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EFFECT OF POYNTING-ROBERTSON FORCE ON THE RESONANT MOTION OF GEOCENTRIC SATELLITE

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ABSTRACT

This paper is to discuss the effects of Poynting-Robertson force on the resonant motion of geocentric satellite. In presence of Poynting-Robertson force the resonances $1:1, 1:2, 1:3, 2:1, 2:3, 3:1, 3:2, 4:1, 4:3, 5:1$ and $5:3$ are occurred. Also discuss the amplitude and time period of the geocentric satellite at all these resonant points.

Keywords: Resonance, Poynting-Robertson Force, Geocentric Satellite, Amplitude and Time Period

INTRODUCTION

Over the last few decades, several authors widely studied three-body problem and restricted three-body problem under diverse perturbation conditions. The perturbation owing to mechanism of dissipation in solar system is several dimensional. Understanding of factors affecting the motion of the bodies of the solar system is necessary for the analysis of solar dynamics. Resonance established in solar system plays a vital role in the solar dynamics. Throughout the integration of equations of motion; a set of cases wherein the periods of revolution are in the ratio of two integers demonstrated by the appearance of small divisors is defined as resonance. Hughes (1980) reported resonance's impact on the orbit of Earrth's satellite because of lunisolar gravity and respective direct solar radiation pressure, whose appearance relies only on the orbital of satellite inclination. Under gas rich condition, the nature of resonance trapping was explored by Weidenschilling and Davis (1985). Further, which was continued by Patterson (1987) for the existence of resonances of any order and exhibiting formation of planetary embryous at two-body external resonances by accretion of infinitesimals caught in these orbits. Bhatnagar and Mehera (1986) verified the motion of a satellite using gravitational forces of several bodies including Moon, Earth and the radiating Sun. Ferraz-Mello (1992) studied "averaging of the elliptic asteroidal problem with a Stokes drag" and with the assistance of Beaugé and Ferraz-Mello (1993) he studied "resonance trapping and Stokes drag dissipation in the primordial solar nebula. The often decrease of semi-major axis due to dissipation and consequent collision between one primary and minor bodies has been studied by Celletti et al. (2011). Quarles et al. (2012) has studied the resonances for co-planar CR3BP for the mass ratio between 0.10 and 0.15 and used the method of maximum Lyaponav exponent to locate the resonant points. They showed that in presence of single resonance, the orbital stability is ensured for high value of resonance.

Sushil et al. (2013) worked on resonance in a geocentric satellite due to Earth's equatorial ellipticity and analysed the effects on amplitude and time period of oscillation on Γ (angle measured from the minor axis of the Earth's equatorial ellipse to the projection of the moon on the plane of equator) and on the other orbital elements of the satellite. Rosemary (2013) has given detail description

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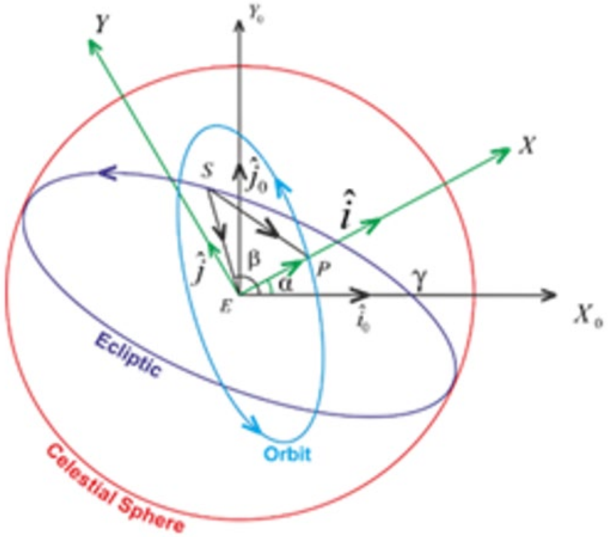
of the perturbation theory to determine the presence of resonance based on approximations to a harmonic oscillation. Kour et al. (2018) worked on resonance in the motion of geocentric satellite due to PR-drag and further in (2019) they have discussed the "Resonance" in the motion of geocentric satellite due to PR-drag and equatorial ellipticity of the Earth. Hassan et al. (2022) studied effects of Stokes drag on the resonant motion of a geocentric satellite and found that time period and amplitude vary with the variation of Stokes Drag parameter. Presently we proposed to extend the work of Hassan et al. (2022) by considering the Combined Effect of Stokes Drag and Earth's equatorial ellipticity on the resonant motion of moon, where the minor axis of the Earth's equatorial section is called ellipticity parameter of the Earth. Here the Stokes-Drag defined by Ferraz-Mello (1992) is under consideration.

