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COMPUTATIONAL ANALYSIS OF BUTTERWORTH AND CHEBYSHEV-I FILTERS USING BILINEAR TRANSFORMATION

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ABSTRACT

Due to the intense demands in advanced telecommunications during the last fifteen years for both higher spectrum band and better accuracy, the digital Infinite Impulse Response (IIR) filter has emerged as the basic component in both digital telecommunication and Digital Signal Processing (DSP) systems. In our research work reported in this paper we conducted meticulous investigation using computer simulation of the digital Infinite Impulse Response (IIR) filter to implement the Butterworth and Chebyshev I procedure with bilinear transformation algorithm aimed at both statistical analysis and computer simulation. Our simulation results reveal the comparative accuracy between digital filters and analog filters of the spectrum response in i) absolute magnitude, ii) the magnitude in decibels (dB) and iii) phase. Conversely the filter selectivity and gain in decibel scale were numerically obtained.

Keywords: Butterworth Analogue and Digital Filters, Chebyshev I Analogue and Digital Filters, Infinite Impulse Response (IIR) Filter, Filter Selectivity, Bilinear Transformation (BLT)

1. INTRODUCTION

Digital signal processing (DSP) experienced phenomenal advances in both research and application in the past few decades due to progresses in digital computer technology and software development. Digital filter is one of the most important and frequently used elements of DSP. It is a frequency selective device which extract the useful part of the input signal within its operating frequency range Mitra (2001) Aggarwal et al. (2015) Infinite impulse response (IIR) -recursive and

Finite impulse response (FIR) -non -recursive are the two broad classes of digital filters. IIR tends to be ideal at lower filter order (less number of coefficients) make it preferable to FIR. Conversely, non-recursive algorithm of FIR filter has greater filter order as compared to IIR filter.

However, diverse methods exist for the design of digital filters. Mostly, five methods are used to design IIR digital filters viz: Bilinear transformation method, Impulse-invariance method, Backward difference method, Step-invariance methods, and Matched-z-transformation Madisetti and Williams (1999) Bilinear transformation method is used in this research for its simplicity and similarity to analogue filter.

The design of IIR digital filters with Butterworth and Chebyshev I filter responses, using MATLAB software are based on the concepts of bilinear transformation and analog filters. So, they are universally used to approximate the piecewise constant magnitude characteristic of ideal LP, HP, BP, and BS filters Kou et al. (2006)

A desired design of IIR filter can be achieved with the support of specifications: the passband frequency (f_p), stopband frequency (f_s), maximum allowable passband ripple (R_p), minimum allowable stopband ripple (R_s) and bandwidth (Bw).

However, the design of IIR filters is more difficult than FIR filters design for their rational transfer functions. Moreover, it is necessary to consider the stability of the filters and linear phase design may be achievable Saini and Kaur (2015)

The necessary algorithm use in converting Analogue to digital filter is given Natarajan (2017)

Figure 1

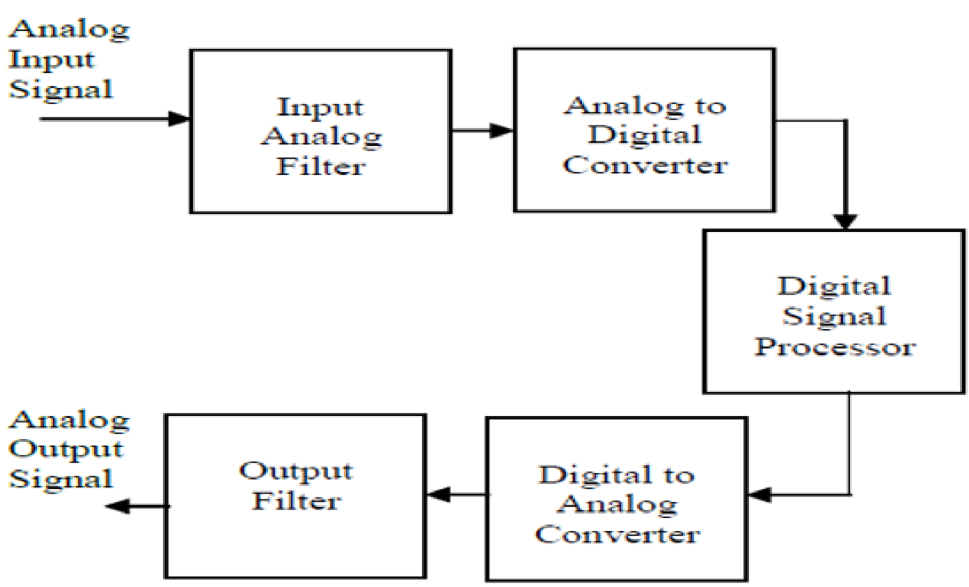

Figure 1 Block diagram of digital signal processing system

2. METHODOLOGY

The Bilinear Transform method applied in designing frequency selective filters yields very efficient results as to other mapping methods listed above for designing IIR digital filters. Digital filters resulting from the bilinear transform will preserve the magnitude response characteristics of the analog filters, at the expense of the time domain properties. This method is better for designing frequency selective IIR digital filters.

This method credited simplicity and similarity of frequency response of IIR digital filters to that prototype analog filter. The bilinear transform requires higher sampling frequency which in turn requires lower sampling rate. It overcomes effects of aliasing that is caused due to analog frequency response containing components at or beyond the half sampling (Nyquist) frequency somewhat degraded by frequency warping. This method is the result of one-to-one mapping from s-plane to z-plane inherent. In addition, the filter roll-off characteristics are sharper using this method.

Furthermore, the Bilinear Transform of analog prototype filter has less limitations and exists in: lowpass, high pass, bandpass and band stop filters.

This design starts with the transfer function of an analog filter, and then a mapping from s (Analog) to z (Digital) plane results in a general form for an IIR filter with an arbitrary number of poles and zeros Verma (2013)

A digital filter can be characterized as a linear time invariant (LTI) discrete system by a constant coefficient difference equation Proakis and Monalakis (2002), Shenoi (2006)

$$\mathrm{y(n)} = \sum_{k=0}^{M} b_m x(n-m) - \sum_{k=1}^{M} a_m y(n-m) \qquad \textbf{Equation 1}$$

where a(k) and b(k) are the forward tap coefficients and feedback tap coefficients respectively.

