Original Article
ISSN (Online): 2456-8651

International Journal of Engineering Science Technologies
January-February 2025 9(1), 1–14

MATHEMATICAL MODELING AND Z-CONTROL MECHANISM TO MITIGATE CHLAMYDIA TRACHOMATIS-INDUCED CONJUNCTIVITIS IN NEWBORNS

Nita H Shah [1], Jalpa N Vaghela [2], Ekta N Jayswal [3]

[1] School of Emerging Science and Technology, Ahmedabad - 380009, Gujarat, India
[2] Department of Mathematics, Gujarat University, Ahmedabad - 380009, Gujarat, India
[3] Nirma University, Ahmedabad, Gujarat, India

Received 15 November 2024
Accepted 06 December 2024
Published 15 January 2025

Corresponding Author
Nita H Shah, nitahshah@gmail.com

DOI 10.29121/IJOEST.v9.i1.2025.660

Funding: This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

ABSTRACT

This study analyzes a mathematical model employing a Z-control mechanism to prevent neonatal Chlamydial conjunctivitis in newborns of infected mothers. Chlamydia trachomatis transmission during delivery can lead to conjunctivitis and severe complications in the lungs and nasopharynx if untreated. The model, based on nonlinear differential equations, evaluates the basic reproduction number at the disease-free equilibrium point. Numerical simulations demonstrate that Z-control effectively stabilizes chaotic oscillations in the system, reducing the risk of conjunctivitis in newborns. The findings highlight that prenatal testing, early treatment of pregnant women, and newborn screening are critical for preventing and controlling Chlamydia transmission. The implementation of Z-control has proven effective in mitigating this epidemic, significantly reducing the incidence of neonatal Chlamydial conjunctivitis.

Keywords: Mathematical Model, Z-Control, Chlamydia Trachomatis, Neonatal Chlamydial Conjunctivitis, Numerical Simulation

1. INTRODUCTION

Chlamydia research is becoming more important in the field of human health because the number of cases is rising quickly every day (CDC. Sexually Transmitted Disease Survelailnce, 2020). Most infected individuals do not exhibit symptoms and do not seek medical treatment. Modelling studies show that between 5 and 30% of women and 10% of men with a confirmed infection have symptoms Korenromp et al. (2002) . Chlamydia is more frequent in adolescents. According to a recent study, people aged 15-24 account for two-thirds of the total chlamydia population Kreisel

et al. (2021). The ratio of Chlamydia affecting young females between the ages of 14 and 24 is 1:20, and the prevalence of the disease continues to increase. Without treatment, a uterine or fallopian tube infection may cause pelvic inflammatory disease (PID), which affects pregnancy and results in 2%–5% of women who are not treated [Haggerty et al. (2010) ,Oakeshott et al. (2010)].

The possibility of vertical transference of Chlamydia from a pregnant woman to her newborn is commonly recognized, but it is not clear how much the infection affects pregnancy and may lead to difficulties after birth. During delivery, pregnant women may transmit Chlamydial infection to their infants "Chlamydia - CDC Fact Sheet". CDC. Nov 19, 2022. . Some infants might get conjunctivitis or pneumonia. According to prospective studies, Chlamydial conjunctivitis affects 18–44 percent of newborns born to Chlamydia-positive mothers, while Chlamydial pneumonia affects 3–16 percent of those same infants Frommell et al. (1979),Heggie et al. (1981),Hammerschlag et al. (1982),Schachter et al. (1986)]. Conjunctivitis is the most common symptom seen in newborns, and 30 to 50% of newborns of women with Chlamydia are affected by infection Hammerschlag (2011).

Neonatal conjunctivitis is the most frequent infection that occurs due to Chlamydial. Symptoms of Chlamydial conjunctivitis are eye redness, irritation, mucous discharge, swollen eyelids, and eyelid crusting. Symptoms may appear between one and three weeks after the infection. 5–12 days after delivery, symptoms may occur. If the amniotic sac ruptures during birth, symptoms usually develop earlier. Newborns with Chlamydial conjunctivitis may have additional infections, and the lungs and nasopharynx (where the back of the nose connects to the mouth) can be infected Centers for Disease Control and Prevention (CDC) Conjunctivitis in Newborn

In severe cases, sexually transmitted infections (STIs) may cause eye damage and conjunctivitis. The eyes might be especially vulnerable to these infections. Even today, STI-related eye disorders remain a leading cause of conjunctivitis in several countries. Chlamydia in the eye might cause neonatal conjunctivitis and serious problems in other parts of the body, like the lungs and nasopharynx, if it is not treated. However, it is preventable, and prompt treatment will help to both cure the infection and avoid its consequences. Due to the continuing high frequency of Chlamydia around the world and the insufficient prenatal screening in many nations, newborns who develop a Chlamydial infection continue to need safe and effective therapy. Prenatal testing and medication are recommended for pregnant women to avoid neonatal chlamydial conjunctivitis. Systemic antibiotics are used to treat this eye infection; topical antibiotics are generally ineffective. Oral tetracycline, oral doxycycline, or oral erythromycin stearate are recommended treatments for 3-6 weeks CDC Sexually Transmitted Disease Survelailnce (2020).

1.1. THE Z-CONTROL MECHANISM

This portion explains the fundamental concept of Z-control mechanisms. The Z-control can be used directly and indirectly in an epidemic system. To control the dynamics of the disease, regulation may be directly supplied to susceptible as well as infected populations at the same time. If we want to control any of the population, indirect control allows for the application of the control to the other population. To achieve the desired dynamics, we may change the susceptible population size in the Z-control system. The design formula assures that the error exponentially goes to zero and moves the system toward the required state. The Z-controller increases system stability by substituting an unstable steady state with a more stable one. Therefore, one of the most successful control methods for epidemiological systems

is Z-control. The goal of this study is to figure out how to control infection from spreading and how to stop the chaotic and periodic oscillations that have been seen.