We divide this paper in five sections. In section 2, the equations of motion of the geocentric satellite in polar form have been established in presence of Poynting-Robertson force in rotating frame relative to the Earth. In Section 3, we have solved first the unperturbed equation of motion and hence the integrable form of the perturbed equation of motion and its solution is established. In section 4, Amplitudes and time periods have been found out by using the generalised formula of Hassan et al. (2022). The manuscript has been concluded in section 5 and ended with the references.

THE EQUATIONS OF MOTION



Let us considering the inertial frame (E, X_0Y_0Z) whose origin at the Earth E and a rotating frame (E, XYZ) relative to the inertial one , where $\overrightarrow{EX_0}$ passes through the vernal equinox γ .Let $\hat{i}_0, \hat{j}_0$ and $\hat{i}$, $\hat{j}$ be the unit vectors along the axes of inertial frame and rotating frame with common unit vector $\hat{k}$ along the vertical axis EZ (not seen in the figure). Let $\overrightarrow{EP} = \vec{r}$ be the position vector of the satellite P, $\overrightarrow{SP} = \vec{\rho}$ be the position of the Sun S relative to the Earth E and $\overrightarrow{SE} = \vec{R}$. If M , m and μ be the masses of the Sun, Earth and the Geocentric Satellite respectively then their mutual gravitational forces are given by

$$\vec{F}_{EP} = -\frac{Gm\mu}{r^3}\vec{r},\ \vec{F}_{SP} = -\frac{GM\mu}{\rho^3}\vec{\rho},\ \vec{F}_{SE} = -\frac{GMm}{R^3}\vec{R} \tag{1}$$

The Poynting-Robertson force applied on the satellite P is given by

$$\vec{P}' = -\frac{\lambda\mu}{\rho^2}\left(\dot{\vec{\rho}} - \vec{\rho}\times\hat{k}\right) - \frac{\lambda\mu}{\rho^4}\left(\vec{\rho}\square\dot{\vec{\rho}}\right)\vec{\rho} \tag{2}$$

where $\lambda \in [0,1)$ is the dissipative constant, we can decompose the above force into two components, first one is the drag component due to the impact of photons of the solar radiation with the satellite and second component represents the Doppler shift of solar radiation that's hit the satellite. Let $\vec{\omega}$ be angular velocity of the rotating frame relative to the inertial frame and the unit vector $\hat{i}$ along the direction of geocentric satellite then the equation of motion of satellite in rotating frame can be written as

$$\ddot{\vec{r}} = \frac{\partial^2 r}{\partial t^2}\hat{\imath} + 2\frac{\partial r}{\partial t}(\omega \times \hat{\imath}) + r\left(\frac{\partial \vec{\omega}}{\partial t} \times \hat{\imath}\right) + r[(\vec{\omega}\hat{\imath}) - (\vec{\omega}\vec{\omega})\hat{\imath}] \tag{3}$$

Let α be the angle of direction of the satellite with the direction of vernal equinox, then $\vec{\omega} = \dot{\alpha}\vec{k}$, where $\dot{\alpha}$ is the angular velocity of the satellite. Thus, the equation (3) reduced to

$$\ddot{\vec{r}} = \left(\frac{\partial^2 r}{\partial t^2} - r\dot{\alpha}^2\right)\hat{\imath} + \left(2\dot{\alpha}\frac{\partial r}{\partial t} + r\ddot{\alpha}\right)\hat{\jmath} \tag{4}$$

In the triangle EPS

$$\ddot{\vec{r}} = \ddot{\vec{\rho}} - \ddot{\vec{R}} = \frac{\vec{F}_{SP} + \vec{F}_{EP} + \vec{P}'}{\mu} - \frac{\vec{F}_{SE}}{m} = -\frac{GM}{\rho^3}\vec{\rho} - \frac{Gm}{r^3}\vec{r} - \frac{\lambda}{\rho^2}\left(\dot{\vec{\rho}} - \vec{\rho} \times \hat{k}\right) - \frac{\lambda}{\rho^4}(\vec{\rho}\dot{\vec{\rho}})\vec{\rho} + \frac{GM}{R^3}\vec{R}.$$