The frequency domain response function can be obtained from its z-transform Balami et al. (2020) Islam and Aktar (2019) Jamal et al. (1996)

$$Y(z) = X(z) \sum_{k=0}^{m} b_k z^{-k} + \sum_{k=0}^{m} b_k z^{-k} \qquad \textbf{Equation 2}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{m} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} \qquad \textbf{Equation 3}$$

The mapping or transformation that relates points on the s-plane to z-planes is defined as Kou and Lee (2001), Shenoi (2006)

$$s = \frac{2}{T}\left(\frac{Z-1}{Z+1}\right) = \frac{2}{T}\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right) \qquad \textbf{Equation 4}$$

where T is the indicated sample period

Therefore, the digital transfer function can be calculated in terms discrete time variable z as follows Patanavijit (2020)

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} \qquad \textbf{Equation 5}$$

The mapping between the continuous-time spectrum (-∞≤Ω≤∞) to the discrete-time spectrum (-π≤ω≤π) can be computed as:

$$\Omega = \frac{2}{T} tan(\omega/2)$$ **Equation 6**

where Ω is the analog frequency and ω is the digital frequency

$$\omega = 2tan^{-1}(\Omega T/2)$$ **Equation 7**

This is a non-linear relation, and it is known as prewarping

Theoretical Design Parameters

δ_p: maximum variation

δ_s: minimum stopband attenuation

R_p: passband ripple in decibel (dB)

R_s: stopband ripple in decibel (dB)

f_p: passband frequency in (Hz)

f_s: stopband frequency in (Hz)

F_s: sampling frequency in (Hz)

T_s: sampling rate in (Hz)

ω_p: digital passband frequency in (rad)

ω_s: digital stopband frequency in (rad)

Ω_p: analogue passband frequency in (rad/sec)

Ω_p: analogue stopband frequency in (rad/sec)

ε: ripple parameter

β: a constant

$\Re$: region of convergence (ROC) of the poles and zeros

F_{ss}: Filter Selectivity

Theoretical Design Equations

The filter order (N) Butterworth filter can be obtained by:

$$\mathrm{N} \geq \frac{log_{10}\left[\sqrt{(10^{\delta_s/10}-1)} \Big/ \sqrt{(10^{\delta_p/10}-1)}\right]}{\log_{10}[\Omega_s/\Omega_p]}$$ **Equation 8**

The filter order (N) Chebyshev I filter can be computed by:

$$\mathrm{N} \geq \frac{\cosh^{-1}\left[\sqrt{(10^{\delta_s/10}-1)} \Big/ \sqrt{(10^{\delta_p/10}-1)}\right]}{\cosh^{-1}[\omega_s/\omega_p]}$$ **Equation 9**

The analogue angular frequency in the passband

$$\Omega_p = 2\pi f_p$$ **Equation 10**

The analogue angular frequency in the stopband

$$\Omega_s = 2\pi f_s \quad \textbf{Equation 11}$$

$$R_p = 20\log_{10}(\delta_p)dB \quad \textbf{Equation 12}$$

$$R_s = 20\log_{10}(\delta_s)dB \quad \textbf{Equation 13}$$

The digital passband cut-off frequency (ω_{cp})

$$\omega_{cp} = \frac{\Omega_p}{\left(10^{R_p/10}-1\right)^{1/2N}} \quad \textbf{Equation 14}$$

The digital stopband cut-off frequency (ω_{cs})

$$\omega_{cs} = \frac{\Omega_s}{\left(10^{R_s/10}-1\right)^{1/2N}} \quad \textbf{Equation 15}$$

The cut-off frequency can be anywhere in the interval

$$\Omega_p/\left(10^{R_p/10}-1\right)^{1/2N} \leq \omega_c \leq \Omega_s/\left(10^{R_s/10}-1\right)^{1/2N} \quad \textbf{Equation 16}$$

The digital frequency is related to analogue frequency in passband and stopband respectively by the relations Scarpa (2013) Ramesh et al. (2009)

$$\omega_p = \Omega_p T_s \quad \textbf{Equation 17}$$

$$\omega_s = \Omega_s T_s \quad \textbf{Equation 18}$$

The modified prewarping frequency in the passband and stopband can be computed by equations Equation 19 and Equation 20 respectively as:

$$\acute{\Omega}_p = \frac{2}{T}\tan(\omega_p/2) \quad \textbf{Equation 19}$$

$$\acute{\Omega}_s = \frac{2}{T}\tan(\omega_s/2) \quad \textbf{Equation 20}$$

The pole position of Butterworth filter can be computed by Abu-hudrouss (2009)

$$S_k = e^{j[\pi/2+(2k+1)\pi/2N]}, k = 0,1,2,3,\ldots,N \ \ (N = 15 \text{ odd}) \quad \textbf{Equation 21}$$

$$\rightarrow S_k = cos\varphi_k + jsin\varphi_k$$ **Equation 22**

Where

$$\varphi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1,2,3, \ldots, N$$ **Equation 23**

The region of convergence $\left(ROC(\Re)\right)$ of the poles and zeros is determined as Abu-hudrouss (2009)

$$\Re = \varepsilon^{-1/N}$$ **Equation 24**

The poles of Chebyshev I filter can be analysed by the relation:

$$s_k = rcos\phi_k + jRsin\phi_k$$ **Equation 25**

Where $\phi_k$is given by equation Equation 23

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1,2,3, \ldots, N$$ **Equation 26**

$$\varepsilon = \sqrt{10^{R_p/10} - 1}$$ **Equation 27**

$$\beta = \left[\frac{\sqrt{1+\varepsilon^2}+1}{\varepsilon}\right]^{1/N}$$ **Equation 28**

The minor (r) axis of the pole can be obtained by the relation:

$$r = \Omega_p \frac{\beta^2-1}{2\beta}$$ **Equation 29**

The major (R) axis of the pole can be computed by the relation:

$$R = \Omega_p \frac{\beta^2+1}{2\beta}$$ **Equation 30**

The transfer function of the filter can be found by the relation Aremu et al. (2013), Prêle et al. (2021), Aremu et al. (2013)

$$H(s) = \frac{KH_0}{(s-s_1)(s-s_2)(s-s_3)\ldots(s-s_N)}$$ **Equation 31**

Where

$$\mathrm{k} = \begin{cases} 1, \text{for N odd} \\ \frac{1}{\sqrt{1+\varepsilon^2}}, \text{ for N even} \end{cases}$$ **Equation 32**

$$H_0 = \begin{cases} 1, \text{for N even} \\ (s-s_1)(s-s_2)\ldots(s-s_N)|_{s=0}, \text{for N odd} \end{cases}$$ **Equation 33**

