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APPLICATIONS OF DIFFERENTIAL CALCULUS TO ARCHITECTURE

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ABSTRACT

Differential calculus constitutes a fundamental mathematical tool in contemporary architectural design and analysis. This documentary research examines the practical applications of differential calculus in architecture, demonstrating how abstract mathematical concepts transform into tangible constructive solutions. The study analyzes the application of mathematical limits to establish design restrictions, guarantee structural safety, and optimize the behavior of high-rise buildings and complex geometries. Derivatives emerge as crucial analytical instruments for calculating curvatures, optimizing organic structures, and analyzing energy efficiency in emblematic works such as the Guggenheim Museum Bilbao. Differential geometry enables the modeling of complex surfaces and non-Euclidean spaces, while integrals facilitate the quantification of geometric properties, structural load analysis, and optimization of thermal behavior in sustainable buildings. The document also explores advanced applications such as multiple integrals, Gaussian series, partial derivatives, and Lebesgue measure theory, evidencing their relevance in parametric design and computational architecture. The results demonstrate that mastery of differential calculus not only improves the technical precision of architectural design but also expands the frontiers of creativity, enabling architects to create habitable, safe, and aesthetically innovative spaces that respond with scientific rigor to contemporary social and environmental needs.

Keywords: Differential Calculus, Architecture, Structural Optimization, Differential Geometry, Parametric Design, Energy Efficiency

1. INTRODUCTION

Differential calculus has transcended the limits of traditional sciences, weaving its influence into disciplines as diverse as physics, economics, engineering, probability, the arts and, particularly significantly, architecture. In the architectural context, calculus reveals itself as more than a mathematical tool: it is a fundamental language that structures thought, providing students with a rigorous method to project, understand, and materialize their ideas.

From the first human constructions to complex contemporary designs, mathematics has been the invisible skeleton that supports architectural creativity. Functions, geometric structures, trigonometric functions, limits, derivatives, and integrals are not simple abstractions, but precise tools that allow for exact modeling of every curve, angle, and volume. This mathematical precision not only improves the structural quality of projects but also expands the frontiers of what is possible to design.

For architects in training, mastery of differential calculus represents more than an academic requirement: it is a passport to a professional future where they will be able to transform social and human needs into habitable, safe, and aesthetically innovative spaces.

Rescala (2012) points out that in the search for foundations for why we teach mathematical analysis, there are: the importance that the mentioned concepts acquire during the course of study as the foundation for developing content in other subjects; the need for these concepts for professional development; and our concern for improving teaching quality by transmitting to students intangible notions that acquire importance in field applications. Without knowledge of these topics, a work cannot be realized. Similarly, Romero (2005) states that the basic principles of statics were discovered from the beginning with great accuracy and have served as the basis for all analysis methods; however, the laboriousness represented by the mathematical calculations required to relate external loads acting on a structure with the reactions they provoke in the supports and with the internal forces that develop in their members is what led to the search for various methods that would facilitate their use. Thus, approximate methods have been introduced that produce results very close to real ones and generally greater than these; the inconvenience arises at the nodes of the frames, because if there is a true value of the moment at the node, when a moment greater than this is obtained in the beams, it will be smaller in the columns, since the laws of equilibrium must always be fulfilled; consequently, the moment in the column is not on the side of safety; therefore, the prudent practice of increasing loads when using these methods must be maintained.

In this way, integral calculus has been visualized as a vital tool for construction and aesthetics in architecture. We can point out that when speaking of the intuitive idea of the limit or the approximation between forces, what arises in architecture, together with physics, is what the future architect must consider when designing for atmospheric events. The main idea of the derivative and its geometry in hyperbolic paraboloids also emerges, which when constructed must take into account the soil and subsoil system. Likewise, velocity and acceleration in designs and integration in the area under the curve and tensor levels for Lebesgue integrals and measure theory. These notional ideas form the principle of the importance of integral calculus for architecture.

Likewise, Serres (2012) has a very clear vision of the importance of integral calculus for university careers such as engineering and architecture. He establishes that the first contact of an architecture student with mathematics begins with the study of form, order, and subsequently space. The architect requires a morphological study of the essential elements of form and space to be able to carry out preliminary design ideas; to successfully accomplish this task, beyond intuition, it is necessary to systematically provide mathematical tools. This systematic study allows establishing hierarchies, path configurations, axes of symmetry, relationship with the environment, spatial relationships, articulations, transformations, proportions, scale, among others.

For this reason, the vision of calculus and its structured system of analysis allows the architect's mind to create a broad vision of analytical structure when designing, taking into account physical, spatial, chemical, biological, and environmental factors. Similarly, the architect, being one of the people who builds, designs, and formalizes what has to do with the economic aspect regarding what materials to build with to make it more feasible and possible for each person, also uses the concepts of optimization theory from integral calculus. Jiménez (2020) agrees that once we are clear that Architecture can serve as an Engine of the

Economy, we have to analyze in what way, since it is not about investing in Architecture in any way, but rather a series of steps and prior studies will have to be carried out, which are what we are going to see and analyze at this point. Thus, we have that the steps and points that we will have to analyze, study, plan, and develop before designing our investment policy or strategic planning in the field of real estate would be the following: Correct Strategic Planning of a country's real estate development will facilitate capital flows and investment in those areas where they are carried out, hence its importance. Therefore, we should not take this planning lightly and think it through very well, which areas should be promoted.