If $\hat{R}$ be the unit vector along $\vec{R}$ and β be the angle of the direction of the sun with the direction of vernal equinox γ then $\hat{R} = \cos\beta\,\hat{\imath}_0 + \sin\beta\,\vec{\jmath}_0$ and $\dot{\beta}^2 = \frac{GM}{R^3}$ implies that

$$\ddot{\vec{r}} = -\frac{Gm}{r^2}\hat{\imath} + \dot{\beta}^2 R(\cos\beta\,\hat{\imath}_0 + \sin\beta\,\vec{\jmath}_0) - \frac{\lambda}{\rho^2}\left(\dot{\vec{\rho}} - \vec{\rho} \times \hat{k}\right) - \frac{\lambda}{\rho^4}(\vec{\rho}\dot{\vec{\rho}})\vec{\rho}$$

$$-\frac{GM}{\rho^3}(r\hat{\imath} + R\cos\beta\,\hat{\imath}_0 + R\sin\beta\,\vec{\jmath}_0). \tag{5}$$

Scalar product of $\hat{i}$ with (4) and (5) and that of $\hat{j}$ with (4)and (5) comparing the results one can find the equations of motions of the satellite in polar form as

$$\frac{\partial^2 r}{\partial t^2} - \dot{\alpha}^2 r + \frac{Gm}{r^2} = R\left(\dot{\beta}^2 - \frac{GM}{\rho^3}\right)\cos(\alpha - \beta) - \frac{\lambda}{\rho^2}\{\dot{\vec{\rho}}\hat{\imath} - (\vec{\rho} \times \hat{k})\hat{\imath}\}$$

$$-\frac{\lambda}{\rho^4}(\vec{\rho}\dot{\vec{\rho}})(\vec{\rho}\hat{\imath}) - \frac{GMr}{\rho^3} \tag{6}$$

$$\frac{\partial}{\partial t}(r^2\dot{\alpha}) = -\dot{\beta}^2 Rr\sin(\alpha - \beta) - \frac{\lambda r}{\rho^2}\{\dot{\vec{\rho}}\hat{\jmath} - (\vec{\rho} \times \hat{k})\hat{\jmath}\} - \frac{\lambda r}{\rho^4}(\vec{\rho}\dot{\vec{\rho}})(\vec{\rho}\hat{\jmath})$$

$$-\frac{GRMr}{\rho^3}\sin(\alpha - \beta)\,(7) \tag{7}$$

These equations are not integrable, so we replace r and $\dot{\alpha}$ by their steady state value r_0 and $\dot{\alpha}_0$ by perturbation technique which can be introduced in (6) and (7) as$\alpha = \dot{\alpha}_0 t \& \beta = \dot{\beta} t$

$$\frac{d^2 r}{dt^2} - \dot{\alpha}^2 r + \frac{Gm}{r^2} = R\left(\dot{\beta}^2 - \frac{GM}{\rho^3}\right)\cos\left(\dot{\alpha}_0 - \dot{\beta}\right)t - \frac{\lambda}{\rho^2}\{\dot{\vec{\rho}}\hat{\imath} - (\vec{\rho} \times \hat{k})\hat{\imath}\}$$

$$-\frac{\lambda}{\rho^4}(\vec{\rho}\dot{\vec{\rho}})(\vec{\rho}\hat{\imath}) - \frac{GMr_0}{\rho^3} \tag{8}$$

$$\frac{d}{dt}(r^2\dot{\alpha}) = -\dot{\beta}^2 R r_0 \sin(\dot{\alpha}_0 - \dot{\beta})\, t - \frac{\lambda r_0}{\rho^2}\{\dot{\vec{\rho}}\hat{\jmath} - (\vec{\rho} \times \hat{k})\hat{\jmath}\} - \frac{\lambda r_0}{\rho^4}(\vec{\rho}\dot{\vec{\rho}})(\vec{\rho}\hat{\jmath})$$

$$-\frac{GRMr}{\rho^3}\sin(\dot{\alpha}_0 - \dot{\beta})\, t \qquad (9)$$

At steady state

$$\dot{\vec{\rho}} = R\dot{\beta}\sin(\dot{\alpha}_0 - \dot{\beta}_0)t\hat{\imath} + R\dot{\beta}\cos(\dot{\alpha}_0 - \dot{\beta}_0)t\hat{\jmath} + \dot{\alpha}_0 r_0\hat{\jmath}$$