$$B_{\mathrm{N}} = \prod_{k=1}^{\mathrm{N}} \left[s^2 - 2s\cos\left(\frac{2\mathrm{k}+\mathrm{N}-1}{2\mathrm{N}}\pi\right) + 1\right] \quad \text{N is even}$$ **Equation 34**

$$B_{\mathrm{N}} = (s+1)\prod_{k=1}^{\frac{\mathrm{N}-1}{2}} \left[s^2 - 2s\cos\left(\frac{2\mathrm{k}+\mathrm{N}-1}{2\mathrm{N}}\pi\right) + 1\right] \quad \text{N is odd}$$ **Equation 35**

$$F_{ss} = \frac{N}{2\sqrt{2\omega_c}}$$ **Equation 36**

Design specifications

$\delta_p = 0.5$

$\delta_s = 34$

$\mathrm{F}_s = 50\ KHz$

$f_p = 10\ KHz$

$f_s = 14\ KHz$

Calculations of Design specifications

$$\Omega_p = 2\pi \mathrm{f_p} = 2\pi \times 10000 = 20000\pi = 6.2832 \times 10^4\ rad/sec$$

$$\Omega_s = 2\pi \mathrm{f_s} = 2\pi \times 14000 = 28000\pi = 8.7965 \times 10^4\ rad/sec$$

$$R_p = 20log_{10}(\delta_p) = 20log_{10}(0.5) = -6.0206\ dB$$

$$R_s = 20log_{10}(\delta_s) = 20log_{10}(34) = 30.6296\ dB$$

$$\mathrm{T_s} = \frac{1}{\mathrm{F_s}} = \frac{1}{50000} = 2 \times 10^{-5}\ Hz$$

$$\omega_p = \frac{\Omega_p}{F_s} = 2\pi\left(\frac{f_p}{F_s}\right) = 2\pi\left(\frac{10\ 000}{50\ 000}\right) = 0.4\pi = 1.2566\ rad$$

$$\omega_s = \frac{\Omega_s}{F_s} = 2\pi\left(\frac{f_S}{F_s}\right) = 2\pi\left(\frac{14\ 000}{50\ 000}\right) = 0.56\pi = 1.7593\ rad$$

$$\varepsilon = [10^{0.1A_\mathrm{p}} - 1]^{1/2} = [10^{0.1\times 6.0206} - 1]^{1/2} = 1.7321$$

Calculation of Design Parameters Using Bilinear Transformation (BLT)

Ripple Parameters

$$R_{\mathrm{p}} = 20log_{10}\delta_p = 20log_{10}(0.5) = -6.0206\ d\mathrm{B}$$

$$R_{\mathrm{s}} = -20log_{10}\delta_s = 20log_{10}(34) = -30.6296\ d\mathrm{B}$$

Prewarping the frequency

$$\mathrm{T} = 1\ \mathrm{sec}$$

$$\acute{\Omega}_{\mathrm{p}} = \frac{2}{\mathrm{T}}\tan\left(\omega_{\mathrm{p}}/2\right) = \frac{2}{1}\tan(0.4\pi/2) = 1.4531\ \ \mathrm{rad/sec}$$

$$\acute{\Omega}_{\mathrm{s}} = \frac{2}{\mathrm{T}}\tan(\omega_{\mathrm{s}}/2) = \frac{2}{1}\tan(0.56\pi/2) = 2.4176\ \ \mathrm{rad/sec}$$

Table1

Table 1 Comparison of Butterworth and Chebyshev I Lowpass Filters

Filter Type	Respons e Type	Filte r Orde r	Filter Selectivit y	Region of Convergen ce (ROC)	Transition Frequency (KHz)			
					Analogue		Digital	
					Passban d	Stopban d	Passban d	Stopban d
Butterwort h	Lowpas s	15	4.7309	0.9391	29	∞	31.5	∞
Chebyshev I	Lowpas s	7	2.2078	0.8741	36.25	∞	36.25	∞

The denominator of the analogue transfer function $H(s)$ of the normalized 15th order Butterworth filter as depicted in Table 2 is obtained using quadratic polynomial equation Equation 35

Table 2

Table 2 Butterworth polynomial quadratic factors for filter order (N=15)

N	B_{N}
0	1
1	$(s+1)$
2	$(s^2+1.4142s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.7654s+1)(s^2+1.8478s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0..5176s+1)(s^2+1.4142s+1)(s^2+1.9319s+1)$
7	$(s+1)(s^2+0.4450s+1)(s^2+1.2470s+1)(s^2+1.8019s+1)$

8	$(s^2+0.3902s+1)(s^2+1.1111s+1)(s^2+1.6629s+1)(s^2+1.9616s+1)$
9	$(s+1)(s^2+0..3473s+1)(s^2+s+1)(s^2+1..5321s+1)(s^2+1.8794s+1)$
10	$(s^2+0.3129s+1)(s^2+0.9080s+1)(s^2+1.4142s+1)(s^2+1.7820s+1)(s^2+1.9754s+1)$
11	$(s+1)(s^2+0.2846s+1)(s^2+0.8308s+1)(s^2+1.3097s+1)(s^2+1.9190s+1)(s^2+2s+1)$
12	$(s^2+0.2611s+1)(s^2+0.7654s+1)(s^2+1.2175s+1)(s^2+1.5867s+1)(s^2+1.9829s+1)(s^2+1.8478s+1)$
13	$(s+1)(s^2+0.2411s+1)(s^2+0.7092s+1)(s^2+1.1361s+1)(s^2+1.4970s+1)(s^2+1.7709s+1)(s^2+1.7709s+1)$
14	$(s^2+0..2239s+1)(s^2+0.6606s+1)(s^2+1.0641s+1)(s^2+1.4142s+1)(s^2+1.6934s+1)(s^2+1.8878s+1)(s^2+1.9874s+1)$
15	$(s+1)(s^2+0.2091s+1)(s^2+0.6180s+1)(s^2+s+1)(s^2+1.3383s+1)(s^2+1.6180s+1)(s^2+1.8271s+1)$

$$B_{\mathrm{N}}=\prod_{k=1}^{\mathrm{N}}\left[s^2-2s\cos\left(\frac{2\mathrm{k}+\mathrm{N}-1}{2\mathrm{N}}\pi\right)+1\right] \quad \text{N is even}$$ **Equation 37**

$$B_{\mathrm{N}}=(s+1)\prod_{k=1}^{\frac{\mathrm{N}-1}{2}}\left[s^2-2s\cos\left(\frac{2\mathrm{k}+\mathrm{N}-1}{2\mathrm{N}}\pi\right)+1\right] \quad \text{N is odd}$$ **Equation 38**

