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Original Article

AN INVENTORY MODEL ESTABLISHING ECONOMIC SUSTAINABILITY FOR DEVELOPING EFFICIENT MARKDOWN POLICIES WITH PRODUCT DETERIORATION CONSIDERATIONS

Archit Jain [1*], **Lakhan Singh** [2]

[1] Scholar, Digamber Jain College, Baraut, Uttar Pradesh, India

[2] Professor and Supervisor, Digamber Jain College, Baraut, Uttar Pradesh, India

ABSTRACT

This study develops a sustainable deteriorating inventory model incorporating multi-variable demand, time-dependent holding cost, inflation, and an optimal markdown policy to maximize annual profit. In modern retail systems, deterioration of products and inflation significantly affect inventory decisions, requiring dynamic pricing and cost-sensitive strategies. The proposed model assumes that demand is a joint function of selling price and on-hand inventory, making it more realistic than constant-demand models. Holding cost is taken as a time-varying function to reflect real operational conditions, while deterioration is also considered to be time-dependent with salvage value for deteriorated items. A single markdown is introduced during the selling cycle to stimulate demand and reduce leftover stock. The analytical framework formulates production, holding, deterioration, and sales revenue costs to derive the annual profit function. Optimal cycle time and markdown timing are obtained using second-order optimality conditions. A numerical example illustrates the applicability of the model, and results show that properly timed markdown policies significantly improve profitability while reducing excess inventory. Sensitivity analysis is conducted to examine the impact of key parameters such as holding cost, deterioration rate, production cost, markdown rate, and inflation. Results reveal that lower holding and deterioration costs enhance profit, while inflation and higher production costs negatively affect system performance. The findings confirm that an efficient markdown policy plays a crucial role in managing deteriorating inventories under inflationary conditions. The model provides valuable decision-making support for retailers dealing with perishable goods. Extensions of this work may include partial backordering, stochastic demand, and trade credit policies.

Keywords: Inventory, Deterioration, Markdown, Inflation, Profitability, Optimization, Demand

INTRODUCTION

The deteriorating inventory model, that has multi-variable demand rate and utilizes reduction policy in boosting the profit, is being studied. Inflation will also rise in this paper; as a result of that, the purchasing power per unit of money will reduce. In this paper, we also assume that the holding costs will be time dependent. The reduction policy can also increase the accumulated amount of profit yet the shortest of the reduction cases is the case of time and price dependence. One of them is a numerical example. Lastly, sensitivity analysis has also been prepared in consideration to some important parameters.

The most important process to control is inventory control, which entails stocks, improved services and other storage space. It involves the planning of the sales and the stock-outs, the maximization of the inventory profit and the removal of the dead stock

*Corresponding Author:

Email address: Archit Jain (architjain1610@gmail.com), Lakhan Singh (drlakhansingh10@gmail.com)

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piling. There are increased challenges faced by the companies when dealing with goods that are becoming deteriorated. The definition of deterioration includes depreciation, destruction, wear and tear of the products as well as obsolescence of the products. Business school scholars have conducted gigantic research on the notion and the majority of its facets were examined. The model developed by Ghare and Schrader (1963) is a first time economic order quantity where the planning time constant rate of demand is finite and the rate of deterioration is constant. In Goyal and Giri (2001), the new tendencies of the inventory modelling growth were taken into account. The system of supply chain where items were deteriorated by reverse logistics was formed partially and congested Singh and Sharma (2018).

It is known that the classical inventory model is characterised by a fixed holding cost. As a matter of fact, material goods require and holding cost might be time dependent. The time would also feature prominently on the inventory systems and as such we do believe that holding costs are also time sensitive. Trying to create the inventory model, Jaggi (2014) created the price-sensitive demand. It factored in functions of holding expenses that were time-related. A model of the quantity of production was suggested based on the speculations by Tayal et al. (2015) in terms of variable holding cost as an outline of non-instantaneous deteriorating products. Singh and Rana have discussed EO model in which the time dependent holding cost and multi-variable demand were addressed Singh and Rana (2020). At inflation profit search, they obtained maximum profit. Singh and Rana Singh and Rana (2020) examined the demand, which is satisfied by a new and recycle product. Inflation was also introduced to them as an efficient element of new product as well as old product.

Retailers have embraced the use of markdown policies to sell (delicate) things. The good consumers buy on distorted times and price of sale. Widyadana and Wee (2007) used depreciating inventory model which involved price-sensitive demand, and price-sensitive use reduction policy to maximize the profit. Wang et al. (2016) have derived the most appropriate markdown policy that can be enforced on the perishable food that the consumer perceives based on the prices, Nagare and Utia Nagare and Dutta (2018) have taken the single-period sensitivity.