$$(\vec{\rho}\dot{\vec{\rho}})(\rho\hat{\imath}) = \left(R{r_0}^2\dot{\beta} + \frac{R^3\dot{\beta}}{4}\right)\sin(\dot{\alpha}_0 - \dot{\beta})\, t + R^2\dot{\beta}\dot{r}_0\sin(2\dot{\alpha}_0 - 2\dot{\beta})\, t - \frac{R^2 r_0}{2}(\dot{\alpha}_0 + \dot{\beta})\sin 2\,\dot{\alpha}_0 t$$

$$-\frac{R^2 r_0}{2}(\dot{\alpha}_0 + \dot{\beta})\sin 2\,\dot{\beta}t - \left(R{r_0}^2\dot{\alpha}_0 + \frac{R^3\dot{\beta}}{2}\right)\sin(\dot{\alpha}_0 + \dot{\beta})\, t + \frac{R^3\dot{\beta}}{4}\sin(3\dot{\alpha}_0 - 3\dot{\beta})\, t$$

$$-\frac{R^3\dot{\beta}}{4}\sin(3\dot{\alpha}_0 - \dot{\beta})\, t - \frac{R^3\dot{\beta}}{4}\sin(3\dot{\beta} - \dot{\alpha}_0)\, t.$$

For central orbit of satellite $r^2\dot{\alpha} =$ constant=h (say) and $r = 1/u$ in (8) we get

$$\frac{d^2u}{d\alpha^2} + u = \frac{Gm}{{r_0}^4{\dot{\alpha}^2}_0} + \frac{R}{{\dot{\alpha}^2}_0}\left(\frac{GM}{\rho^3} - \dot{\beta}^2\right)u^2\cos(\dot{\alpha}_0 - \dot{\beta})\, t + \frac{\lambda R u^2}{{\dot{\alpha}_0}^2\rho^2}(\dot{\beta} + 1)\sin(\dot{\alpha}_0 - \dot{\beta})\, t$$

$$+\frac{\lambda u^2}{\rho^4{\dot{\alpha}^2}_0}\left[\left(R{r_0}^2\dot{\beta} + \frac{R^3\dot{\beta}}{4}\right)\sin(\dot{\alpha}_0 - \dot{\beta})\, t + R^2\dot{\beta}\dot{r}_0\sin(2\dot{\alpha}_0 - 2\dot{\beta})\, t\right.$$

$$-\frac{R^2 r_0}{2}(\dot{\alpha}_0 + \dot{\beta})\sin 2\,\dot{\alpha}_0 t - \frac{R^2 r_0}{2}(\dot{\alpha}_0 + \dot{\beta})\sin 2\,\dot{\beta}t - \left(R{r_0}^2\dot{\alpha}_0 + \frac{R^3\dot{\beta}}{2}\right)\sin(\dot{\alpha}_0 + \dot{\beta})\, t$$

$$\left.+\frac{R^3\dot{\beta}}{4}\sin(3\dot{\alpha}_0 - 3\dot{\beta})\, t - \frac{R^3\dot{\beta}}{4}\sin(3\dot{\alpha}_0 - \dot{\beta})\, t - \frac{R^3\dot{\beta}}{4}\sin(3\dot{\beta} - \dot{\alpha}_0)\, t\right] + \frac{GMr_0u^2}{\rho^3{\dot{\alpha}_0}^2}. \qquad (10)$$

RESONANCE IN THE MOTION OF THE SATELLITE

The complete solution of the unperturbed equation of motion $\frac{d^2u}{d\alpha^2} + u = \frac{Gm}{{r_0}^4{\dot{\alpha}^2}_0}$is given by $\boldsymbol{\frac{l}{r} = 1 + e\cos(\alpha - \psi)}$, where $l = a(1 - e^2)$ and e, ψ are constants**.** Thus $\boldsymbol{u = \frac{1 + e\cos(\alpha - \psi)}{a(1 - e^2)}}$.

Let us consider $\alpha - \psi = \dot{\alpha}_0 t = nt(say)$ where n be the frequency of the satellite.

Since eccentricity $e < 1$, so $(1 + e\cos n\, t)^{h_1} \approx 1 + h_1 e\cos n\, t$.