1.2. LITERATURE SURVEY

In the literature, there are various applications of Z-control to problems in epidemiology and population dynamics. Alzahrani et al. Alzahrani et al. (2018) suggested a model in which a predator interacts with prey that is infected with an infectious disease, and they demonstrated that Z-control can reduce chaotic behaviour in solutions. To prevent the extinction of species and promote their coexistence, Zhang et al. Zhang et al. (2016)used the Lotka-Volterra model using the Z-control method. Samanta et al. Samanta (2018) demonstrated that a disease can be controlled when the reproduction number of the uncontrolled system is more than one by applying the Z–control to a planar SI-like epidemic model. Lacitignola et al. Lacitignola et al. (2016) studied an epidemic model with backward transcritical bifurcation, demonstrating that the disease may spread even when smaller than one. They also showed that the Z-control may abolish the backward bifurcation under some conditions.

In the literature of epidemiology, McGregor and French McGregor & French (1991)researched Chlamydial infection during pregnancy, problems include early membrane rupture, low birth weight, preterm delivery, and stillbirth. Infection with Chlamydia trachomatis is also implicated in postcesarean section, postabortal, and postpartum maternal infections. They conclude that treating Chlamydial infection during pregnancy prevents newborn morbidity. Preventing and controlling C. trachomatis infections in pregnant females and treating newborn Chlamydial conjunctivitis may help. According to Hammerschlag Hammerschlag (2011), prenatal screening and treatment of pregnant women appears to be an effective method for reducing neonatal Chlamydial infection. In addition, investigate the detection and treatment of both Chlamydial and Gonococcal infections in newborns. Darville Darville (2005) analysed newborns and young infants with conjunctivitis or pneumonia, their clinical aspects, diagnosis, treatment, and preventive strategies. Chlamydia-induced newborn conjunctivitis causes eyelid erythema and purulent ocular discharge. Conjunctivitis may arise between 5 and 14 days after delivery. Positive test results should diagnose conjunctivitis. Researchers investigated the growth cycle of Chlamydia trachomatis, neonatal conjunctivitis or pneumonia, and Chlamydial treatment. Zikic et al. (2018) researched antibiotic therapies for newborn Chlamydial conjunctivitis, such as oral erythromycin, azithromycin, or trimethoprim.

In this study, we construct a model to better understand how Chlamydial infection affects newborns during childbirth. Untreated neonatal eye infections may cause conjunctivitis. The article is organized as follows: Sections 2 include the calculations of the existence of an equilibrium point and the reproduction number, while Sections 3 include the of Z-control mechanism. The computing of a numerical simulation that it uses in the study of Chlamydia is discussed in the fourth section of the article, and a short conclusion is provided in the last section.

2. MATHEMATICAL FORMULATION AND DESCRIPTION OF THE MODEL

This model is developed using three state variables and a compartment that, if a pregnant woman (W_P) is infected with Chlamydia trachomatis (C_T), it increases

the risk of neonatal conjunctivitis (N_C) during childbirth. Here, we have taken into consideration the saturated incidence, which is denoted by the formula which α_2 represents a force of infection and a represents saturation constant.

Figure 1

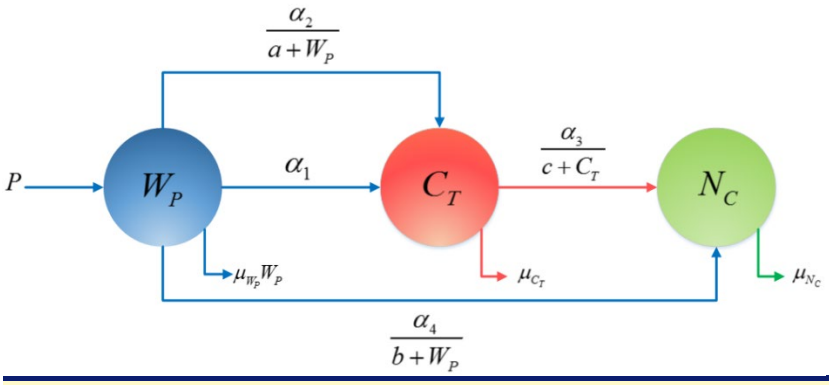

Figure 1 Diagram for the Transmission of a Chlamydia Model

Interpretation of additional parameters: P is the pregnancy rate; α_1 is the rate of Chlamydia infection in pregnant females; α_3 is the rate of neonatal conjunctivitis due to Chlamydia-infected pregnant women, and α_4 is the rate of conjunctivitis in newborn, and c are half-saturation constants; c_1 and c_2 are the conversion efficiency of Chlamydia trachomatis and pregnant women, respectively. μ_{W_P} is maternal mortality rate; μ_{C_T} is pregnant women recovered from Chlamydia, and μ_{N_C} is the rate of infants who do not have conjunctivitis.

Figure 1 depicts the model diagram, which is made up of a dynamic system of differential equations that illustrate the rate of change over time. Described as follows:

$$\frac{dW_P}{dt} = PW_P - \alpha_1 W_P C_T - \frac{\alpha_2 W_P C_T}{a+W_P} - \frac{\alpha_4 W_P N_C}{b+W_P} - \mu_{W_P} {W_P}^2$$

$$\frac{dC_T}{dt} = \alpha_1 W_P C_T + \frac{\alpha_2 W_P C_T}{a+W_P} - \frac{\alpha_3 C_T N_C}{c+C_T} - \mu_{C_T} C_T \qquad (1)$$

$$\frac{dN_C}{dt} = \frac{c_1 \alpha_3 C_T N_C}{c+C_T} + \frac{c_2 \alpha_4 W_P N_C}{b+W_P} - \mu_{N_C} N_C.$$

with $W_P > 0$, and $C_T, N_C \geq 0$.

