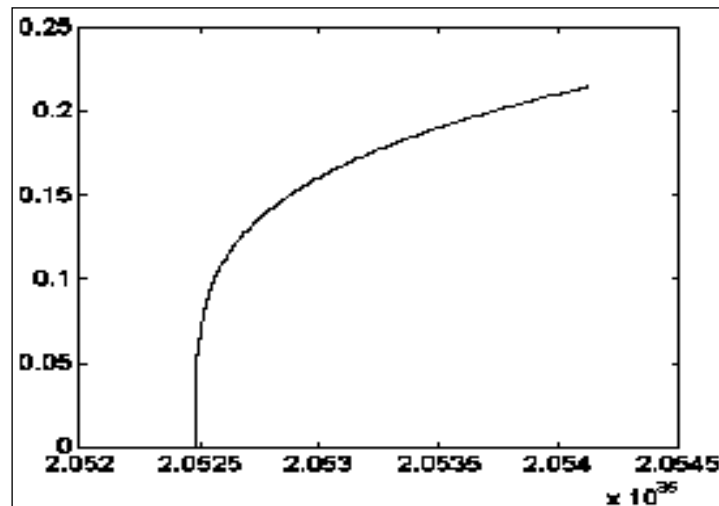


Figure 1

In our case, it can be observed that quadrant (1) is only partially covered, which outlines the instability character of the internal grinding process in this direction, a fact that was shown also in specialized literature.

The solutions of the vibration movement can be written under the general form [9]:

$$v_3(t, x_1) = A(t) + A'(t) \cdot x_1 \tag{6}$$

where:  $A(t) = a_1 \sin(\omega_1 t + \phi_1) + a_2 e^{-\sigma_2 t} \sin(\omega_2 t + \phi_2) + a_3 e^{\sigma_3 t} \sin(\omega_3 t + \phi_3)$

and  $A'(t) = a'_1 \sin(\omega_1 t + \phi'_1) + a'_2 e^{-\sigma_2 t} \sin(\omega_2 t + \phi'_2) + a'_3 e^{\sigma_3 t} \sin(\omega_3 t + \phi'_3)$

with  $\sigma_2 = \sigma_3$  and  $\omega_2 = \omega_3$  (7)

Based on these results, it was realized a simulation for the three components of the vibration movements denoted by  $C_1(t)$ ,  $C_2(t)$  and  $C_3(t)$ , in which:

$$\omega_1 = 7 \cdot 10^{11} \text{ sec}^{-1}$$

- according to Fig. 2 (a, b, c), we have the real case:

$$\omega_2 = \omega_3 = 0.42 \text{ sec}^{-1}$$

$$\sigma_2 = \sigma_3 = 0.42$$

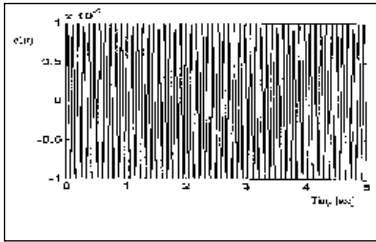


Figure 2a

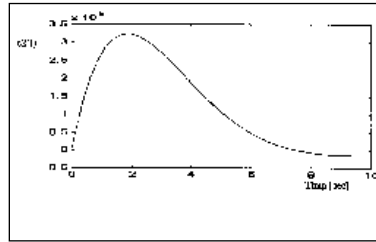


Figure 2b

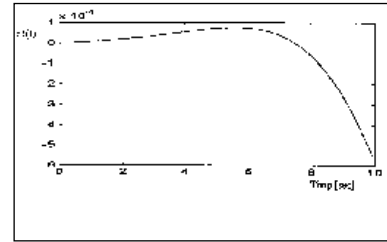


Figure 2c

$$\omega_1 = 7 \text{ sec}^{-1}$$

- according to Fig. 3 (a, b, c), we have the real case:

$$\omega_2 = \omega_3 = 4.2 \text{ sec}^{-1}$$

$$\sigma_2 = \sigma_3 = 0.42$$

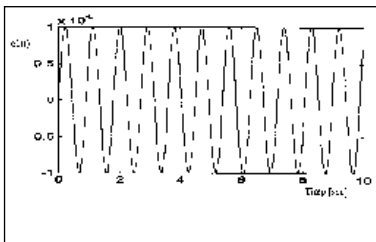


Figure 3a

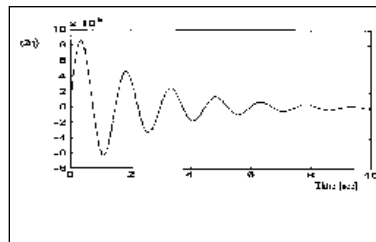


Figure 3b

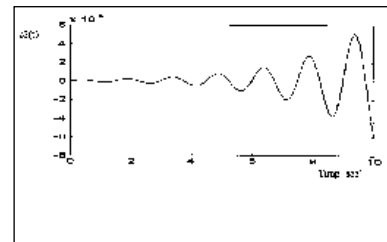


Figure 3c

$$\omega_1 = 70 \text{ sec}^{-1}$$

- according to Fig. 4 (a, b, c), we have the real case:

$$\omega_2 = \omega_3 = 42 \text{ sec}^{-1}$$

$$\sigma_2 = \sigma_3 = 0.42$$

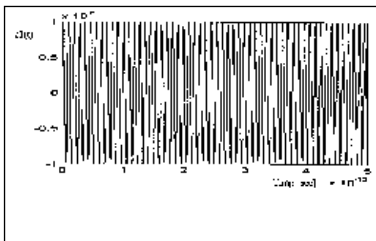


Figure 4a

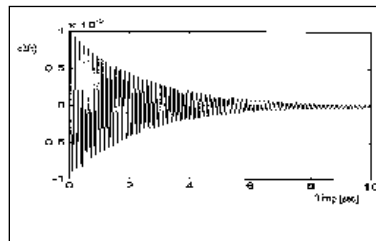


Figure 4b

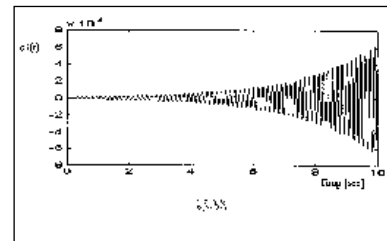


Figure 4c

The analysis of the main spindle vibrations in transversal direction at internal grinding shows the specific characteristic of this grinding process, that is the dynamic instability which determines an increase of the amplitude level in a short period, due to the loading of the abrasive tool, which leads to a deterioration in the surface quality and consequently to processing stop.

### 3. CONCLUSIONS

- 1) Stability analysis of the spindle in transversal direction is performed based on the 1<sup>st</sup> degree equation system that describe the transversal vibration movement.
- 2) Using Mihailov's stability criterion emphasizes the partial instability characteristic of the process in this direction, a fact also emphasized in other works from specialty literature.
- 3) Finally, there is presented a simulation of the three components of the vibration movement  $C_1(t)$ ,  $C_2(t)$ ,  $C_3(t)$  for representative values of the frequency.

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### CONFLICT OF INTEREST

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