CROWDING DISTANCE BASED PARTICLE SWARM OPTIMIZATION ALGORITHM FOR SOLVING OPTIMAL REACTIVE POWER DISPATCH PROBLEM

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Abstract

In this paper, Crowding Distance based Particle Swarm Optimization (CDPSO) algorithm has been proposed to solve the optimal reactive power dispatch problem. Particle Swarm Optimization (PSO) is swarm intelligence-based exploration and optimization algorithm which is used to solve global optimization problems. In PSO, the population is referred as a swarm and the individuals are called particles. Like other evolutionary algorithms, PSO performs searches using a population of individuals that are updated from iteration to iteration. The crowding distance is introduced as the index to judge the distance between the particle and the adjacent particle, and it reflects the congestion degree of no dominated solutions. In the population, the larger the crowding distance, the sparser and more uniform. In the feasible solution space, we uniformly and randomly initialize the particle swarms and select the no dominated solution particles consisting of the elite set. After that by the methods of congestion degree choosing (the congestion degree can make the particles distribution more sparse) and the dynamic \( \varepsilon \) infeasibility dominating the constraints, we remove the no dominated particles in the elite set. Then, the objectives can be approximated. Proposed crowding distance based Particle Swarm Optimization (CDPSO) algorithm has been tested in standard IEEE 30 bus test system and simulation results shows clearly the improved performance of the projected algorithm in reducing the real power loss and static voltage stability margin has been enhanced.

Keywords: Optimal Reactive Power; Transmission Loss; Crowding Distance Based Particle Swarm Optimization.


1. Introduction

Reactive power optimization plays a key role in optimal operation of power systems. Many numerical methods [1-7] have been applied to solve the optimal reactive power dispatch problem.
The problem of voltage stability plays a strategic role in power system planning and operation [8]. So many Evolutionary algorithms have been already proposed to solve the reactive power flow problem [9-11]. In [12, 13], Hybrid differential evolution algorithm and Biogeography Based algorithm has been projected to solve the reactive power dispatch problem. In [14, 15], a fuzzy based technique and improved evolutionary programming has been applied to solve the optimal reactive power dispatch problem. In [16, 17] nonlinear interior point method and pattern based algorithm has been used to solve the reactive power problem. In [18-20], various types of probabilistic algorithms utilized to solve optimal reactive power problem. In this paper, Crowding Distance based Particle Swarm Optimization (CDPSO) algorithm has been proposed to solve the optimal reactive power dispatch problem. Particle Swarm Optimization (PSO) [21] has been used efficaciously in solving many optimization problems, for its simplicity and fast convergence rate. Swarm intelligence is the subdivision of artificial intelligence and based on collective behaviour of self-organized system [22-31]. The crowding distance is introduced as the index to judge the distance between the particle and the adjacent particle, and it reflects the congestion degree of no dominated solutions. In the population, the larger the crowding distance, the sparser and more uniform. In the feasible solution space, we uniformly and randomly initialize the particle swarms and select the no dominated solution particles consisting of the elite set. After that by the methods of congestion degree choosing (the congestion degree can make the particles distribution more sparse) and the dynamic $\epsilon$ infeasibility dominating the constraints, we remove the no dominated particles in the elite set. Then, the objectives can be approximated. Proposed crowding distance based Particle Swarm Optimization (CDPSO) algorithm has been tested in standard IEEE 30 bus test system and simulation results shows clearly the improved performance of the projected algorithm in reducing the real power loss and static voltage stability margin has been enhanced.

2. Voltage Stability Evaluation

2.1. Modal Analysis for Voltage Stability Evaluation

Modal analysis is one among best methods for voltage stability enhancement in power systems. The steady state system power flow equations are given by.

$$
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
J_{p\theta} & J_{pV} \\
J_{q\theta} & J_{QV}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix}
$$

(1)

Where

$\Delta P =$ Incremental change in bus real power.