Table 3

Table 3 Digital Transfer Functions Calculated Using BLT Algorithm

Designed Filter	Transfer Function
Butterworth	$H(s)=\frac{1}{\left[\begin{array}{c}s^{15}+9.5664s^{14}+45.7584s^{13}+\\144.8378s^{12}+338.6618s^{11}+\\618.7556s^{10}+1045.6s^{9}+1102.2s^{8}+\\1102.2s^{7}+911.6186s^{6}+618.7267s^{5}+\\338.6392s^{4}+144.8249s^{3}+45.7531s^{2}+\\9.5650s+0.9998\end{array}\right]}$
Chebyshev I	$H(s)=\frac{0.0446}{\left[\begin{array}{c}s^{7}+0.4435s^{6}+1.3501s^{5}+0.5840s^{4}+\\1.8575s^{3}+0.5107s^{2}+0.4844s+0.0446\end{array}\right]}$

Table 4

Table 4 Digital Transfer Functions Calculated Using Z-Transform

Designed Filter	Transfer Function
Butterworth	$H(z)=\frac{[z^{-14}]}{\left[\begin{array}{c}1+9.566z^{-1}+45.76z^{-2}+\\144.8z^{-3}+338.7z^{-4}+\\618.8z^{-5}+1046z^{-6}+\\1102z^{-7}+1102z^{-8}+\\911.6z^{-9}+618.7z^{-10}+\\338.6z^{-11}+144.8z^{-12}+\\45.753z^{-13}+9.565z^{-14}+z^{-15}\end{array}\right]}$
Chebyshev I	$H(z)=\frac{[0.0446z^{-6}]}{\left[\begin{array}{c}1+0.4435z^{-1}+1.35z^{-2}+0.584z^{-3}+1.857z^{-4}+\\05107z^{-5}+0.4844z^{-6}+0.0446z^{-7}\end{array}\right]}$

Design Procedures of Butterworth Filter

1) Determine the filter order (N) using equation Equation 8

$$N \geq \frac{log_{10}\left[\sqrt{(10^{\delta_s/10}-1)}\Big/\sqrt{(10^{\delta_p/10}-1)}\right]}{\log_{10}[\Omega_s/\Omega_p]} = \frac{log_{10}\left[\sqrt{(10^{34/10}-1)}\Big/\sqrt{(10^{0.5/10}-1)}\right]}{log_{10}[2\pi\times 14000/2\pi\times 10000]}$$

$N \geq 14.7590$

$\therefore N = 15$

2) Determine the cut-off frequency (ω_c) from the expressions of constraints using Equation 14 and Equation 15

$$\omega_{cp} = \frac{\Omega_p}{\left(10^{R_p/10}-1\right)^{1/2N}} = \frac{20000\pi}{\left(10^{6.0206/10}-1\right)^{1/2\times 30}} = 6.0573 \times 10^4\ Hz$$

$$\omega_{cs} = \frac{\Omega_s}{\left(10^{R_s/10}-1\right)^{1/2N}} = \frac{28000\pi}{\left(10^{30.6296/10}-1\right)^{1/2\times 30}} = 8.4802 \times 10^4\ Hz$$

$$\therefore \omega_c = \omega_{cp} = 6.0573 \times 10^4\ Hz$$

3) For the computed N determine the denominator polynomial of normalized H(s) from the Table 2 which was computed using equation B_N i.e., Equation 36 which is computed using equation Equation 22
4) The numerator of the transfer function is obtained using equation (33).

$$H_0 = \begin{cases} 1, \text{for N even} \\ (s-s_1)(s-s_2)\ldots(s-s_N)|_{s=0}, \text{for N odd} \end{cases}$$

Since $(N = 15)$ odd

$$\begin{aligned}(s-s_1)(s-s_2)\ldots(s-s_N)|_{s=0} &= \big(s-(-0.1045 \pm j0.9945)\big)\big(s-(-0.3090 \pm j0.9511)\big)\big(s \\ &- (-0.5000 \pm j0.8660)\big)\big(s-(-0.6691 \pm j0.7431)\big)\big(s \\ &- (-0.8090 \pm j0.5878)\big)\big(s-(-0.9135 \pm j0.4067)\big)\big(s \\ &- (-0.9781 \pm j0.2079)\big)\big(s-(-1.0000 + j0.0000)\big)\end{aligned}$$

$\rightarrow (s^2 + 0.2090s + 1.0000)(s^2 + 0.6180s + 1.0001)(s^2 + s + 1.0000)\ (s^2 + 1.3382s + 0.9999)(s^2 + 1.6180s + 1.0000)(s^2 + 1.8270s + 0.9999)(s^2 + 1.9562s + 0.9999)(s + 1.0000)$

$$\begin{aligned}\rightarrow (s^{15} &+ 9.5664s^{14} + 45.7584s^{13} + 144.8378s^{12} + 338.6618s^{11} + 618.7556s^{10} + 1045.6s^9 \\ &+ 1102.2s^8 + 1102.2s^7 + 911.6186s^6 + 618.7267s^5 + 338.6392s^4 \\ &+ 144.8249s^3 + 45.7531s^2 + 9.5650s + 0.9998)\end{aligned}$$

Setting (s=0)

$\rightarrow ((0)^{15} + 9.5664(0)^{14} + 45.7584(0)^{13} + 144.8378(0)^{12} + 338.6618(0)^{11} + 618.7556(0)^{10} + 1045.6(0)^{9} + 1102.2(0)^{8} + 1102.2(0)^{7} + 911.6186(0)^{6} + 618.7267(0)^{5} + 338.6392(0)^{4} + 144.8249(0)^{3} + 45.7531(0)^{2} + 9.5650(0)^{1} + 0.9998)0.9998$

$\therefore\ H_0$ $(zeros)$ is $= 0.9998 \cong 1$

$$\mathrm{k} = \begin{cases} 1, \text{for N odd} \\ \frac{1}{\sqrt{1+\varepsilon^2}}, \ \text{for N even} \end{cases}$$