Hence by using n and $\frac{d^2u}{dt^2} = {\dot{\alpha}^2}_0\frac{d^2u}{d\alpha^2}$ in equation (10), then we get the perturbed equation of the motion of the satellite is

$$\frac{d^2u}{dt^2} + n^2u = M_1 + M_2\cos n\, t + M_3\cos\dot{\beta}\, t + M_4\sin n\, t + M_5\sin\dot{\beta}\, t + M_6\sin 2\,\dot{\beta}t + M_7\sin 3\,\dot{\beta}t + M_8\sin 2\,nt$$

$$+M_9\sin 3\,nt + M_{10}\cos(n - \dot{\beta})\, t + M_{11}\sin(n - \dot{\beta})\, t + M_{12}\sin(n + \dot{\beta})\, t + M_{13}\sin(n - 2\dot{\beta})\, t$$

$$+M_{14}\sin\left(n+2\dot{\beta}\right)t+M_{15}\sin\left(n-3\dot{\beta}\right)t+M_{16}\sin\left(2n-\dot{\beta}\right)t+M_{17}\cos\left(2n-\dot{\beta}\right)t$$

$$+M_{18}\sin\left(2n+\dot{\beta}\right)t+M_{19}\sin\left(2n-2\dot{\beta}\right)t+M_{20}\sin\left(2n-3\dot{\beta}\right)t+M_{21}\sin\left(3n-\dot{\beta}\right)t$$

$$+M_{22}\sin\left(3n-2\dot{\beta}\right)t+M_{23}\sin\left(3n-3\dot{\beta}\right)t+M_{24}\sin\left(4n-\dot{\beta}\right)t+M_{25}\sin\left(4n-3\dot{\beta}\right)t. \qquad (11)$$

Where

$$M_1=\frac{Gm}{{r_0}^4}+\frac{GMr_0}{\rho^3a^2(1-e^2)^2},M_2=\frac{2GMr_0e}{\rho^3a^2(1-e^2)^2},M_3=\frac{Re\left(\frac{G\mu}{\rho^3}-\dot{\beta}^2\right)}{a^2(1-e^2)^2},$$

$$M_4=-\frac{\lambda R^2r_0e\left(\dot{\alpha}_0+\dot{\beta}\right)}{2\rho^4a^2(1-e^2)^2},M_5=-\frac{\lambda eR\left(8{r_0}^2\dot{\alpha}_0+3R^2\dot{\beta}+4\rho^2\dot{\beta}+4\rho^2\right)}{4\rho^4a^2(1-e^2)^2},$$

$$M_6=-\frac{\lambda R^2r_0\left(\dot{\alpha}_0+\dot{\beta}\right)}{2\rho^4a^2(1-e^2)^2},M_7=-\frac{\lambda R^3\dot{\beta}e}{4\rho^4a^2(1-e^2)^2},M_8=-\frac{\lambda R^2r_0\left(\dot{\alpha}_0+\dot{\beta}\right)}{2\rho^4a^2(1-e^2)^2}=M_6,$$

$$M_9=-\frac{\lambda R^2r_0e\left(\dot{\alpha}_0+\dot{\beta}\right)}{2\rho^4a^2(1-e^2)^2}=M_4,M_{10}=\frac{R\left(\frac{G\mu}{\rho^3}-\dot{\beta}^2\right)}{a^2(1-e^2)^2},M_{11}=\frac{\lambda R\left[(4{r_0}^2+R^2+4\rho^2)\dot{\beta}+4\rho^2\right]}{4\rho^4a^2(1-e^2)^2},$$

$$M_{12}=-\frac{\lambda\left(2R{r_0}^2\dot{\alpha}_0+R^3\dot{\beta}\right)}{2\rho^4a^2(1-e^2)^2},M_{13}=\frac{\lambda R^2r_0e\left(\dot{\alpha}_0+3\dot{\beta}\right)}{2\rho^4a^2(1-e^2)^2},M_{14}=M_6=M_8,$$

$$M_{15}=\frac{\lambda R^3\dot{\beta}}{4\rho^4a^2(1-e^2)^2},M_{16}=\frac{\lambda Re\left[({r_0}^2+\rho^2)\dot{\beta}+\rho^2\right]}{\rho^4a^2(1-e^2)^2},M_{17}=\frac{Re\left(\frac{G\mu}{\rho^3}-\dot{\beta}^2\right)}{a^2(1-e^2)^2}=M_3,$$