$\Delta Q =$ Incremental change in bus reactive Power injection

$\Delta \theta =$ incremental change in bus voltage angle.

$\Delta V =$ Incremental change in bus voltage Magnitude

$J_{p\theta} , J_{PV} , J_{Q\theta} , J_{QV}$ jacobian matrix are the sub-matrixes of the System voltage stability is affected by both P and Q.

To reduce (1), let $\Delta P = 0$, then.

$$
\Delta Q = (J_{QV} - J_{Q\theta})P_0^{-1}J_{pV}\Delta V = J_R\Delta V
$$

(2)
\[ \Delta V = J^{-1} - \Delta Q \]  

(3)

Where

\[ J_R = (J_{QV} - J_{Q}P^{-1}J_{PV}) \]  

(4)

\( J_R \) is called the reduced Jacobian matrix of the system.

2.2. Modes of Voltage Instability

Voltage Stability characteristics of the system have been identified by computing the Eigen values and Eigen vectors.

Let

\[ J_R = \xi \wedge \eta \]  

(5)

Where,

\( \xi \) = right eigenvector matrix of \( J_R \)
\( \eta \) = left eigenvector matrix of \( J_R \)
\( \wedge \) = diagonal eigenvalue matrix of \( J_R \) and

\[ J_{R^{-1}} = \xi \wedge^{-1} \eta \]  

(6)

From (5) and (8), we have

\[ \Delta V = \xi \wedge^{-1} \eta \Delta Q \]  

(7)

Or

\[ \Delta V = \sum \frac{\xi_i \eta_i}{\lambda_i} \Delta Q \]  

(8)

Where \( \xi_i \) is the ith column right eigenvector and \( \eta \) the ith row left eigenvector of \( J_R \). \( \lambda_i \) is the ith Eigen value of \( J_R \).

The ith modal reactive power variation is,

\[ \Delta Q_{mi} = K_i \xi_i \]  

(9)

Where,

\[ K_i = \sum \xi_{ij}^2 - 1 \]  

(10)

Where \( \xi_{ji} \) is the jth element of \( \xi_i \)
The corresponding ith modal voltage variation is

\[ \Delta V_{mi} = \frac{1}{\lambda_i} \Delta Q_{mi} \]  

(11)

If \( |\lambda_i| = 0 \) then the ith modal voltage will collapse.

In (10), let \( \Delta Q = e_k \) where \( e_k \) has all its elements zero except the kth one being 1. Then,

\[ \Delta V = \sum_i \frac{\eta_{1k} \xi_1}{\lambda_1} \]  

(12)

\( \eta_{1k} \) kth element of \( \eta_1 \)

\( V-Q \) sensitivity at bus k

\[ \frac{\partial V_k}{\partial Q_k} = \sum_i \frac{\eta_{1k} \xi_1}{\lambda_1} = \sum_i \frac{P_{ki}}{\lambda_1} \]  

(13)

3. Problem Formulation

The objectives of the reactive power dispatch problem is to minimize the system real power loss and maximize the static voltage stability margins (SVSM).

3.1. Minimization of Real Power Loss

Minimization of the real power loss (Ploss) in transmission lines is mathematically stated as follows.

\[ P_{\text{loss}} = \sum_{k=1}^{n} g_k (V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}) \]  

(14)

Where \( n \) is the number of transmission lines, \( g_k \) is the conductance of branch \( k \), \( V_i \) and \( V_j \) are voltage magnitude at bus \( i \) and bus \( j \), and \( \theta_{ij} \) is the voltage angle difference between bus \( i \) and bus \( j \).

3.2. Minimization of Voltage Deviation

Minimization of the voltage deviation magnitudes (VD) at load buses is mathematically stated as follows.

Minimize \( VD = \sum_{k=1}^{nl} |V_k - 1.0| \)  

(15)

Where \( nl \) is the number of load busses and \( V_k \) is the voltage magnitude at bus \( k \).

3.3. System Constraints

Objective functions are subjected to these constraints shown below.