$\therefore \mathrm{k} = 1$

5) The transfer function can be obtained using equation (30), (31) and (32) as:

$$H(s) = \frac{KH_0}{(s-s_1)(s-s_2)(s-s_3)\ldots(s-s_N)} = \frac{1}{\left[\begin{array}{c} s^{15}+9.5664s^{14}+45.7584s^{13}+ \\ 144.8378s^{12}+338.6618s^{11}+ \\ 618.7556s^{10}+1045.6s^{9}+1102.2s^{8}+ \\ 1102.2s^{7}+911.6186s^{6}+618.7267s^{5}+ \\ 338.6392s^{4}+144.8249s^{3}+45.7531s^{2}+ \\ 9.5650s+0.9998 \end{array}\right]}$$

The digital transfer function is calculated using equation (5) (Patanavijit, 2020):

$$H(z) = H(s)|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$H(z) = \frac{[1]}{\left[\begin{array}{c} \left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{15}+9.5660\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{14}+ \\ 45.7584\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{13}+144.8375\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{12}+ \\ 338.661\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{11}+618.7556\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{10}+ \\ 1045.6\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{9}+1102.2\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{8}+ \\ 1102.2\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{7}+911.6186\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{6}+ \\ 618.7267\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{5}+338.6392\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{4}+ \\ 144.8249\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{3}+45.753\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^{2}+ \\ 9.5650\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)+0.9998 \end{array}\right]}$$

The analytical solution of digital transfer function using z- transform is given as:

$$\therefore H(z) = \frac{[z^{-14}]}{\begin{bmatrix} 1+9.566z^{-1}+45.76z^{-2}+ \\ 144.8z^{-3}+338.7z^{-4}+ \\ 618.8z^{-5}+1046z^{-6}+ \\ 1102z^{-7}+1102z^{-8}+ \\ 911.6z^{-9}+618.7z^{-10}+ \\ 338.6z^{-11}+144.8z^{-12}+ \\ 45.753z^{-13}+9.565z^{-14}+z^{-15} \end{bmatrix}}$$

The difference equation of the digital transfer function can be computed using equation Equation 1 as:

$$\mathrm{y(n)} = \sum_{k=0}^{M} b_m x(n-m) - \sum_{k=1}^{M} a_m y(n-m)$$

$$\begin{aligned} \rightarrow y(\mathrm{n}) = x(k-14) &+ 9.566y(k-1) + 45.76y(k-2) + 144.8y(k-3) + 338.7y(k-4) \\ &+ 618.8y(k-5) + 1046y(k-6) + 1102y(k-7) + 1102y(k-8) \\ &+ 911.6y(k-9) + 618.7y(k-10) + 338.6y(k-11) + 144.8y(k-12) \\ &+ 45.75y(k-13) + 9.565y(k-14) + y(k-15) \end{aligned}$$

6) Adjust the gain of the filter by the desired amplification factor, if needed.

The dc gain of the filter: $\omega T = 0 \rightarrow z = 1$ The dc gain: which gives |G (1)|=1

$$G(z) = H(z)|_{z=0}$$

$$G(z) = \frac{[1]}{\begin{bmatrix} z^{15}+9.566z^{14}+45.76z^{13}+ \\ 144.8z^{12}+338.7z^{11}+618.8z^{10}+ \\ 1046z^{9}+1102z^{8}+911.6z^{7}+ \\ 618.7z^{6}+911.6z^{5}+618.7z^{4}+ \\ 144.8z^{3}+45.753z^{2}+9.656z^{1}+1 \end{bmatrix}}$$

$$|G(1)| = \frac{[1]}{\begin{bmatrix} (0)^{15}+9.566(0)^{14}+45.76(0)^{13}+144.8(0)^{12}+ \\ 338.7(0)^{11}+618.8(0)^{10}+1046(0)^{9}+1102(0)^{8}+ \\ 911.6(0)^{7}+618.7(0)^{6}+911.6(0)^{5}+618.7(0)^{4}+ \\ 144.8(0)^{3}+45.753(0)^{2}+9.656(0)^{1}+1 \end{bmatrix}} = 1$$

$$\therefore |G(1)| = 1$$

3 dB cutoff frequency $\omega T = 0.4\pi$

Thus,

$$G\left(e^{j0.4\pi}\right) = \frac{[1]}{\begin{bmatrix} \left(\cos(6\pi)+jsin(6\pi)\right)+ \\ 9.565\left(\cos(5.6\pi)+jsin(5.6\pi)\right)+ \\ 45.76\left(\cos(5.2\pi)+jsin(5.2\pi)\right)+ \\ 144.8\left(\cos(4.8\pi)+jsin(4.8\pi)\right)+ \\ 338.7\left(\cos(4.4\pi)+jsin(4.4\pi)\right)+ \\ 618.8\left(\cos(4\pi)+jsin(4\pi)\right)+ \\ 1046\left(\cos(3.6\pi)+jsin(3.6\pi)\right)+ \\ 1102\left(\cos(3.2\pi)+jsin(3.2\pi)\right)+ \\ 1102\left(\cos(2.8\pi)+jsin(2.8\pi)\right)+ \\ 911.6\left(\cos(2.4\pi)+jsin(2.4\pi)\right)+ \\ 618.7\left(\cos(2\pi)+jsin(2\pi)\right)+ \\ 338.6\left(\cos(1.6\pi)+jsin(1.6\pi)\right)+ \\ 144.8\left(\cos(1.2\pi)+jsin(1.2\pi)\right)+ \\ 45.753\left(\cos(0.8\pi)+jsin(0,8\pi)\right)+ \\ 9.565\left(\cos(0.4\pi)+jsin(0.4\pi)\right)+1 \end{bmatrix}} = -0.0088 - j0.0021$$

$$\therefore \left|G(e^{j0.4\pi})\right| = \sqrt{(-0.0088)^2 + (-0.0021)^2} = 0.0197$$

$$\text{Gain } dB = 20log_{10}(0.0197) = -34.1107\ dB$$

The filter Selectivity of lowpass Butterworth filter is obtained using equation (35)

$$F_{ss} = \frac{N}{2\sqrt{2\omega_c}} = \frac{15}{2\sqrt{2\times 1.2566}} = 4.7309$$

