WHAT WILL HAPPEN TO NUMBER OF THE INFECTED IF ACCURACY OF THE PCR TEST IMPROVES?

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ABSTRACT

Among the strategies to tackle the COVID-19, much attention is paid to “test and isolation” advocated by Romer (2000), Peto et al (2020) etc., in addition to vaccination and development of medicines to treat COVID-19. According to these articles, what is necessary is a targeted version of lockdown, that is, to test everyone regularly and isolate the small fraction of the population who test positive. There is, however, concern that the PCR test is not always effective, so that, in the present paper, we investigate the effects of improvement of the PCR test by constructing a simple intertemporal theoretical model. Main result we obtain is that improvement of the PCR test could increase the number of the infected individuals at first, so that we should improve the accuracy of the PCR test much enough to reduce the number of the infected individuals.

1. INTRODUCTION

Among the strategies to tackle the COVID-19, much attention is paid to “test and isolation” advocated by Romer (2020), Peto et al. (2020) etc., in addition to vaccination and development of medicines to treat COVID-19. According to these articles, what is necessary is a targeted version of lockdown, that is, to test everyone regularly and isolate the small fraction of the population who test positive.

There is, however, concern that the PCR test is not always effective, so that, in the present paper, we investigate the effects of improvement of the PCR test by constructing a simple intertemporal theoretical model, extending theoretical models such as Fujita (2020a), Fujita (2020b), Fujita (2020c), Fujita (2021a), Fujita (2021b). The present paper is also inspired by numerous empirical studies, which revealed the impacts of the COVID-19 on various economic aspects such as consumption, stock market, uncertainty, tax policy, leadership styles and so on Baker et al. (2020), Baker et al. (2020), Baker et al. (2020), Watanabe (2020), Bandara and Weerasooriya (2021), Ramadhanti and Kularajasingham (2021). In the present paper, we investigate what will happen to number of the infected if accuracy of the PCR test improves. Different from Fujita (2021a), which analyzed the consequence of prevention measures such as vaccination, and Fujita (2021b), which focused on the treatment of the patients who had positive PCR
test, the present paper focuses on the improvement of the PCR test by incorporating the antibodies.

Structure of this paper is as follows. Section 2 lays out the basic model and section 3 derives the number of the infected individuals in the steady state. Concluding remarks are made in section 4.

2. BASIC MODEL

Let us consider an intertemporal economy where time passes continuously and the time horizon is infinite. The economy consists of $N (>0)$ individuals, of whom $x(t)$ individuals in period $t$ are infected with the coronavirus but do not present symptoms. We assume that every individual takes the PCR test in every period, which is incomplete in a sense that $p \times 100\%$ of the infected individuals $x(t)$, i.e., $px(t)$, are falsely judged to be negative. We also assume that those who tested positive get treatments and are allowed to go out for work, shopping etc., but can be infected again in the same period.

As a characteristic of the present paper, we assume that, in period $t$, among those who are not infected, $\theta px(t)$ individuals develop antibodies, where $\theta$ is a positive constant, meaning the more the infected individuals, the more individuals develop antibodies.

By assuming each of those who are infected but judged to be negative transmits the virus to $\alpha$ individuals in period $t$, of whom a fraction of $1-(1+\theta)px(t)/N$ is newly infected since $px(t)/N$ is infected and $\theta px(t)/N$ has developed antibody already, we can express the motion of number of the infected individuals as:

$$x(t+1) = px(t) + px(t)\alpha \left(1 - \frac{(1+\theta)px(t)}{N}\right). \quad (1)$$

3. RESULTS AND DISCUSSIONS

Letting $x^*$ denote the steady state number of the infected individuals and substituting $x(t+1) = x(t) = x^*$ into equation (1), we have the condition $x^*$ should satisfy as

$$x^* = px^*[1 + \alpha \left(1 - \frac{(1+\theta)px^*}{N}\right)], \quad (2)$$

and by solving $x^*$ with respect to (2), we obtain

$$x^* = \frac{((1+\alpha)p-1)N}{\alpha(1+\theta)p^2}. \quad (3)$$

Since $x^*$, the steady state number of the infected people, should be less than $N-\theta px^*$, steady state number of those who can be infected, we have the condition $x^*$ should satisfy as

$$x^* \leq \frac{N}{1+\theta p}, \quad (4)$$
If (4) is satisfied, the following inequality is also satisfied since \( p \), the failure probability, is less than 1:

\[
x^* \leq \frac{N}{(1+\theta)p'}
\]  

(5)

which is derived by the condition that \( 1 - \frac{(1+\theta)px^*}{N} \) in equation (2) should be nonnegative.

Considering (3)-(5), we can draw a relationship between \( p \) and \( x^* \) typically as a graph, which is the combination of (3), curve-segment AB, and \( x^* = \frac{N}{1+\theta p} \), which is the part of (4)), curve-segment BC, as in Figure 1 on the \( p-x^* \) plane.

**Figure 1** Relationship between failure probability and steady state number of the infected

**Proposition:**

The steady state number of the infected people, \( x^* \), increases if the failure probability of PCR test decreases from 1

From this proposition, we can say that we should improve the accuracy of the PCR test much enough to reduce the number of the infected individuals since the improvement of the PCR test increases the number of the infected individuals at first.

**4. CONCLUSIONS**

The present study theoretically investigated the effects of improvement of the PCR test, to obtain the result that accuracy of the PCR test should be improved much enough to reduce the number of the infected individuals since the improvement of the PCR test increases the number of the infected individuals at first.

It is necessary to examine the robustness of our results in a more general framework that incorporates the economic activities. It is also of interest to investigate the compound effects of the PCR test, the vaccination and the treatment of COVID-19. We will conduct such analyses in the next paper.
REFERENCES