2.1. EXISTENCE OF EQUILIBRIUM POINTS AND REPRODUCTION NUMBER

Determination of equilibrium points by solving (1) stated above:

1) $E_0 = \left\{ \frac{P}{\mu_{W_P}}, 0, 0 \right\}.$

2) $E_1 = \left\{ \frac{b\mu_{N_C}}{c_2\alpha_4 - \mu_{N_C}}, 0, \frac{bc_2(P\alpha_4 c_2 - b\mu_{W_P}\mu_{N_C} - P\mu_{N_C})}{(\alpha_4 c_2 - \mu_{N_C})^2} \right\}.$

3) $E_2 = \left\{ r_1, \frac{(P\alpha_1 - \mu_{W_P}(\mu_{C_T} - \alpha_2))r_1 + a(P\alpha_1 - \mu_{W_P}\mu_{C_T})}{\alpha_1(a\alpha_1 + r_1\alpha_1 + \alpha_2)}, 0 \right\}$

$where\ r_1 = RootOf\{\alpha_1 x^2 + (a\alpha_1 + \alpha_2 - \mu_{C_T})x - a\mu_{C_T}\}$

4) $E^*_{end} = \{W_P^*, C_T^*, N_C^*\}$

$$W_P^* = Rootof \begin{pmatrix} (\alpha_3 c_1 \mu_{W_P} + \alpha_4 c_2 \mu_{W_P} - \mu_{W_P}\mu_{N_C})x^3 + (a\alpha_3 c_1 \mu_{W_P} + a\alpha_4 c_2 \mu_{W_P} + \alpha_1\alpha_4 cc_1 - \alpha_1\alpha_4 cc_2 \\ +\alpha_3 bc_1 \mu_{W_P} - P\alpha_3 c_1 - P\alpha_4 c_2 - a\mu_{W_P}\mu_{N_C} + \alpha_1 c\mu_{N_C} - b\mu_{W_P}\mu_{N_C} + P\mu_{N_C})x^2 \\ +(a\alpha_1\alpha_4 cc_1 - a\alpha_1\alpha_4 cc_2 + a\alpha_3 bc_1 \mu_{W_P} - Pa\alpha_3 c_1 - Pa\alpha_4 c_2 - P\alpha_3 bc_1 \\ +a\alpha_1 c\mu_{N_C} - ab\mu_{W_P}\mu_{N_C} + \alpha_1 bc\mu_{N_C} + \alpha_2\alpha_4 cc_1 - \alpha_2\alpha_4 cc_2 - \alpha_4 cc_1\mu_{C_T} + Pa\mu_{N_C} \\ +Pb\mu_{N_C} + \alpha_2 c\mu_{N_C})x - Pa\alpha_3 bc_1 + a\alpha_1 bc\mu_{N_C} - a\alpha_4 cc_1\mu_{C_T} + Pab\mu_{N_C} + \alpha_2 bc\mu_{N_C} \end{pmatrix}$$

Assume $W_P^* = r_2$

$$C_T^* = -\frac{c(\alpha_4 c_2 r_2 - b\mu_{N_C} - \mu_{N_C} r_2)}{\alpha_3 bc_1 + \alpha_3 c_1 r_2 + \alpha_4 c_2 r_2 - b\mu_{N_C} - \mu_{N_C} r_2}$$