The region of convergence $\left(ROC(\Re)\right)$ of Butterworth lowpass filter can be obtained using equation (24)

$$\Re = \varepsilon^{-1/N} = (2.5651)^{-1/15} = 0.9391$$

Design Procedures of Chebyshev I Filter

1) Determine the filter equation (N) using equation Equation 9

$$N \geq \frac{\cosh^{-1}\left[\sqrt{\left(10^{\delta_s/10}-1\right)}\Big/\sqrt{\left(10^{\delta_p/10}-1\right)}\right]}{\cosh^{-1}[\omega_s/\omega_p]} = \frac{\cosh^{-1}\left[\sqrt{\left(10^{34/10}-1\right)}\Big/\sqrt{\left(10^{0.5/10}-1\right)}\right]}{\cosh^{-1}[2\pi\times 14000/2\pi\times 10000]} = 6.5271$$

$$N \geq 6.5271$$
$$\therefore N = 7$$

2) Determine ω_c from the expressions of constraints using Equation 14 and Equation 15 For the computed N determine the denominator (pole) polynomial of normalized H(s) The normalized denominator (pole) polynomial of the transfer function is obtained using Equation 23 Equation 25 Equation 26 Equation 27 Equation 28 and Equation 29

$$\varepsilon = [10^{0.1R_\text{p}} - 1]^{1/2} = [10^{0.1\times 6.0206} - 1]^{1/2} = 1.7321$$

$$\beta = \left[\frac{\sqrt{1+\varepsilon^2}+1}{\varepsilon}\right]^{1/N} = \left[\frac{\sqrt{1+1.7321^2}+1}{1.7321}\right]^{1/7} = 1.0816$$

The minor axis (r) is computed as:

$$r = \omega_p \frac{\beta^2-1}{2\beta} = 1.2566\,\frac{(1.0816)^2-1}{2\times 1.0816} = 0.0987$$

The major axis (R) is calculated by:

$$R = \omega_p \frac{\beta^2+1}{2\beta} = 1.2566\,\frac{(1.0816)^2+1}{2\times 1.0816} = 1.2605$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1,2,3,\dots,N \qquad (N = 7 \text{ odd})$$

$$\phi_k = \frac{4\pi}{7}, \frac{5\pi}{7}, \frac{6\pi}{7}, \frac{7\pi}{7}, \frac{8\pi}{7}, \frac{9\pi}{7} \text{ and } \frac{10\pi}{7}$$

$$s_k = rcos\phi_k + jRsin\phi_k = 0.0987cos\phi_k + j1.2605sin\phi_k$$

The denominator (pole) of the transfer function using denominator of Equation 31

$s_k = -0.0220 \pm j1.2289, -0.0615 \pm j\ 0.9855, -0.0889\ j\ 0.5469$ and $-0.0987 +\ j0.0000$

$$(s - s_k) = (s - s_1)(s - s_2)(s - s_3)\dots(s - s_N)$$

$$\rightarrow \big(s - (-0.0220 \pm j1.2289)\big)\big(s - (-0.0615 \pm j\ 0.9855)\big)\big(s\ (-0.0889 \pm j\ 0.5469\ )\big)\big(s - (-0.0987 + j0.0000)\big)$$

$$\rightarrow ((s + 0.0220)^2 - (-j1.2289)^2)((s + 0.0615)^2 - (-j0.985)^2)((s + 0.0889)^2 - (-j0.5469)^2)(s + 0.0987)$$

$$\rightarrow (s^2 + 0.0440s + 1.5107)(s^2 + 0.1230s + 0.9740)(s^2 + 0.1778s + 0.3070)\ (s + 0.0987)$$

Thus, the denominator of the transfer function is:

$$(s^7 + 0.4435s^6 + 1.3501s^5 + 0.5840s^4 + \ 1.8575s^3 + 0.5107s^2 + 0.4844s + 0.0446)$$

Since $(N = 7)$ is odd the numerator of the transfer function in Equation 31 can be obtained using Equation 33

$$H_0 = (s - s_1)(s - s_2)(s - s_3) \dots (s - s_N)|_{s=0}$$

$\rightarrow H_0 = (s^7 + 0.4435s^6 + 1.3501s^5 + 0.5840s^4 + \ 1.8575s^3 + 0.5107s^2 + 0.4844s + 0.0446)|_{s=0} = ((0)^7 + 0.4435(0)^6 + 1.3501(0)^5 + 0.5840(0)^4 + \ 1.8575s^3 + 0.5107(0)^2 + 0.4844(0) + 0.0446) = 0.0446$

$$\therefore H_0 = 0.0446$$

The transfer function of the filter can be evaluated using Equation 33 Equation 32 Equation 31

$$\rightarrow H(s) = \frac{\mathrm{k}H_0}{(s-s_1)(s-s_2)(s-s_3)\dots(s-s_N)} = \frac{0.0446}{\begin{pmatrix} s^7+0.4435s^6+1.3501s^5+0.5840s^4+ \\ 1.8575s^3+0.5107s^2+0.4844s+0.0446 \end{pmatrix}}$$

$$\therefore H(s) = \frac{0.0446}{\begin{pmatrix} s^7+0.4435s^6+1.3501s^5+0.5840s^4+ \\ 1.8575s^3+0.5107s^2+0.4844s+0.0446 \end{pmatrix}}$$

The analytical solution of digital transfer function using z- transform is obtained using Equation 5

$$H(z) = H(s)|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$\mathrm{H}(z) = \frac{[0.0446]}{\begin{bmatrix} \left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^7 + 0.4435\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^6 + \\ 1.3501\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^5 + 0.5840\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^4 + \\ 1.8575\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^3 + 0.5107\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + \\ 0.4844\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right] + 0.5107\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + \\ 0.0446 \end{bmatrix}}$$

$$H(z) = \frac{[0.0446z^{-6}]}{\begin{bmatrix} 1+0.4435z^{-1}+1.35z^{-2}+0.584z^{-3}+1.857z^{-4}+ \\ 05107z^{-5}+0.4844z^{-6}+0.0446z^{-7} \end{bmatrix}}$$

- The dc gain of the filter: $\omega T = 0 \rightarrow z = 1$ The dc gain: which gives |G (0) |=1

$$G(z) = H(z)|_{z=0}$$

$$\rightarrow G(z) = \frac{[0.0446]}{\begin{bmatrix} z^7+0.4435z^6+1.35z^5+0.584z^4+1.857z^3+ \\ 05107z^2+0.4844z^1+0.0446 \end{bmatrix}}$$

$$\rightarrow |G(0)| = \frac{[0.0446]}{\begin{bmatrix}(0)^7+0.4435(0)^6+1.35(0)^5+0.584(0)^4+\\1.857(0)^3+0.5107(0)^2+0.4844(0)^1+0.0446\end{bmatrix}}$$