Load flow equality constraints:
\[ P_{Gi} - P_{Di} - V_i \sum_{j=1}^{nb} v_j \left[ G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right] = 0, i = 1, 2, ..., nb \]  

\[ Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{nb} v_j \left[ G_{ij} \sin \theta_{ij} + B_{ij} \cos \theta_{ij} \right] = 0, i = 1, 2, ..., nb \]  

where, \( nb \) is the number of buses, \( PG \) and \( QG \) are the real and reactive power of the generator, \( PD \) and \( QD \) are the real and reactive load of the generator, and \( Gij \) and \( Bij \) are the mutual conductance and susceptance between bus \( i \) and bus \( j \).

Generator bus voltage (\( VGi \)) inequality constraint:

\[ V_{Gi}^{\text{min}} \leq V_{Gi} \leq V_{Gi}^{\text{max}}, i \in ng \]  

Load bus voltage (\( VLi \)) inequality constraint:

\[ V_{Li}^{\text{min}} \leq V_{Li} \leq V_{Li}^{\text{max}}, i \in nl \]  

Switchable reactive power compensations (\( QCi \)) inequality constraint:

\[ Q_{Ci}^{\text{min}} \leq Q_{Ci} \leq Q_{Ci}^{\text{max}}, i \in nc \]  

Reactive power generation (\( QGi \)) inequality constraint:

\[ Q_{Gi}^{\text{min}} \leq Q_{Gi} \leq Q_{Gi}^{\text{max}}, i \in ng \]  

Transformers tap setting (\( Ti \)) inequality constraint:

\[ T_{i}^{\text{min}} \leq T_{i} \leq T_{i}^{\text{max}}, i \in nt \]  

Transmission line flow (\( SLi \)) inequality constraint:

\[ S_{Li}^{\text{min}} \leq S_{Li} \leq S_{Li}^{\text{max}}, i \in nl \]  

Where, \( nc, ng \) and \( nt \) are numbers of the switchable reactive power sources, generators and transformers.

### 4. Particle Swarm Optimization (PSO)

PSO is a population based optimization tool, where the system is initialized with a population of random particles and the algorithm searches for optima by updating generations. Suppose that the search space is \( D \)-dimensional. The position of the \( i \)-th particle can be represented by a \( D \)-dimensional vector \( X_i = (x_{i1}, x_{i2}, ..., x_{iD}) \) and the velocity of this particle is \( V_i = (v_{i1}, v_{i2}, ..., v_{iD}) \). The best previously visited position of the \( i \)-th particle is represented by \( P_i = (p_{i1}, p_{i2}, ..., p_{iD}) \) and the global best position of the swarm found so far is denoted by \( P_g = (p_{g1}, p_{g2}, ..., p_{gD}) \).
\((p_{g1}, p_{g2}, \ldots, p_{gD})\). The fitness of each particle can be evaluated through putting its position into a designated objective function. The particle's velocity and its new position are updated as follows:

\[
v_{id}^{t+1} = \omega v_{id}^t + c_1 r_1^t (p_{id}^t - x_{id}^t) + c_2 r_2^t (p_{gd} - x_{id}^t) \tag{24}
\]

\[
x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1} \tag{25}
\]

Where \(d \in \{1,2,\ldots,D\}, i \in \{1,2,\ldots,N\}\) \(N\) is the population size, the superscript \(t\) denotes the iteration number, \(\omega\) is the inertia weight, \(r_1\) and \(r_2\) are two random values in the range \([0,1]\), \(c_1\) and \(c_2\) are the cognitive and social scaling parameters which are positive constants. These both equations are used to update the velocity and position of a particle in the exploration space. The equation (24) is used to balance the search abilities of the particle in the search space. The equation (25) uses the velocity obtained in first equation to get the new position of the particle.