$\boldsymbol{N_C^* = \frac{n_1}{m_1}}$, where

$n_1 = (-\alpha_1((-\alpha_3(-r_2\mu_{W_P} + P)c_1 + \mu_{N_C}(\alpha_1 c - r_2\mu_{W_P} + P))(-\alpha_3 c_1 + \mu_{N_C})(c_1 - c_2)a - \mu_{N_C}(\alpha_3 c(\alpha_1 r_2 + \alpha_2 - \mu_{C_T})c_1^2$
$+(-c(\alpha_1 r_2 + \alpha_2 - \mu_{C_T})\mu_{N_C} - \alpha_3 c_2(\alpha_1 cr_2 - r_2^2\mu_{W_P} + \mathrm{Pr}_2 + \alpha_2 c - c\mu_{C_T}))c_1 + \mu_{N_C} c_2(\alpha_1 cr_2 - r_2^2\mu_{W_P} + \mathrm{Pr}_2 + \alpha_2 c)))b^3$
$+((-\alpha_3(-r_2\mu_{W_P} + P)c_1 + \mu_{N_C}(\alpha_1 c - r_2\mu_{W_P} + P))\alpha_1(-c_1\alpha_3 - c_2\alpha_4 + \mu_{N_C})(c_1 - c_2)a^2 + (-\alpha_3^2(\alpha_1 r_2 - \alpha_2 + \mu_{C_T})$
$(-r_2\mu_{W_P} + P)c_1^3 + 2\alpha_3((((3\alpha_1 c/2) - r_2\mu_{W_P} + P)\mu_{C_T} - (\alpha_1^2 cr_2/2) + (-r_2^2\mu_{W_P} + \mathrm{Pr}_2 - \alpha_2 c)\alpha_1 - \alpha_2(-r_2\mu_{W_P} + P))$
$\mu_{N_C} + (1/2)(c_2((\alpha_1 r_2 - \alpha_2)(-r_2\mu_{W_P} + P)\alpha_3 + ((-\alpha_1 c + r_2\mu_{W_P} - P)\mu_{C_T} + \alpha_2(-r_2\mu_{W_P} + P))\alpha_4)))c_1^2$
$+(((3\alpha_1 c + r_2\mu_{W_P} - P)\mu_{C_T} + \alpha_1^2 cr_2 + (r_2^2\mu_{W_P} - \mathrm{Pr}_2 + 2\alpha_2 c)\alpha_1 + \alpha_2(-r_2\mu_{W_P} + P)\mu_{N_C}{}^2 + 2c_2(((-\alpha_1 c + (r_2\mu_{W_P}/2)$
$-(P/2))\mu_{C_T} + (\alpha_1^2 cr_2/2) + \alpha_1\alpha_2 c + \alpha_2(-r_2\mu_{W_P} + P) - (((-2\alpha_1 c + r_2\mu_{W_P} - P)\mu_{C_T} + \alpha_1^2 cr_2 + 2\alpha_1\alpha_2 c$
$+\alpha_2(-r_2\mu_{W_P} + P)\alpha_1/2)\mu_{N_C} - c_2^2(-\alpha_1 c\mu_{C_T} + \alpha_2(-r_2\mu_{W_P} + P))\alpha_3\alpha_4)c_1 - c_2\mu_{N_C}((-\alpha_1 c + r_2\mu_{W_P} - P)\mu_{C_T}$
$+\alpha_1^2 cr_2 + (-r_2^2\mu_{W_P} + \mathrm{Pr}_2 + 2\alpha_2 c)\alpha_1 + \alpha_2(-r_2\mu_{W_P} + P))(-c_2\alpha_4 + \mu_{N_C}))a - \mu_{N_C}(\alpha_3 c(\alpha_2 - \mu_{C_T})(\alpha_1 r_2 + \alpha_2 - \mu_{C_T})c_1^2$
$+(-c(\alpha_2 - \mu_{C_T})(\alpha_1 r_2 + \alpha_2 - \mu_{C_T})\mu_{N_C} - c_2(((r_2^2\mu_{W_P} - \mathrm{Pr}_2 - \alpha_2 c)\mu_{C_T} + \alpha_2(\alpha_1 cr_2 - r_2^2\mu_{W_P} + \mathrm{Pr}_2 + \alpha_2 c))\alpha_3$
$-\alpha_4 c(\alpha_2 - \mu_{C_T})(\alpha_1 r_2 + \alpha_2 - \mu_{C_T})))c_1 + c_2(\alpha_2 - \mu_{C_T})(-c_2\alpha_4 + \mu_{N_C})(\alpha_1 cr_2 - r_2^2\mu_{W_P} + \mathrm{Pr}_2 + \alpha_2 c)))b^2$
$+a((-\alpha_3(\alpha_1 r_2 + \mu_{C_T})(-r_2\mu_{W_P} + P)c_1^2 + (((2\alpha_1 c - r_2\mu_{W_P} + P)\mu_{C_T} + \alpha_1 r_2(-r_2\mu_{W_P} + P))\mu_{N_C}$
$+c_2(\alpha_1 r_2(-r_2\mu_{W_P} + P)\alpha_3 - \alpha_4\mu_{C_T}(\alpha_1 c - r_2\mu_{W_P} + P)))c_1 - c_2\mu_{C_T}(-c_2\alpha_4 + \mu_{N_C})(-c_1\alpha_3 - c_2\alpha_4 + \mu_{N_C})a$
$+\alpha_3^2 r_2(\alpha_2 - \mu_{C_T})(-r_2\mu_{W_P} + P)c_1^3 - 2\alpha_3((-c\mu_{C_T}{}^2 + ((1/2)\alpha_1 cr_2 + r_2^2\mu_{W_P} - \mathrm{Pr}_2 + \alpha_2 c)\mu_{C_T}\alpha_2 r_2(-r_2\mu_{W_P} + P))\mu_{N_C}$
$+((c_2(\alpha_2 r_2(-r_2\mu_{W_P} + P)\alpha_3 - (\alpha_2 - \mu_{C_T})\alpha_4(c\mu_{C_T} + r_2(-r_2\mu_{W_P} + P))))/2))c_1^2 + ((-2c\mu_{W_P}{}^2 + (\alpha_1 cr_2 + r_2^2\mu_{W_P}$
$-\mathrm{Pr}_2 + 2\alpha_2 c)\mu_{C_T} + \alpha_2 r_2(-r_2\mu_{W_P} + P)\mu_{N_C} + (((-r_2^2\mu_{W_P} + \mathrm{Pr}_2 + \alpha_2 c)\mu_{C_T} + \alpha_2 r_2(-r_2\mu_{W_P} + P))\alpha_3 - \alpha_4 c\mu_{C_T}$
$(\alpha_1 r_2 + \alpha_2 - \mu_{C_T}))c_2)(-c_2\alpha_4 + \mu_{N_C})c_1 - c_2\mu_{C_T}(-c_2\alpha_4 + \mu_{N_C}{}^2)(\alpha_1 cr_2 - r_2^2\mu_{W_P} + \mathrm{Pr}_2 + \alpha_2 c))b$
$+(-\alpha_3 r_2(-r_2\mu_{W_P} + P)c_1 + (c\mu_{C_T} + r_2(-r_2\mu_{W_P} + P))(-c_2\alpha_4 + \mu_{N_C}))\mu_{C_T}a^2(-c_1\alpha_3 - c_2\alpha_4 + \mu_{N_C})c_1.$

and

$$m_1 = ((-\alpha_1(-c_1\alpha_3 + \mu_{N_C})(-c_1\alpha_3 + \alpha_3c_2 + \mu_{N_C})b^2 + (\alpha_1(-c_1\alpha_3 + \alpha_3c_2 + \mu_{N_C})a - \alpha_3(\alpha_2 - \mu_{C_T})c_1$$
$$+(\alpha_2 - \mu_{C_T})\mu_{N_C} + \alpha_2\alpha_3c_2)(-c_1\alpha_3 - c_2\alpha_4 + \mu_{N_C})b + a\mu_{C_T}(-c_1\alpha_3 - c_2\alpha_4 + \mu_{N_C})^2)$$
$$(-b\mu_{N_C} + a(\alpha_4c_1 - c_2\alpha_4 + \mu_{N_C}))).$$

2.2. BASIC REPRODUCTION NUMBER

The next-generation matrix technique calculated the edge value for the vertical transference of Chlamydia from pregnant females to their newborns to ensure model stability. The matrix's spectral radius around the disease-free equilibrium point determines the system's reproduction number Diekmann et al. (1990).