$$\therefore |G(0)| = 1$$

- 3 dB cut-off frequency $\omega T = 0.4\pi$

Thus,

$$\mathrm{G}(e^{j0.4\pi}) = \frac{[0.0446]}{\begin{bmatrix}(\cos(2.8\pi)+jsin(2.8\pi))+\\0.4435(\cos(2.4\pi)+jsin(2.4\pi))+\\1.35(\cos(2\pi)+jsin(2\pi))+\\0.584(\cos(1.6\pi)+jsin(1.6\pi))+\\1.857(\cos(1.2\pi)+jsin(1.2\pi))+\\0.5107(\cos(0.8\pi)+jsin(0.8\pi))+\\0.4844(\cos(0.4\pi)+jsin(0.4\pi))+\\0.0446\end{bmatrix}} = 0.0145 + j0.0134$$

$$\therefore |G(e^{j0.4\pi})| = \sqrt{(0.0145)^2 + (0.0134)^2} = 0.0197$$

$$\text{Gain } dB = 20log_{10}(0.0197) = -34.1107\ dB$$

The filter Selectivity of lowpass Chebyshev I filter is obtained using equation (35)

$$F_{ss} = \frac{N}{2\sqrt{2\omega_c}} = \frac{7}{2\sqrt{2\times 1.2566}} = 2.2078$$

The region of convergence $(ROC(\Re))$ of Chebyshev I lowpass filter can be obtained using equation (24)

$$\Re = \varepsilon^{-1/N} = (2.5651)^{-1/7} = 0.8741$$

The difference equation was obtained as:

$$y(k) = 0.0446x(k-6) + 0.4435y(k-1) + 1.35y(k-2) + 0.584y(k-3) + 1.875y(k-4) + 0.5107y(k-5) + 0.4844y(k-6) + 0.0446y(k-7)$$

3. RESULTS AND DISCUSSION

3.1. RESULTS

By means of Bilinear transformation techniques, the designed Butterworth filter and Chebyshev-I digital IIR filter where the passband gain $(0 \le \omega \le 0.4\pi)$ between $(0\ dB)$ and $(-6.0206\ dB)$, and stopband gain $(0.56\pi \le \omega \le \pi)$ has attenuation of $(-30.6296\ dB)$ and sampling period $(T_s = 1)$. The filter selectivity (F_{ss}) of the lowpass filter: Butterworth and Chebyshev I were found to be (4.7309) and (2.2078) respectively. Also, the cutoff frequency (ω_c) of both the filters was computed to be (6.0573×10^4 Hz). The simulation results were depicted below:

Figure 2

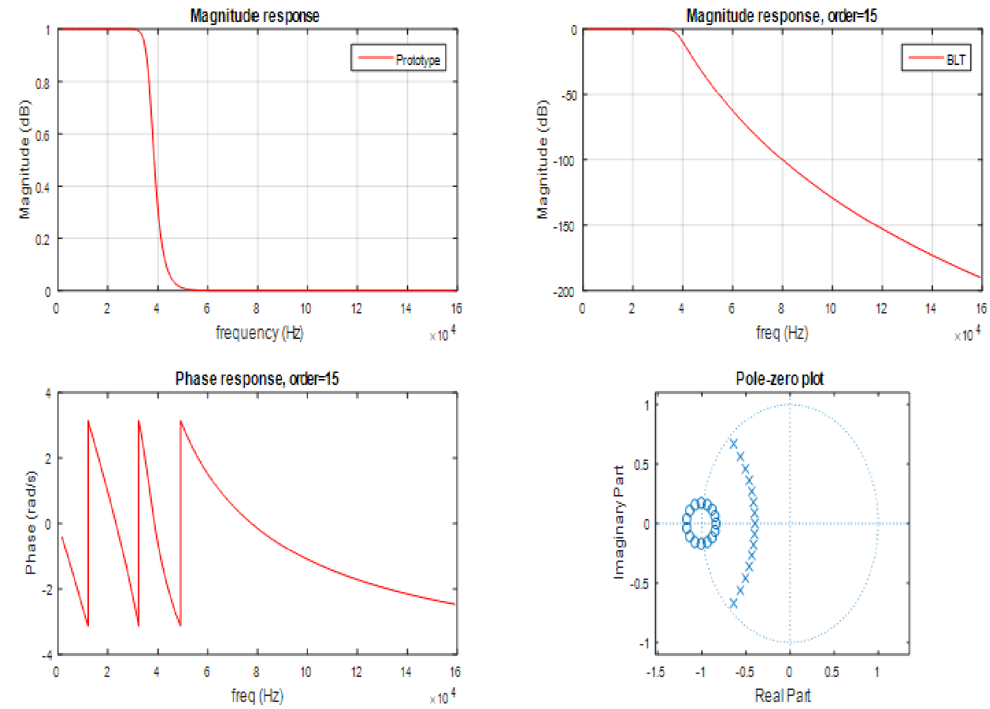

Figure 2 Frequency Response (Magnitude and Phase) and Zero-pole plot of Butterworth Lowpass Filter

Figure 3

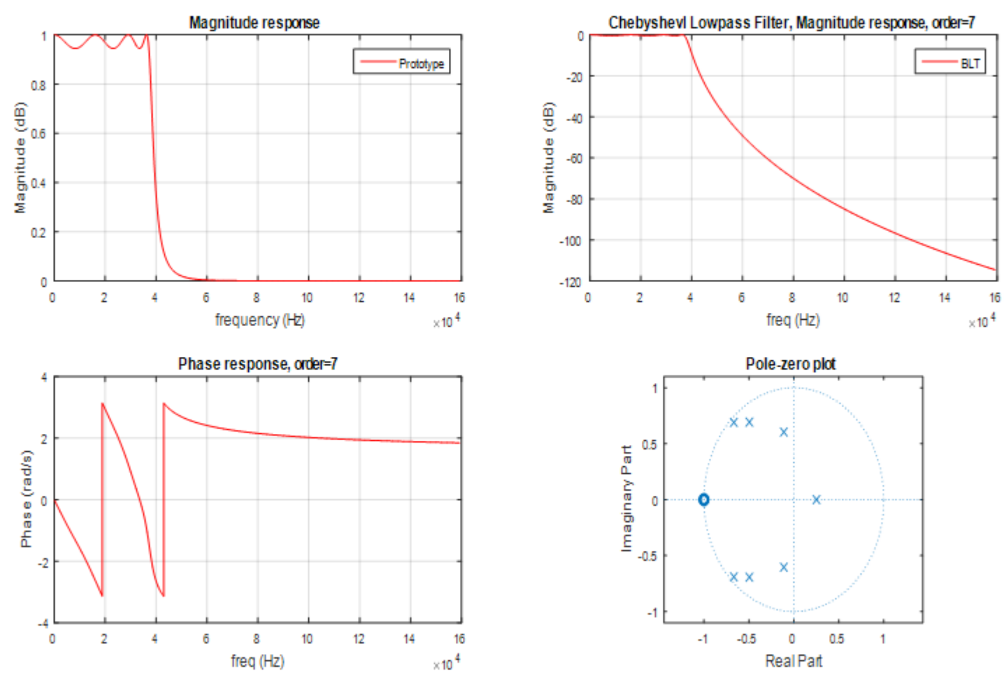

Figure 3 Frequency Response (Magnitude and Phase) and Zero-pole plot of Chebyshev I Lowpass Filter