The determinants of the demand of a commodity in the competitive market are seen to be extremely numerous due to its competitive nature. The multiplex demand of a commodity is one of such factors. Multi-variable demand of commodity is significant to the demand of the product. In Omar and Zulkipli (2014), the demand was assumed to be deterministic and in a positive relationship with the display marker of the items. Singhal and Singh (2017) examined a chain of commodities supplying system which is broken in terms of time and quality and various needs in the market.

ASSUMPTIONS AND NOTATION

ASSUMPTIONS

- The demand rate is deterministic and is a function of both price and on-hand inventory level:

$$D = a(\gamma m)^t + bI(t), \text{ where } a, \gamma, \text{and } \delta > 0 \text{with } 0 < \gamma, \delta < 1$$

- The holding cost is time-dependent where:

$$h(t) = u + vt, u, v > 0$$

- The deterioration is time-dependent where 0<a<1
- There is a salvage value on the deteriorated units.
- All items are mandatory to be sold.
- Inflation and time-value of the currency are considered.
- The deduction value is applied only once in a planning period, and the reduction value is known.
- The production time is relative to the cycle time where $t_p = \eta T$..
- Reduction time varies between $T - t$, which is equivalent to $t = \lambda(T - t)$.

NOTATIONS

- $I(t)$: Inventory level at time t
- a: Deterioration rate
- p: Constant production rate
- r: Inflation rate

- γ: Reduction rate
- ϵ: Increase price rate
- 1: Production percentage
- λ: Reduction percentage
- m: Initial price
- O: Unit ordering cost
- F_c: Salvage value related to deteriorating units during cycle time where 0<β<1
- C_p: Unit production cost

MODEL FORMULATION

The production and supply start instantaneously, and the production ends at a time with the inventory level Qreached. We assume there is no deterioration during the production uptime. In the interval $(t, t+\tau)$, inventory level declines due to demand and deterioration. At the time $(t+\tau)$, a markdown is offered to grow the demand rate. The position of the inventory at any time over a period $(0, T)$is governed by the following differential equation:

Figure 1

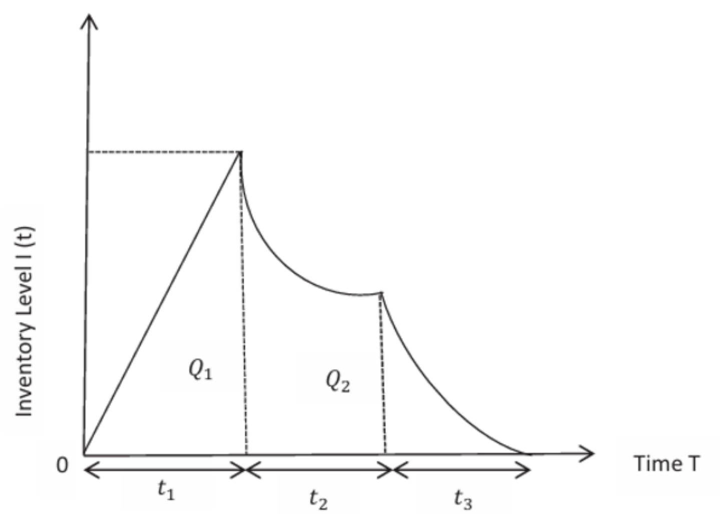

Figure 1 Graphical Representation of the System

$$\dot{I}(t) = P - (am^{-\epsilon} + \delta I(t)), \qquad 0 \le t \le t_1 \tag{1}$$

With γ=1(no markdown) and initial boundary condition $I(0) = 0$,

$$\dot{I}(t) + \alpha t I(t) = -(am^{-\epsilon} + \delta I(t)), 0 \le t \le t_2 \tag{2}$$

With γ=1(no markdown) and boundary condition $I(0) = Q_1$,

$$\dot{I}(t) + \alpha t I(t) = -(a(\gamma m)^{-\epsilon} + \delta I(t)), 0 \le t \le t_3 and 0 < \gamma < 1 \tag{3}$$

With boundary condition $I(0) = Q_2$.