Let, $X = (W_P(t), C_T(t), N_C(t))$ be revised as $X' = F(X) - V(X)$ where $F(X)$ denotes the new infection rate appearing in the compartment and $V(X)$ denotes disease transference rate.

$$F(X) = \begin{bmatrix} \alpha_1 W_P C_T + \frac{\alpha_2 W_P C_T}{a+W_P} \\ \frac{c_1\alpha_3 C_T N_C}{c+C_T} + \frac{c_2\alpha_4 W_P N_C}{b+W_P} \\ 0 \end{bmatrix} \text{and} V(X) =$$

$$\begin{bmatrix} \frac{\alpha_3 C_T N_C}{c + C_T} + \mu_{C_T} C_T \\ \mu_{N_C} N_C \\ -PW_P + \alpha_1 W_P C_T + \frac{\alpha_2 W_P C_T}{a + W_P} + \frac{\alpha_4 W_P N_C}{b + W_P} + \mu_{W_P} {W_P}^2 \end{bmatrix}$$

$D(F(E_0)) = \begin{bmatrix} f & 0 \\ 0 & 0 \end{bmatrix}$ and $D(V(E_0)) = \begin{bmatrix} v & 0 \\ J_1 & J_2 \end{bmatrix}$ are determined by computing the Jacobian

matrices at E_0. where, f and v are matrices of size 3 defined as $f = \left(\frac{\partial F_i(E_0)}{\partial X_j}\right)$ and $v = \left(\frac{\partial V_i(E_0)}{\partial X_j}\right)$.

$$f = \begin{bmatrix} \left(\alpha_1 W_P + \frac{\alpha_2 W_P}{a+W_P}\right) & 0 & \left(\alpha_1 C_T + \frac{\alpha_2 C_T}{a+W_P} - \frac{\alpha_2 W_P C_T}{(a+W_P)^2}\right) \\ \left(\frac{c_1\alpha_3 N_C}{c+C_T} - \frac{c_1\alpha_3 C_T N_C}{(c+C_T)^2}\right) & \left(\frac{c_1\alpha_3 C_T}{c+C_T} + \frac{c_2\alpha_4 W_P}{b+W_P}\right) & \left(\frac{c_2\alpha_4 N_C}{b+W_P} - \frac{c_2\alpha_4 W_P N_C}{(b+W_P)^2}\right) \\ 0 & 0 & 0 \end{bmatrix} \text{and}$$

$$v = \begin{bmatrix} \left(\frac{\alpha_3 N_C}{c+C_T} - \frac{\alpha_3 C_T N_C}{(c+C_T)^2} + \mu_{C_T}\right) & \left(\frac{\alpha_3 C_T}{c+C_T}\right) & 0 \\ 0 & \mu_{N_C} & 0 \\ \left(\alpha_1 W_P + \frac{\alpha_2 W_P}{a+W_P}\right) & \left(\frac{\alpha_4 W_P}{b+W_P}\right) & \left(\begin{matrix} -P + \alpha_1 C_T + \frac{\alpha_2 C_T}{a+W_P} - \frac{\alpha_2 W_P C_T}{(a+W_P)^2} + \frac{\alpha_4 N_C}{b+W_P} \\ -\frac{\alpha_4 W_P N_C}{(b+W_P)^2} + 2\mu_{W_P} W_P \end{matrix}\right) \end{bmatrix}$$

v is a non-singular matrix in this case. Now, calculate the next generation matrix (fv^{-1}) and the model's reproduction number is the greatest modulus of eigenvalues for (fv^{-1}). $R_0 =$

$$\frac{P\left((b(a\alpha_1+\alpha_2)\mu_{N_C}+\alpha_4 ac_2\mu_{C_T})\mu_{W_P}^2+\left(((a+b)\alpha_1+\alpha_2)\mu_{N_C}+\alpha_4 c_2\mu_{C_L}\right)\right)P\mu_{W_P}+P^2\alpha_1\mu_{N_C}}{\mu_{W_P}(\mu_{W_P}a+P)\mu_{C_L}(\mu_{W_P}b+P)\mu_{N_C}}$$

With $R_0 = 0.020$, the reproductive number is less than one. This means each infected pregnant woman transmits, on average, only 2% of neonatal conjunctivitis cases to newborns during her infectious period.

3. Z-CONTROL

In this paper, controlling the Chlamydial infection can be conducted via a Z-control mechanism in pregnant women. It is an effective mathematical technique for disease control. The influence of Chlamydial infection on infants is studied using this mechanism. When using indirect Z-control, the controller is applied to the variable that is being constrained by other variables. Error-based dynamic differential equations are stabilized using this approach. According to this indirect Z-control theory, the preceding model is modified by the addition of the control variable to the Chlamydia trachomatis class, as shown below:

$$\begin{aligned}
\frac{dW_P}{dt} &= PW_P - \alpha_1 W_P C_T - \frac{\alpha_2 W_P C_T}{a+W_P} - \frac{\alpha_4 W_P N_C}{b+W_P} - \mu_{W_P} {W_P}^2 \\
\frac{dC_T}{dt} &= \alpha_1 W_P C_T + \frac{\alpha_2 W_P C_T}{a+W_P} - \frac{\alpha_3 C_T N_C}{c+C_T} - \mu_{C_T} C_T - u(t) C_T \\
\frac{dN_C}{dt} &= \frac{c_1 \alpha_3 C_T N_C}{c+C_T} + \frac{c_2 \alpha_4 W_P N_C}{b+W_P} - \mu_{N_C} N_C
\end{aligned} \tag{2}$$

The indirect control variable in this scenario is $u(t)$. To make a model stable, the chaotic oscillations must be slowed down with Z-control. It finds an expression for input that leads to the required state by causing the error function $e(t)$ goes to zero exponentially. This forces the actual output $y(t)$ to reach the required output $y_d(t)$, which is defined by the error function $e(t) = y(t) - y_d(t) \to 0 ast \to \infty$. If the $e(t)$ satisfies the following differential equation, it tends to zero:

$$\dot{e}(t) = -\lambda e(t) \tag{3}$$

Equation (3) is referred to as the design formula and $\lambda > 0 \in R$ is the design parameter, in which the convergence rate is measured using λ . Our aim is to reduce the rate of Chlamydia infection among pregnant women to the desired state $W_{Pd}(t)$. *i.e.,* $W_P(t) \to W_{Pd}(t)$.. $e(t)$ is defined as the difference between the actual and expected output. $e_1 = W_P(t) - W_{Pd}(t)$. Using the design formula provided by the Z-type dynamic approach, we have $\dot{W}_P(t) - \dot{W}_{Pd}(t) = -\lambda(W_P(t) - W_{Pd}(t))$. Now let's construct another error function as $e_2 = \dot{e}_1 + \lambda e_1$, hence $e_2 = \dot{W}_P(t) - \dot{W}_{Pd}(t) + \lambda(W_P(t) - W_{Pd}(t))$.