$$M_{18}=-\frac{\lambda e\left(2R{r_0}^2\dot{\alpha}_0+R^3\dot{\beta}\right)}{2\rho^4a^2(1-e^2)^2},M_{19}=\frac{\lambda R^2r_0\dot{\beta}}{\rho^4a^2(1-e^2)^2},M_{20}=\frac{\lambda R^3\dot{\beta}e}{2\rho^4a^2(1-e^2)^2},$$

$$M_{21}=-\frac{\lambda R^3\dot{\beta}}{4\rho^4a^2(1-e^2)^2}=-M_{15},M_{22}=\frac{\lambda R^2r_0\dot{\beta}e}{\rho^4a^2(1-e^2)^2},M_{23}=-M_{21}=-M_{15},$$

$$M_{24}=-\frac{\lambda R^3\dot{\beta}e}{4\rho^4a^2(1-e^2)^2},M_{25}=\frac{\lambda R^3\dot{\beta}e}{4\rho^4a^2(1-e^2)^2}=-M_{24}$$

The solution of equation (11) is given by

$$u=M\cos(nt-\xi)+\frac{M_1}{n^2}+\frac{M_2t\sin nt}{2n}+\frac{M_3\cos\dot{\beta}t}{n^2-\dot{\beta}^2}+\frac{M_4t\cos nt}{2n}+\frac{M_5\sin\dot{\beta}t}{n^2-\dot{\beta}^2}+\frac{M_6\sin2\dot{\beta}t}{n^2-4\dot{\beta}^2}$$

$$+\frac{M_7\sin3\dot{\beta}t}{n^2-9\dot{\beta}^2}-\frac{M_8\sin2nt}{3n^2}-\frac{M_9\sin3nt}{8n^2}+\frac{M_{10}\cos(n-\dot{\beta})t}{n^2-(n-\dot{\beta})^2}+\frac{M_{11}\sin(n-\dot{\beta})t}{n^2-(n-\dot{\beta})^2}$$

$$+\frac{M_{12}\sin(n+\dot{\beta})t}{n^2-(n+\dot{\beta})^2}+\frac{M_{13}\sin(n-2\dot{\beta})t}{n^2-(n-2\dot{\beta})^2}+\frac{M_{14}\sin(n+2\dot{\beta})t}{n^2-(n+2\dot{\beta})^2}+\frac{M_{15}\sin(n-3\dot{\beta})t}{n^2-(n-3\dot{\beta})^2}$$

$$+\frac{M_{16}\,sin(\,2n-\dot{\beta})t}{n^2-(2n-\dot{\beta})^2}+\frac{M_{17}\,cos(\,2n-\dot{\beta})t}{n^2-(2n-\dot{\beta})^2}+\frac{M_{18}\,sin(\,2n+\dot{\beta})t}{n^2-(2n+\dot{\beta})^2}+\frac{M_{19}\,sin(\,2n-2\dot{\beta})t}{n^2-(2n-2\dot{\beta})^2}$$

$$+\frac{M_{20}\,sin(\,2n-3\dot{\beta})t}{n^2-(2n-3\dot{\beta})^2}+\frac{M_{21}\,sin(\,3n-\dot{\beta})t}{n^2-(3n-\dot{\beta})^2}+\frac{M_{22}\,sin(\,3n-2\dot{\beta})t}{n^2-(3n-2\dot{\beta})^2}+\frac{M_{23}\,sin(\,3n-3\dot{\beta})t}{n^2-(3n-3\dot{\beta})^2}$$

$$+\frac{M_{24}\,sin(4n-\dot{\beta})t}{n^2-(4n-\dot{\beta})^2}+\frac{M_{25}\,sin(4n-3\dot{\beta})t}{n^2-(4n-3\dot{\beta})^2}. \qquad (12)$$

Where ξ is constant of integration. On vanishing the denominator of any term of equation (12) we get some points at which motion becomes indeterminate and hence resonance occurs at these points. Thus, the resonances occur at the points $n=\dot{\beta}, n=2\dot{\beta}, n=3\dot{\beta}, 2n=\dot{\beta}$ $2n=3\dot{\beta}, 3n=\dot{\beta}, 3n=2\dot{\beta}, 4n=\dot{\beta}, 4n=3\dot{\beta}, 5n=\dot{\beta}$and $5n=3\dot{\beta}$. All the resonances 1: 1,1: 2,1: 3,2: 12: 3,3: 1,3: 2,4: 1,4: 3,5: 1 and 5: 3 occur due to Poynting-Robertson force.