Solution of the above differential equations are given by:

$$I(t) = \frac{(P-am^{-\epsilon})}{\delta}(1-e^{-\delta t})0 \le t \le t_1 \tag{4}$$

$$I(t) = \left(Q_1 - am^{-\epsilon}\left(t + \frac{\delta t^2}{2} + \frac{at^3}{6}\right)\right)e^{-\left(\delta t + \frac{at^2}{2}\right)}, 0 \le t \le t_2 \tag{5}$$

$$I(t) = \left(Q_2 - a(\gamma m)^{-\epsilon}\left(t + \frac{\delta t^2}{2} + \frac{at^3}{6}\right)\right)e^{-\left(\delta t + \frac{at^2}{2}\right)}, 0 \le t \le t_3 \tag{6}$$

Where, using the boundary condition and continuity condition, the inventory level

$$Q_1 = \frac{(P-am^{-\epsilon})}{\delta}(1-e^{-\delta t_1}) \tag{7}$$

$$Q_2 = a(\gamma m)^{-\epsilon}\left(t_3 + \frac{\delta {t_3}^2}{2} + \frac{a{t_3}^3}{6}\right) \tag{8}$$

Ordering Cost = O/T

Production Cost = $\frac{C_p t_1}{T}$

Holding cost = $\int_0^{t_1} T(t)e^{-rt}dt + \int_0^{t_2} T(t)e^{-rt}dt + \int_0^{t_3} T(t)e^{-rt}dt$

Deterioration cost = $\int_0^{t_1} atI(t)e^{-rt}dt + \int_0^{t_2} atI(t)e^{-rt}dt + \int_0^{t_3} atI(t)e^{-rt}dt$

Sales Revenue cost = $\int_0^{t_1} De^{-rt}dt + \int_0^{t_2} De^{-rt}dt + \int_0^{t_3} De^{-rt}dt$

Annual profit = Sales revenue cost – Holding cost– Ordering cost – Production cost– Deterioration cost

$$AP = \left| \begin{array}{l} \frac{m}{t}\left[\begin{array}{l} a(\gamma m)^{-\epsilon}\left(t_1 - \frac{rt_1^2}{2} + t_2 - \frac{rt_2^2}{2} + t_3 - \frac{rt_3^2}{2} + \delta\left(\frac{t_3^2}{2} - \frac{rt_3^3}{6} - \frac{at_3^4}{24}\right) + \frac{\delta a t_3^4}{8}\right) + \delta am^{-\epsilon} \\ \left(\left(t_2 - \frac{rt_2^2}{2} - \frac{at_2^4}{8}\right) + a\delta am^{-\epsilon}\left(\frac{t_2^4}{24} - \frac{rt_2^5}{30}\right) + \delta(P - am^{-\epsilon})\left(\frac{t_1^2}{2} - \frac{rt_1^3}{3} + t_1t_2 - \frac{rt_1t_2^2}{2} - \frac{at_1t_2^3}{6}\right)\right) \end{array}\right] \\ -\frac{u}{T}\left[\begin{array}{l} (P - am^{-\epsilon})\left(\frac{t_1^2}{2} - \frac{rt_1^3}{3} + t_1t_2 - \frac{rt_1t_2^2}{2} - \frac{rt_1t_2^2}{2} - \frac{at_1t_2^3}{6}\right) - am\left(\frac{t_2^2}{2} - \frac{rt_2^3}{3} - \frac{\delta t_2^3}{6} - \frac{r\delta t_2^4}{8} - \frac{at_2^4}{12}\right) \\ +a(\gamma m)^{-\epsilon}\left(\frac{t_3^2}{2} - \frac{rt_3^3}{6} - \frac{\delta t_3^3}{6} - \frac{rt_3^4}{6} - \frac{at_3^4}{4} - \frac{r\delta t_3^4}{8}\right) \end{array}\right] \\ -\frac{v}{T}\left[\begin{array}{l} (P - am^{-\epsilon})\left(\frac{t_1^3}{3} - \frac{rt_1^4}{4} + \frac{t_1t_2^2}{2} - \frac{rt_1t_2^3}{3} - \frac{\delta t_1t_2^3}{3} - \frac{at_1t_2^4}{8}\right) - am^{-\epsilon}\left(\frac{t_2^3}{3} - \frac{rt_2^4}{4} - \frac{\delta t_2^4}{8}\right) \\ +a(\gamma m)^{-\epsilon}\left(\frac{t_3^3}{6} - \frac{rt_3^4}{12} + \frac{\delta t_3^4}{24}\right) \end{array}\right] - \frac{O}{T} - \frac{PC_Pt_1}{T} \\ -\frac{C_d}{T}\left[\begin{array}{l} (P - am^{-\epsilon})\left(\frac{at_1^3}{3} - \frac{art_1^4}{4} + \frac{at_1t_2^2}{2} - \frac{art_1t_2^3}{3} - \frac{a\delta t_1t_2^3}{3}\right) - am^{-\epsilon}\left(\frac{at_2^3}{3} - \frac{rat_2^4}{4} - \frac{a\delta t_2^4}{8}\right) \\ +a(\gamma m)^{-\epsilon}\left(\frac{at_3^3}{6} - \frac{rat_3^4}{12} + \frac{a\delta t_3^4}{12} + \frac{a\delta t_3^4}{8}\right) \end{array}\right] \end{array} \right|$$