Figure 4

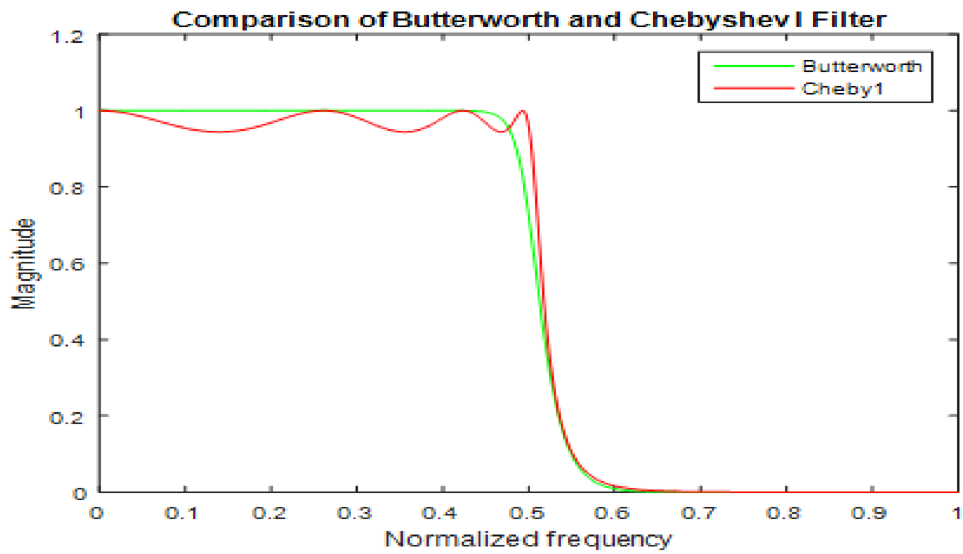

Figure 4 Comparison of Transition band of Butterworth and Chebyshev I Filter with Filter order of N=15 and N=7 respectively in absolute magnitude scale

Figure 5

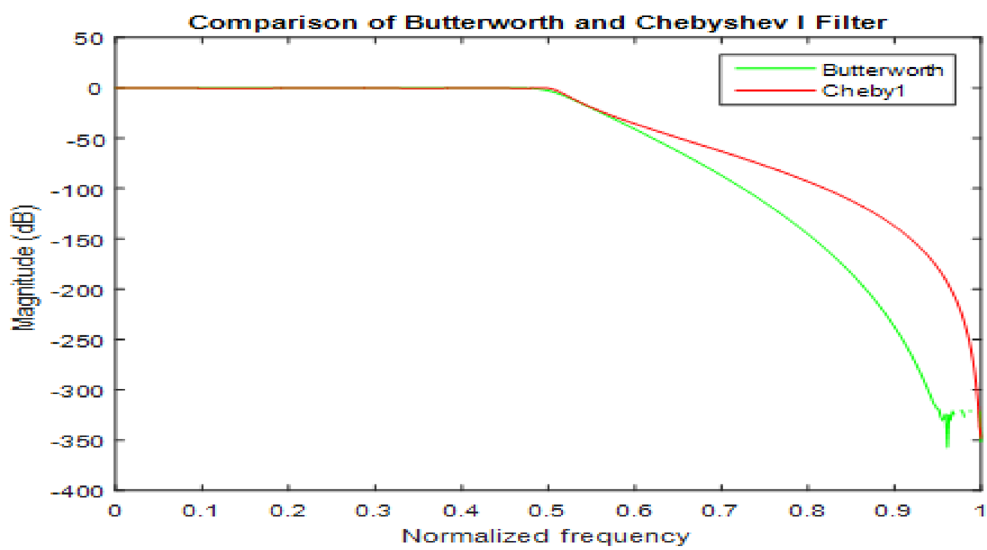

Figure 5 Comparison of Transition band of Butterworth and Chebyshev I Filter with Filter order of N=15 and N=7 respectively in dB scale

4. DISCUSSION

Using the given specifications, the filters designed were stable as proved by the pole-zero plots depicted in Figure 2 Figure 3 The filter order and the polynomial terms of the transfer function of Chebyshev I filter in Table 1 has lower values as compared to the Butterworth filter. The transitions proved that Chebyshev I filter has sharper roll off than the Butterworth filter as depicted in Figure 4 in terms of magnitude scale and Figure 5 in terms of decibel scale. Meanwhile the phase responses also proved that IIR digital filters has nonlinear phase.

The achieved magnitude and phase response plots with respect to frequency presented in Figure 2 and Figure 3 discloses that the designed IIR digital filters employing BLT possess flat passband for Butterworth while Chebyshev I filter possess ripple in the passband and both filters possess maximally flat stopband characteristics in which they were in correspondence with prototype analogue IIR filter as depicted in the same Figures.

The 3 dB cutoff frequency $(\omega T = 0.4\pi)$ of the designed filters were at the point $\left(\left|G\left(e^{j0.4\pi}\right)\right| = 0.0197 = -34.1107\ dB\right)$ with the same bandwidth of 7.8957E4 and d.c gain of 1 for both filters.

5. CONCLUSION

The achieved results were analysed to observe the performance of the filters. The obtained lowpass IIR digital filters using BLT meet the stability gauge as all the poles lie within the unit circle. Conversely, BLT has justified the potentiality of the proposed algorithm for the design of lowpass digital IIR filter to prototype analogue filter and it has fast convergence rate in term of the number of function calculations to achieve the universal solution due to its high sampling frequency which in turn has low sampling time. The efficiency of MATLAB has also been justified for depicting prototype analogue filter. The comparative analysis Table 1 shows that the Chebyshev I is better choice to reduce implementation cost. It also proved that the attenuation of the digital filter in the passband is not zero while the attenuation of the digital filter in the stopband have infinite value.

CONFLICT OF INTERESTS

None.

ACKNOWLEDGMENTS

None.

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