5. Proposed Crowding Distance based Particle Swarm Optimization (CDPSO) Algorithm

Let \(W_{d}^{next}\) next \(wd\) and \(W_{d}^{last}\) last \(wd\) be the next and the last adjacent particles of the \(d\)-th particle \(W_{d}\), respectively. The crowding distance of the \(d\)-th individual particle \(W_{d}\) is defined as,

\[
\sigma_d = \sum_{m=1}^{p} \frac{F_m(W_{d}^{next}) - F_m(W_{d}^{last})}{f_{m}^{max} - f_{m}^{min}} \tag{26}
\]

where \(p\) is the number of the objectives, \(F_m(W_{d}^{next}) - F_m(W_{d}^{last})\) and \(F_m(W_{d})\) are respectively the fitness values of \(m\)-th objective, \(f_{m}^{max} - f_{m}^{min}\) are the maximum and minimum values of the \(m\)-th objective, respectively.

The crowding distance is introduced as the index to judge the distance between the particle and the adjacent particle, and it reflects the congestion degree of no dominated solutions. In the population, the larger the crowding distance, the sparser and more uniform.

The infeasibility threshold \(\eta\) is defined as

\[
\eta = \begin{cases} 
\eta_0 (1 - 5t/4N), & t \leq 0.8N \\
0, & t > 0.8N 
\end{cases} \tag{27}
\]

Where \(\eta_0\) is an initial value allowed by constraint violation degree, \(t\) is the current evolution generation, and \(N\) is the maximal evolution generation.

The degree of the individual particle violating the constraints is defined as,

\[
C(W_{d}) = \sum_{r=1}^{d} \max (H_{5r}(W_{d}), 0) + \sum_{j=1}^{m} \max (|H_{1j}(W_{d})| - \delta, 0) + \sum_{r=1}^{l} \max (|H_{3r}(W_{d}) - H_{2r}(W_{d}) - \sqrt{H_{2r}^2(W_{d}) + H_{3r}^2(W_{d}) + \epsilon}| - \delta, 0) + \max (|H_{4}(W_{d})| - \delta, 0) \tag{28}
\]
where $\delta$ is the tolerance of the equation constraints, which reflects the degree of the strictness on the equation constraints. In the feasible solution space, we uniformly and randomly initialize the particle swarms and select the non-dominated solution particles consisting of the elite set. After that by the methods of congestion degree choosing (the congestion degree can make the particles distribution more sparse) and the dynamic $\epsilon$ infeasibility dominating the constraints, we remove the non-dominated particles in the elite set. Then, the objectives can be approximated.

**Step 1.** Give the particle population size $M$ (including the position $x$ and the velocity $v$) and the maximal evolution generation $N$. Select the initial infeasibility threshold $\eta_0$, and let $t=0$.

**Step 2.** Update each particle in the particle group:

- **Step 2.1.** Archive the non-dominated solutions of the particle swarm in the external elite set and calculate the congestion distance and the degree of the individual particle violating the constraints $C(w)$ on each non-dominated solution in the external elite set. The distances are made in descending order. Then randomly select one particle as the global optimal position $P_g$ from the archived elite set.

- **Step 2.2.** Update the velocity and position of the particle. If the position of a particle exceeds the preset boundary, the position of the particle is equal to its boundary value and its velocity is multiplied by $-1$ to search the particle in the opposite direction.

**Step 3.** Update the external elitist set: Compare the updated non-dominated solutions of the particle swarm with the non-dominated solutions in the external elites, and decide whether the non-dominated solutions in the particle swarm should be archived in the external elite set. If the solution in the particle swarm satisfies the domination relation, it needs to judge whether the external elitist set is full: if it is not full, the non-dominated solution is archived directly; otherwise, the following steps are adopted:

- **Step 3.1.** Archive all non-dominated solutions of the external elite set in descending order according to the congestion distance.

- **Step 3.2.** Randomly pick a particle in the $M$ particles of the sorted set and replace it with the particle that needs to be archived.

**Step 4.** Update the local optimal position of the particle:

- **Step 4.1.** Update the global optimum position if the position of the particle updated dominates its historical optimal position.