AMPLITUDE AND TIME PERIOD

By using Brown and Shook (1933) and Hassan et al. (2022) the generalization formula of the amplitude A and the time period T at the resonant point $m_1 n = m_2 \dot{\beta}$ where $m_1,\ m_2 \in N$ for the equation is of the form $\frac{d^2u}{dt^2}+n^2u=M_s\phi$ and $s \in N$ are

$$A=\frac{\sqrt{2}\,cos\frac{m_1}{m_2}\beta}{\sqrt{|M_s|n_0}}\,and T=\frac{2\sqrt{2}\pi\,cos\frac{m_1}{m_2}\beta}{\sqrt{|M_s|n_0}} \qquad (13)$$

It's to be noted that any value of s may or may not represent the corresponding value of A and T using the result of (13) the amplitude and time period at different resonant points are cited in the table.

Resonant Point	Amplitude	Time Period	s
$n=\dot{\beta}$	A_1	T_1	5
	A_2	T_2	13
	A_3	T_3	16
	A_4	T_4	20
	A_5	T_5	22
	A_6	T_6	25
$n=2\dot{\beta}$	A_7	T_7	6
	A_8	T_8	19
$n=3\dot{\beta}$	A_9	T_9	7
$2n=\dot{\beta}$	A_{10}	T_{10}	11
	A_{11}	T_{11}	21
	A_{12}	T_{12}	22
$2n=3\dot{\beta}$	A_{13}	T_{13}	15
$3n=\dot{\beta}$	A_{14}	T_{14}	16
$3n=2\dot{\beta}$	A_{15}	T_{15}	19
$4n=\dot{\beta}$	A_{16}	T_{16}	21
$4n=3\dot{\beta}$	A_{17}	T_{17}	23
$5n=\dot{\beta}$	A_{18}	T_{18}	24
$5n=3\dot{\beta}$	A_{19}	T_{19}	25

Where

$$A_1 = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda eR\left(8{r_0}^2\dot{\alpha}_0 + 3R^2\dot{\beta} + 4\rho^2\dot{\beta} + 4\rho^2\right)n_0}} T_1 = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda eR\left(8{r_0}^2\dot{\alpha}_0 + 3R^2\dot{\beta} + 4\rho^2\dot{\beta} + 4\rho^2\right)n_0}}$$

$$A_2 = \frac{2\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda R^2 r_0\left(\dot{\alpha}_0 + 3\dot{\beta}\right)en_0}} T_2 = \frac{4\pi\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda R^2 r_0\left(\dot{\alpha}_0 + 3\dot{\beta}\right)en_0}}$$

$$A_3 = \frac{\sqrt{2}\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda Re\left[({r_0}^2 + \rho^2)\dot{\beta} + \rho^2\right]n_0}} T_3 = \frac{2\sqrt{2}\pi\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda Re\left[({r_0}^2 + \rho^2)\dot{\beta} + \rho^2\right]n_0}}$$

$$A_4 = \frac{2\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda R^3\dot{\beta}en_0}} T_4 = \frac{4\pi\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda R^3\dot{\beta}en_0}}$$

$$A_5 = \frac{\sqrt{2}\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda R^2\dot{\beta}r_0 en_0}} T_5 = \frac{2\sqrt{2}\pi\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda R^2\dot{\beta}r_0 en_0}}$$

$$A_6 = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda R^3\dot{\beta}en_0}} T_6 = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos\beta}{\sqrt{\lambda R^3\dot{\beta}en_0}}$$

$$A_7 = \frac{2\rho^2 a(1-e^2)\cos 2\beta}{\sqrt{\lambda R^2\left(\dot{\alpha}_0 + \dot{\beta}\right)r_0 n_0}} T_7 = \frac{4\pi\rho^2 a(1-e^2)\cos 2\beta}{\sqrt{\lambda R^2\left(\dot{\alpha}_0 + \dot{\beta}\right)r_0 n_0}}$$

$$A_8 = \frac{\sqrt{2}\rho^2 a(1-e^2)\cos 2\beta}{\sqrt{\lambda R^2\dot{\beta}r_0 en_0}} T_8 = \frac{2\sqrt{2}\pi\rho^2 a(1-e^2)\cos 2\beta}{\sqrt{\lambda R^2\dot{\beta}r_0 en_0}}$$