Note that Annual Profit is a function of t1, t2 and t3. We optimize the AP function by following Srivastava and Gupta (2013) procedure where we rewrite,

$$t_1 = \eta T, t_2 = \lambda(T - t_1) and t_3 = T - (t_1 + t_2) = (1 - \gamma)(1 - \eta)T$$

Srivastava and Gupta (2013) only varies T in order to find their optimal solution. However, in our case we are able to find a better solution by varying T and γ. Substitute t1, t2 and t3 into equation (9), then we obtain new equation w.r.t T and γ.

SOLUTION PROCEDURE

This sector determines the optimum values of T and λ which maximize the total profit AP(T, λ). The necessary condition for maximizing the AP are:

$$\frac{\partial AP}{\partial T} = 0 \text{and} \frac{\partial AP}{\partial \lambda} = 0$$

Also satisfied with the following conditions:

$$\frac{\partial^2 AP}{\partial T^2} < 0, \frac{\partial^2 AP}{\partial \lambda^2} < 0 \text{and} \frac{\partial^2 AP}{\partial T^2} \cdot \frac{\partial^2 AP}{\partial \lambda^2} - (\frac{\partial^2 AP}{\partial T\, \partial \lambda})^2 < 0$$

Numerical Example: In this section, a numerical example is deliberately given to illustrate the model. The following criteria are given below, which are used in the examples.

$$O = 100, m = 35, P = 200, u = 9, v = 3, a = 10000, \alpha = 0.005, \eta = 0.2, \beta = 0.15, C_d = 3.5, \gamma = 0.8, C_p = 10, \epsilon = 1.25, r = 0.075, \delta = 0.9$$

With these values, the solutions of the system were found as follows:

$$AP = 8054.67, T = 6.9183, \lambda = 0.4916, Q_1 = 114.19, Q_2 = 992.91$$

Figure 2

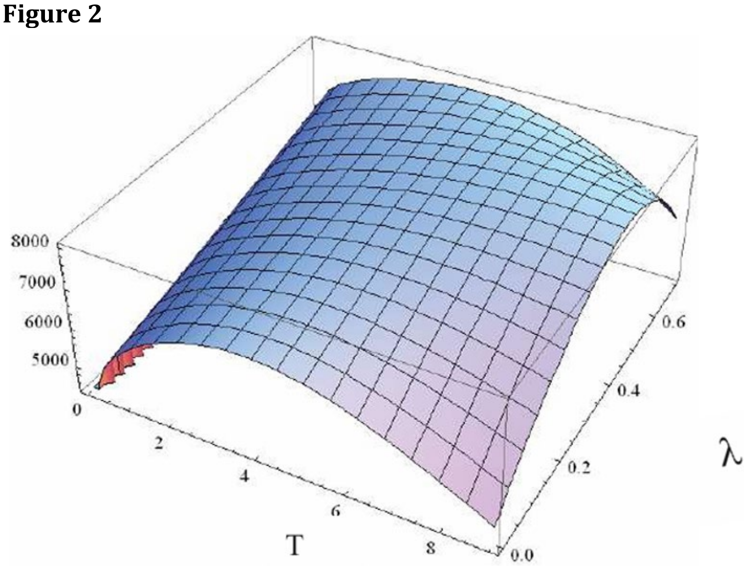

Figure 1 Total Cost Versus the T and γ .

SENSITIVITY ANALYSIS"

In order to achieve more awareness on the issue of cost, all the parameters of +30 percent to -30 percent are run in a sensitivity analysis. Table 1 shows the result. Based on the sensitivity analysis, it is possible to conclude the following insights:

Table 1

Table 1 Effect of Parameters

Parameters	Change %	T	λ	Q1	Q2	AP
V	30%	5.996	0.5503	98.973	661.3	8000.4
	20%	6.272	0.5314	103.53	752.97	8012.3
	10%	6.571	0.5122	108.46	859.7	8029.8
	-10%	7.35	0.4684	121.32	1171.8	8090
	-20%	7.96	0.439	131.39	1452.2	8142.5
	-30%	9.251	0.3938	152.7	2144	8231.7
	30%	7.907	0.4318	130.51	1737.3	9403.1
	20%	7.687	0.445	126.88	1511.6	8941.1
		7.379	0.4634	121.8	1267	8489.6
	-10%	6.041	0.5503	99.71	635.74	7649
	-20%	0.0022	3896.2	0.0363	1520.8	3.98489×10^{7}
	-30%	0.00056	-14710	0.00924	1063.4	1.4025×10^{8}
δ	30%	6.918	0.4916	114.19	992.84	7934.6
	20%	6.918	0.4916	114.19	992.84	7974.6
	10%	6.918	0.4916	114.19	992.84	8014.6
	0%	6.918	0.4916	114.19	992.84	8094.6
	-10%	6.918	0.4916	114.19	992.84	8134.6
	-20%	6.918	0.4916	114.19	992.84	8174.6
	-30%	6.912	0.492	114.09	990.28	8054.2
C_p	30%	6.914	0.4918	114.12	991.33	8054.3
	20%	6.916	0.4917	114.16	992.09	8054.5
	10%	6.92	0.4915	114.22	993.59	8054.8
	0%	6.921	0.4915	114.24	993.82	8054.9
	-10%	6.924	0.4913	114.29	995.1	8055.1
Cd	30%	6.892	0.4903	113.76	990.96	8048.5
	20%	6.9	0.4907	113.76	991.54	8050.5
	10%	6.909	0.4912	114.04	992.04	8052.6
	0%	6.927	0.4921	114.34	993.33	8056.7
	-10%	6.936	0.4926	114.49	993.82	8058.8
	-20%	6.945	0.493	114.49	994.61	8060.9
α	30%	7.224	0.4355	119.24	1254.5	7842.4
	20%	7.007	0.4583	119.24	1119.2	7906.4
	10%	6.929	0.4761	114.3	1043.2	7977.6
	0%	6.95	0.5058	114.72	956.7	8137.4

	-10%	7.013	0.519	115.76	930.31	8226.1
	-20%	7.103	0.531	117.24	910.32	8321.2
R	30%	7.224	0.4355	119.24	1254.5	7842.4
	20%	7.007	0.4583	119.24	1119.2	7906.4
	10%	6.929	0.4761	114.3	1043.2	7977.6
	-10%	6.95	0.5058	114.72	956.7	8137.4
	-20%	7.013	0.519	115.76	930.31	8226.1
	-30%	7.103	0.531	117.24	910.32	8321.2

OBSERVATIONS

- In Table 1 it is resolute on the hypothesis that it is the decrease in the parameter of the cost of holding vcan, which in truth seeks to reduce the total cost of the organism. The greater the increase of the v fades with increase of the cycle time T and the larger the inventory levels Q1,Qincrease with increase of the cycle time T, thus, the less the v, the less the markdown percentage 8.
- As shown in Table 1, increasing the parameter 80 of the markdown percentage +30 to markdown percentage -10 decreases the cycle time T, annual profit, and negative changes in the inventory level Q 1,Q 2 and negative variation in the inventory level 1Q 2Q level with a negative change in the markdown percentage. Nevertheless, 0.20 -20% and -30% is the unclear answer.
- As a fact is known, parameter C pdecreasing can lead towards the working well to decrease the annual income of the system in table 1. It is necessary to mention that there are no changes in other parameters because of C-pdecreases.
- It is clear that the reduction of the cost deterioration Cdwould bring T,Q 1,Q 2 and maximal annual profit of the system, as shown in table 1. But it does decline by a small percentage in the percent of markdown 4by the fall of Cd.
- As shown in Table 1, all the parameters are on the rise as the value of alpha decreases. The rdecreases, but in table 1, optimum annual profit is also better and the markdown percentage of the system λ gets better. Also, cycle time Tand extent of stock Q decreases or increases marginally. In this regard, the lower the quality of inventory Q2, the lower Q2 of inventory.

CONCLUSION

This paper presents a lifetime deteriorating inventory model that has a multi-variable demand rate. When the inventory and profit want to be maximum, then reduction policy would be used. As this paper shows, the monetary expansion in the business world is a good thing because it is evident in the sensitivity analysis where effect of monetary expansion is directly pointed to be the optimal markdown time and optimal cost. Cost is taken to be a variable function because to bring the study closer to reality which increases as time progresses. It is also possible to determine that policy makers must be very keen to make the reduction rate dependent. The proposed model can be generalized in cases of partial backorder, stochastic demand and the other one is permitable delay in payment.

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