- **Step 4.2.** If the updated particle position does not dominate its historical optimum position, according to 50% chance to retain its best position in history. When the degree of all the particles in the non-dominated set violating the constraints $C(w)$ is zero, the algorithm terminates and we get the approximate Pareto optimal solutions $w^*$ and the values $F(w^*)$. Otherwise, go to Step 5.

**Step 5.** If $t \geq N$ we get the approximate Pareto optimal solutions $w^*$ and the values $F(w^*)$. Otherwise, set $t = t + 1$, and go to Step 2.

### 6. Simulation Results

The efficiency of the proposed Crowding Distance based Particle Swarm Optimization (CDPSO) algorithm is demonstrated by testing it on standard IEEE-30 bus system. The IEEE-30 bus system has 6 generator buses, 24 load buses and 41 transmission lines of which four branches are (6-9), (6-10), (4-12) and (28-27) - are with the tap setting transformers. The lower voltage magnitude limits at all buses are 0.95 p.u. and the upper limits are 1.1 for all the PV buses and 1.05 p.u. for all the PQ buses and the reference bus. The simulation results have been presented in Tables 1, 2,
And in the Table 5 shows the proposed algorithm powerfully reduces the real power losses when compared to other given algorithms. The optimal values of the control variables along with the minimum loss obtained are given in Table 1. Corresponding to this control variable setting, it was found that there are no limit violations in any of the state variables.

<table>
<thead>
<tr>
<th>Control variables</th>
<th>Variable setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>1.042</td>
</tr>
<tr>
<td>V2</td>
<td>1.045</td>
</tr>
<tr>
<td>V5</td>
<td>1.041</td>
</tr>
<tr>
<td>V8</td>
<td>1.031</td>
</tr>
<tr>
<td>V11</td>
<td>1.000</td>
</tr>
<tr>
<td>V13</td>
<td>1.030</td>
</tr>
<tr>
<td>T11</td>
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</tr>
<tr>
<td>T12</td>
<td>1.00</td>
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<tr>
<td>T15</td>
<td>1.00</td>
</tr>
<tr>
<td>T36</td>
<td>1.00</td>
</tr>
<tr>
<td>Qc10</td>
<td>2</td>
</tr>
<tr>
<td>Qc12</td>
<td>2</td>
</tr>
<tr>
<td>Qc15</td>
<td>2</td>
</tr>
<tr>
<td>Qc17</td>
<td>0</td>
</tr>
<tr>
<td>Qc20</td>
<td>2</td>
</tr>
<tr>
<td>Qc23</td>
<td>3</td>
</tr>
<tr>
<td>Qc24</td>
<td>3</td>
</tr>
<tr>
<td>Qc29</td>
<td>2</td>
</tr>
<tr>
<td>Real power loss</td>
<td>4.1034</td>
</tr>
<tr>
<td>SVSM</td>
<td>0.2476</td>
</tr>
</tbody>
</table>

Optimal Reactive Power Dispatch problem (ORPD) together with voltage stability constraint problem was handled in this case as a multi-objective optimization problem where both power loss and maximum voltage stability margin of the system were optimized simultaneously. Table 2 indicates the optimal values of these control variables. Also it is found that there are no limit violations of the state variables. It indicates the voltage stability index has increased from 0.2476 to 0.2488, an advance in the system voltage stability. To determine the voltage security of the system, contingency analysis was conducted using the control variable setting obtained in case 1 and case 2. The Eigen values equivalents to the four critical contingencies are given in Table 3. From this result it is observed that the Eigen value has been improved considerably for all contingencies in the second case.