$$A_9 = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos 3\beta}{\sqrt{\lambda R^3\dot{\beta}en_0}} T_9 = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos 3\beta}{\sqrt{\lambda R^3\dot{\beta}en_0}}$$

$$A_{10} = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos\frac{\beta}{2}}{\sqrt{\lambda R\left[(4{r_0}^2 + R^2 + 4\rho^2)\dot{\beta} + 4\rho^2\right]n_0}} T_{10} = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{\beta}{2}}{\sqrt{\lambda R\left[(4{r_0}^2 + R^2 + 4\rho^2)\dot{\beta} + 4\rho^2\right]n_0}}$$

$$A_{11} = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos\frac{\beta}{2}}{\sqrt{\lambda R^3\dot{\beta}n_0}} T_{11} = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{\beta}{2}}{\sqrt{\lambda R^3\dot{\beta}n_0}}$$

$$A_{12} = \frac{\sqrt{2}\rho^2 a(1-e^2)\cos\frac{\beta}{2}}{\sqrt{\lambda R^2 \dot{\beta} r_0 e n_0}} T_{12} = \frac{2\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{\beta}{2}}{\sqrt{\lambda R^2 \dot{\beta} r_0 e n_0}}$$

$$A_{13} = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos\frac{3\beta}{2}}{\sqrt{\lambda R^3 \dot{\beta} n_0}} T_{13} = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{3\beta}{2}}{\sqrt{\lambda R^3 \dot{\beta} n_0}}$$

$$A_{14} = \frac{\sqrt{2}\rho^2 a(1-e^2)\cos\frac{\beta}{3}}{\sqrt{\lambda R \dot{\beta} {r_0}^2 e n_0}} T_{14} = \frac{2\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{\beta}{3}}{\sqrt{\lambda R^2 \dot{\beta} {r_0}^2 e n_0}}$$

$$A_{15} = \frac{\sqrt{2}\rho^2 a(1-e^2)\cos\frac{2\beta}{3}}{\sqrt{\lambda R^2 \dot{\beta} r_0 e n_0}} T_{15} = \frac{2\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{2\beta}{3}}{\sqrt{\lambda R^2 \dot{\beta} r_0 e n_0}}$$

$$A_{16} = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos\frac{\beta}{4}}{\sqrt{\lambda R^3 \dot{\beta} n_0}} T_{16} = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{\beta}{4}}{\sqrt{\lambda R^3 \dot{\beta} n_0}}$$

$$A_{17} = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos\frac{3\beta}{4}}{\sqrt{\lambda R^3 \dot{\beta} n_0}} T_{17} = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{3\beta}{4}}{\sqrt{\lambda R^3 \dot{\beta} n_0}}$$

$$A_{18} = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos\frac{\beta}{5}}{\sqrt{\lambda R^3 \dot{\beta} e n_0}} T_{18} = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{\beta}{5}}{\sqrt{\lambda R^3 \dot{\beta} e n_0}}$$

$$A_{19} = \frac{2\sqrt{2}\rho^2 a(1-e^2)\cos\frac{3\beta}{5}}{\sqrt{\lambda R^3 \dot{\beta} e n_0}} T_{19} = \frac{4\sqrt{2}\pi\rho^2 a(1-e^2)\cos\frac{3\beta}{5}}{\sqrt{\lambda R^3 \dot{\beta} e n_0}}.$$

CONCLUSION

In section 1, of this manuscript, the previous works have been cited. In section 2, the polar equations of motion of the geocentric satellite have been established in presence of Poynting-Robertson force in rotating frame relative to the Earth. To reduce the chances of non-integrability of the equations of motion, we used perturbation technique by taking the steady state values of the position vector and angular velocity of satellite. In section 3, we have solved first the unperturbed equation of motion. The solution of perturbed equation (11) of motion in equation (12). By making denominator of any term from 5th to 26th to zero u becomes infinity and hence the motion of the satellite becomes indeterminate. Thus $n = \dot{\beta}, n = 2\dot{\beta}, n = 3\dot{\beta}, 2n = \dot{\beta}, 2n = 3\dot{\beta}, 3n = \dot{\beta}, 3n = 2\dot{\beta}, 4n = \dot{\beta}, 4n = 3\dot{\beta}, 5n = \dot{\beta}$ and $5n = 3\dot{\beta}$ are eleven resonances of the problem all of them are occurred due to Poynting-Robertson force. In section 4, we have found the amplitudes and time periods at all the resonant points which are occurred due to Poynting-Robertson force.

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