Let's assume that error functions e_1 and e_2 decline over time exponentially. i.e., $\dot{e}_1(t) = -\lambda e_1(t)$ and $\dot{e}_2(t) = -\lambda e_2(t)$. Using the design formula and error function, we can define the control variable as follows:

$$\ddot{W}_P(t) - \ddot{W}_{Pd}(t) + \lambda\big(\dot{W}_P(t) - \dot{W}_{Pd}(t)\big) = -\lambda\big[\dot{W}_P(t) - \dot{W}_{Pd}(t) + \lambda(W_P(t)\, W_{Pd}(t))\big]$$

$$\Rightarrow P\dot{W}_P - \alpha_1 \dot{W}_P C_T - \alpha_1 W_P \dot{C}_T - \frac{\alpha_2 \dot{W}_P C_T}{a+W_P} - \frac{\alpha_2 W_P \dot{C}_T}{a+W_P} + \frac{\alpha_2 W_P C_T \dot{W}_P}{(a+W_P)^2} - \frac{\alpha_4 \dot{W}_P N_C}{b+W_P} - \frac{\alpha_4 W_P N_C}{b+W_P} + \frac{\alpha_4 W_P \dot{W}_P N_C}{(b+W_P)^2}$$

$$-2\mu_{W_P}W_P\dot{W}_P - \ddot{W}_{Pd}(t) + \lambda\left(\dot{W}_P(t) - \dot{W}_{Pd}(t)\right) = -\lambda\left[\dot{W}_P(t) - \dot{W}_{Pd}(t) + \lambda(W_P(t) - W_{Pd}(t))\right]$$

$$\Rightarrow \alpha_1 W_P\dot{C}_T + \frac{\alpha_2 W_P\dot{C}_T}{a + W_P}$$

$$= P\dot{W}_P - \alpha_1\dot{W}_P C_T - \frac{\alpha_2\dot{W}_P C_T}{a + W_P} + \frac{\alpha_2 W_P C_T\dot{W}_P}{(a + W_P)^2} - \frac{\alpha_4\dot{W}_P N_C}{b + W_P} - \frac{\alpha_4 W_P N_C}{b + W_P} + \frac{\alpha_4 W_P\dot{W}_P N_C}{(b + W_P)^2}$$

$$-2\mu_{W_P}W_P\dot{W}_P - \ddot{W}_{Pd}(t) + \lambda\left(\dot{W}_P(t) - \dot{W}_{Pd}(t)\right) + \lambda\left[\dot{W}_P(t) - \dot{W}_{Pd}(t) + \lambda(W_P(t) - W_{Pd}(t))\right]$$

$$\dot{C}_T\left(\alpha_1 W_P + \frac{\alpha_2 W_P}{a + W_P}\right) = P\dot{W}_P - \alpha_1\dot{W}_P C_T - \frac{\alpha_2\dot{W}_P C_T}{a + W_P} + \frac{\alpha_2 W_P C_T\dot{W}_P}{(a + W_P)^2} - \frac{\alpha_4\dot{W}_P N_C}{b + W_P} - \frac{\alpha_4 W_P N_C}{b + W_P}$$

$$+\frac{\alpha_4 W_P\dot{W}_P N_C}{(b + W_P)^2} - 2\mu_{W_P}W_P\dot{W}_P - \ddot{W}_{Pd}(t) + 2\lambda\left(\dot{W}_P(t) - \dot{W}_{Pd}(t)\right) + \lambda^2(W_P(t) - W_{Pd}(t)) \Rightarrow \dot{C}_T(t)$$

$$= \frac{(a + W_P)}{W_P(\alpha_1(a + W_P) + \alpha_2)}\begin{bmatrix} P\dot{W}_P - \alpha_1\dot{W}_P C_T - \frac{\alpha_2\dot{W}_P C_T}{a + W_P} + \frac{\alpha_2 W_P C_T\dot{W}_P}{(a + W_P)^2} - \frac{\alpha_4\dot{W}_P N_C}{b + W_P} \\ -\frac{\alpha_4 W_P N_C}{b + W_P} + \frac{\alpha_4 W_P\dot{W}_P N_C}{(b + W_P)^2} - 2\mu_{W_P}W_P\dot{W}_P - \ddot{W}_{Pd}(t) \\ +2\lambda\left(\dot{W}_P(t) - \dot{W}_{Pd}(t)\right) + \lambda^2(W_P(t) - W_{Pd}(t)) \end{bmatrix}$$

We can formulate a Z-type controller by putting in the system:

$$\frac{dC_T}{dt} = \alpha_1 W_P C_T + \frac{\alpha_2 W_P C_T}{a + W_P} - \frac{\alpha_3 C_T N_C}{c + C_T} - \mu_{C_T} C_T - u(t) C_T$$

$$u(t)=\alpha_1 W_P+\frac{\alpha_2 W_P}{a+W_P}-\frac{\alpha_3 N_C}{c+C_T}-\mu_{C_T}-\frac{(a+W_P)}{W_P\left(\alpha_1(a+W_P)+\alpha_2\right)}\left[\begin{array}{l} P\dot{W}_P-\alpha_1\dot{W}_P C_T-\frac{\alpha_2\dot{W}_P C_T}{a+W_P} \\ +\frac{\alpha_2 W_P C_T\dot{W}_P}{\left(a+W_P\right)^2}-\frac{\alpha_4\dot{W}_P N_C}{b+W_P} \\ -\frac{\alpha_4 W_P N_C}{b+W_P}+\frac{\alpha_4 W_P\dot{W}_P N_C}{\left(b+W_P\right)^2} \\ -2\mu_{W_P} W_P\dot{W}_P-\ddot{W}_{Pd}(t) \\ +2\lambda\left(\dot{W}_P(t)-\dot{W}_{Pd}(t)\right) \\ +\lambda^2\left(W_P(t)-W_{Pd}(t)\right) \end{array}\right] \quad (4)$$

Theorem 4.1: When the preceding model is related to a Z-type controller, the tracking error exponentially goes to zero from a positive beginning state $[W_P(0), C_T(0), N_C(0)]^T$ for a continuously differentiable and bounded required state $W_{Pd}(t)$.