<table>
<thead>
<tr>
<th>Control Variables</th>
<th>Variable Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>1.047</td>
</tr>
<tr>
<td>V2</td>
<td>1.048</td>
</tr>
<tr>
<td>V5</td>
<td>1.042</td>
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</table>
Table 3: Voltage Stability under Contingency State

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Contingency</th>
<th>ORPD Setting</th>
<th>VSCRPD Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28-27</td>
<td>0.1419</td>
<td>0.1434</td>
</tr>
<tr>
<td>2</td>
<td>4-12</td>
<td>0.1642</td>
<td>0.1650</td>
</tr>
<tr>
<td>3</td>
<td>1-3</td>
<td>0.1761</td>
<td>0.1772</td>
</tr>
<tr>
<td>4</td>
<td>2-4</td>
<td>0.2022</td>
<td>0.2043</td>
</tr>
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</table>

Table 4: Limit Violation Checking Of State Variables

<table>
<thead>
<tr>
<th>State variables</th>
<th>limits</th>
<th>ORPD</th>
<th>VSCRPD</th>
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<tbody>
<tr>
<td></td>
<td>Lower</td>
<td>upper</td>
<td></td>
</tr>
<tr>
<td>Q1</td>
<td>-20</td>
<td>152</td>
<td>1.3422</td>
</tr>
<tr>
<td>Q2</td>
<td>-20</td>
<td>61</td>
<td>8.9900</td>
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<tr>
<td>Q5</td>
<td>-15</td>
<td>49.92</td>
<td>25.920</td>
</tr>
<tr>
<td>Q8</td>
<td>-10</td>
<td>63.52</td>
<td>38.820</td>
</tr>
<tr>
<td>Q11</td>
<td>-15</td>
<td>42</td>
<td>2.9300</td>
</tr>
<tr>
<td>Q13</td>
<td>-15</td>
<td>48</td>
<td>8.1025</td>
</tr>
<tr>
<td>V3</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0372</td>
</tr>
<tr>
<td>V4</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0307</td>
</tr>
<tr>
<td>V6</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0282</td>
</tr>
<tr>
<td>V7</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0101</td>
</tr>
<tr>
<td>V9</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0462</td>
</tr>
<tr>
<td>V10</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0482</td>
</tr>
<tr>
<td>V12</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0400</td>
</tr>
<tr>
<td>V14</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0474</td>
</tr>
<tr>
<td>V15</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0457</td>
</tr>
<tr>
<td>V16</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0426</td>
</tr>
<tr>
<td>V17</td>
<td>0.95</td>
<td>1.05</td>
<td>1.0382</td>
</tr>
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</table>

Real power loss 4.9896
SVSM 0.2488
Table 5: Comparison of Real Power Loss

<table>
<thead>
<tr>
<th>Method</th>
<th>Minimum loss</th>
</tr>
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<tbody>
<tr>
<td>Evolutionary programming [32]</td>
<td>5.0159</td>
</tr>
<tr>
<td>Genetic algorithm [33]</td>
<td>4.665</td>
</tr>
<tr>
<td>Real coded GA with Lindex as SVSM [34]</td>
<td>4.568</td>
</tr>
<tr>
<td>Real coded genetic algorithm [35]</td>
<td>4.5015</td>
</tr>
<tr>
<td>Proposed CDPSO method</td>
<td>4.1034</td>
</tr>
</tbody>
</table>

7. Conclusion

Crowding Distance based Particle Swarm Optimization (CDPSO) algorithm has been successfully solved optimal reactive power dispatch problem. The crowding distance is introduced as the index to judge the distance between the particle and the adjacent particle, and it reflects the congestion degree of no dominated solutions. In the population, the larger the crowding distance, the sparser and more uniform. In the feasible solution space, we uniformly and randomly initialize the particle swarms and select the no dominated solution particles consisting of the elite set. After that by the methods of congestion degree choosing (the congestion degree can make the particles distribution more sparse) and the dynamic ε infeasibility dominating the constraints, we remove the no dominated particles in the elite set. Then, the objectives can be approximated. Proposed crowding distance based Particle Swarm Optimization (CDPSO) algorithm has been tested in standard IEEE 30 bus test system and simulation results shows clearly the improved performance of the projected algorithm in reducing the real power loss and static voltage stability margin has been enhanced.

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