Proof: A Lyapunov function is defined as:

$$L=\frac{1}{2}\bar{e}^T\bar{e}=\frac{1}{2}(e_1^2+e_2^2)\geq 0 \quad (5)$$

Where $\bar{e}=[e_1,e_2]^T=[\lambda(W_P(t)-W_{Pd}(t)),\dot{W}_P(t)-\dot{W}_{Pd}(t)+\lambda(W_P(t)-W_{Pd}(t))]^T$ is the controller-equipped modified model's error vector.

Given that $L>0$for $e\neq 0$and that $L=0$, only in the circumstance that L is positive definite and $\bar{e}=0$ (i.e.,$e_1=0 and e_2=0$). The derivative over time is given by, $L'=\frac{dL}{dt}=e_1\dot{e}_1+e_2\dot{e}_2=-\lambda(e_1^2+e_2^2)\leq 0.$ (6)

L' is negatively definite since $L'<0$ exists if and only if $e\neq 0$. If $e\to\infty$, then function $L\to\infty$.

The Lyapunov theory of stability says that the error vector will eventually reach zero. Hence, the error function is equivalent to

$L'=-\lambda(e_1^2+e_2^2)=-\lambda e^T e=-2\lambda L.$

Thus, the error vector exponentially converges to zero and $L=L(0)exp(-2\lambda L),\forall t\geq 0,\lambda$, is the rate of convergence. Thus, a Z-control mechanism can get the system to the desired state at the optimum rate by causing the error function to converge to zero.

4. NUMERICAL SIMULATION

In this section, we simulate the results using MATLAB. A numerical simulation has been analysed to demonstrate how the model can produce varying outcomes based on the observations that are used. The simulation's parameter values are listed below.

Table 1

Table 1 Parameters of the Model

Parameter	P	α_1	α_2	α_3	α_4	a	b	c	C_1	C_2	μ_{W_P}	μ_{C_T}	μ_{N_C}

Value	0.018	0.06	0.02	0.01	0.01	0.1	10	0.1	1	10	0.01	0.03	0.01

Figure 2

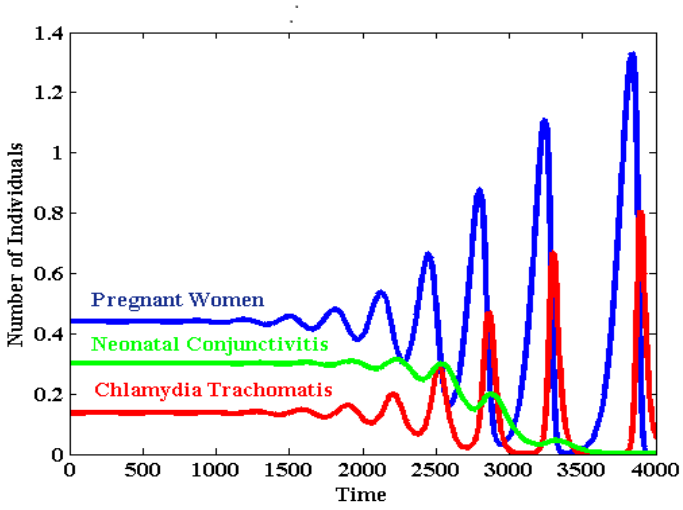

Figure 2 The Transmission Rate in the Compartments

The transmission rate in compartments is seen in Figure 2. It may be established that infected pregnant women initially do not show symptoms and do not seek medical attention. So, initially, the condition is steady, but as it worsens, it causes all compartments to oscillate predictably. For prevention and control of this condition, newborn screening and treatment may be considered. As a result, the number of newborns with Chlamydial conjunctivitis is going down, as shown in the figure.

Figure 3

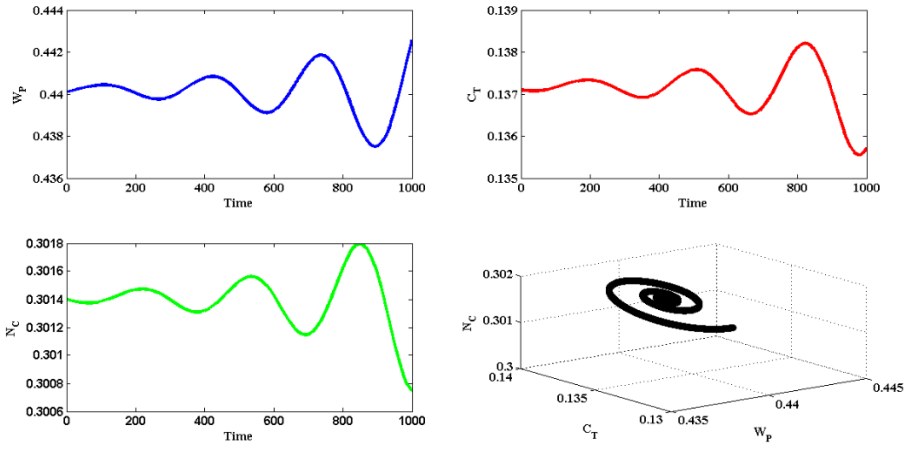

Figure 3 (a). Oscillation in Compartments of the Model

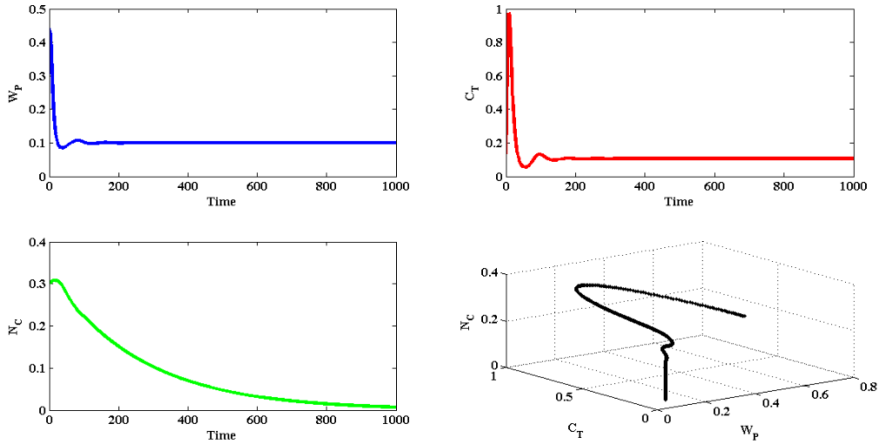

Figure 3 (b). Z-control implementation in the model

Figure 3 (a) depicts the compartments' cyclical nature. This shows that pregnant women always have the risk of Chlamydia infection during pregnancy and can have children who have neonatal conjunctivitis as a result of the infection. Each figure indicates periodic phenomena relative to all compartments, and it demonstrates that, with time, the frequency of periodic oscillations at the endemic equilibrium point increases. Observing a scatter diagram for each compartment reveals the compartment's periodic nature. It is suggested that oscillations in the compartments stop when Z-type control is applied. Figure 3 (b) depicts a dynamic system that is stable and controllable, demonstrating that Chlamydial infection in pregnant women can be avoided and controlled. It will help cure the infection and stop complications of neonatal Chlamydial conjunctivitis, which affect a newborn's vision.

Figure 4

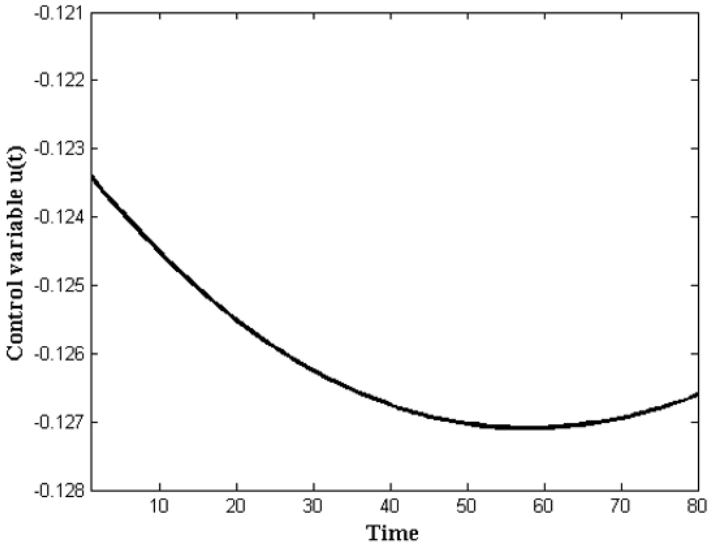

Figure 4 Behaviour of Control Variable

Figure 5

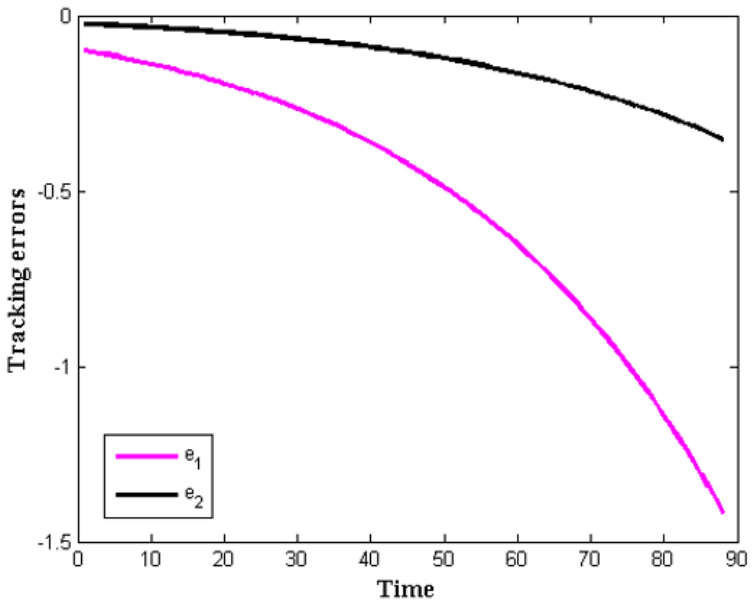

Figure 5 Function of Error

A Z-control is applied to the model, and the resulting change in the control variable is shown in Figure 4. Whereas Figure 5 illustrates the variation of tracking errors utilized in the design formula. The graph demonstrates the fact that the influence of error functions is effective in stabilizing the system.

5. CONCLUSION

To slow the spread of infectious diseases, it is necessary to employ a variety of innovative and effective control techniques. The goal of the various disease control strategies, which vary based on the disease mechanism of transmission, is to reduce present disease prevalence and prevent future outbreaks. The model is constructed as a nonlinear system of differential equations with Z-control to capture the dynamics of STDs. Using numerical simulation, we can determine the conditions at which hypersensitive people require medical intervention. The most effective strategy for avoiding chlamydial infection in newborns is prenatal testing and treatment of expecting mothers. The simulation also hints at the importance of periodicity in determining infection rates and chaotic behavior. When it comes to controlling periodic oscillations in a model, we have found that Z-control works well. The least amount of therapy necessary to achieve a disease-free system was determined by using Z-control. The Z-control is a good way to deal with this model of an epidemic.

DISCLOSURE OF FUNDING

The second author is funded by the SHODH (Scheme of Developing High-Quality Research) education department, Gujarat state. [Student AisheCode: 2021013856]

DATA AVAILABILITY STATEMENT

The datasets utilized in this paper comprise a combination of obtained, assumed, calculated.

Due to the diverse nature of data sources and methods used in this study, the raw datasets may not be directly accessible or downloadable.

6. AUTHOR CONTRIBUTION

1Nita H Shah – Supervision, Formal Analysis

2Jalpa N Vaghela – Writing (Original Draft), writing (review & editing), Methodology, Modelling,

3Ekta Jayswal –Conceptualization

CONFLICT OF INTERESTS

None.

ACKNOWLEDGMENTS

All authors are thankful to DST-FIST file # MSI-097 for technical support to the department of Gujarat University.

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