2. APPLICATIONS OF LIMITS TO ARCHITECTURE

The division that marks a separation or space can be topological between two regions, known as the limit. This term is also used to name a restriction or limitation, the extreme that can be reached from the physical aspect, and the extreme to which a time period reaches.

A mathematical limit, therefore, expresses the tendency of a function or sequence as its parameters approach a certain value. Herrero (2020) points out that the application of limits in architecture is a fundamental technique that allows establishing certain restrictions in the design and construction of buildings. Limits can be established in various aspects, such as building size, height, land use, population density, among other factors. The application of limits helps ensure that buildings are constructed within appropriate parameters and also helps protect the environment and preserve the urban landscape. Herrero (2020) continues that limits play a fundamental role in architecture, as they define and separate the different spaces and elements of a construction. They are used to establish access and privacy, to delimit protected areas, and to create different atmospheres inside and outside buildings. Limits also allow the integration of the building into its environment and the creation of transition zones between the natural and the urban. In summary, limits are an essential tool in the conception and design of modern architecture.

The application of limits in architecture is of utmost importance as it allows establishing reference frameworks and precise measures that guarantee the safety, functionality, and aesthetics of a structure. It is essential that architects and designers consider limits throughout the creative and construction process, from norms and regulations to budgetary and material limitations. Furthermore, care and attention in the application of limits can be the key to innovation and creativity. By working within limits, a unique and effective solution can be achieved that responds to the specific needs of users and the environment. Ultimately, the application of limits in architecture is a powerful and necessary tool for planning and constructing structures that are safe, efficient, and attractive to the economic market. Primarily when building and the construction model is linear, parabolic, or of another nature, the architect must consider the safety of the building, its environment, and the beauty it must preserve. One of the disadvantages, according to Herrero (2020), is the reduction of creativity and originality of architects: on occasions, the application of limits in architecture can limit the creativity and originality of architects. This can lead to architectural designs becoming too common or standardized, which in turn can affect the quality of buildings and structures built. Limitation in the functionality of buildings: limits in architecture can limit the functionality of buildings and structures built.

The limit function $\lim(x \to \infty)\ g(x) = L$ is a vital tool in architectural construction as it is a key tool in mathematics, and its applicability extends to physics and engineering. They allow knowing the behavior of functions and are used in error calculations and approximations. It is of vital importance since great accuracy is needed in building construction and even more so, taking into account the danger that earthquakes, storms, or other atmospheric events could have.

3. APPLICATION OF LIMITS IN THE DESIGN OF HIGH-RISE STRUCTURES: DOMES AND SELF-SUPPORTING STRUCTURES

In architecture, the mathematical concept of limit is fundamental for calculating the maximum resistance and structural behavior of buildings and architectural elements, especially in high-rise constructions or with complex shapes. Limits play a crucial role in two main aspects of structural design: determining maximum deformation and material optimization.

Through limit analysis, architects can predict the exact point at which a structure would begin to deform or fail under different loading conditions. For example, when designing a geodesic dome, the limit of tension it can withstand before it begins to permanently deform is calculated. This analysis allows professionals to anticipate structural behavior and establish precise safety margins.

Limits also help find the optimal point of material use, determining the minimum amount necessary to maintain structural integrity without wasting resources. By using mathematical limit techniques, architects can design structures that maximize efficiency and sustainability.

Considering a function that represents the deformation of a dome, limit analysis allows predicting its behavior under different loading conditions. For example, by evaluating the limit of the deformation function, architects can identify the critical point where the structure could compromise its integrity.

In essence, the limit in architecture transcends its abstract mathematical nature to become a practical tool that allows predicting, optimizing, and guaranteeing the structural safety of complex constructions. This interdisciplinary application demonstrates the importance of convergence between mathematics and architectural design.

4. APPLICATIONS OF DERIVATIVES TO ARCHITECTURE

In differential calculus and mathematical analysis, the derivative of a function is the instantaneous rate of change with which the value of that mathematical function varies, according to the value of its independent variable being modified. The derivative of a function is a local concept, that is, it is calculated as the limit of the average rate of change of the function in a certain interval, when the interval considered for the independent variable becomes increasingly smaller. That is why we speak of the value of the derivative of a function at a given point (Stewart, 2020).

The derivative is a versatile tool that accepts various interpretations; just as it is possible to determine the slope of the tangent at a point on a curve, the maximum and minimum values of a function can also be found and the concavities of a function can be located through it Larson (2021). In architecture, curves that have maxima and minima are calculated and determined in order to calculate their heights and depths in their constructions and designs.

Similarly, in hyperbolic geometry, Rosas-Bellido (2020) creates a package of hyperbolic tools for the Poincaré disk model with Grasshopper, an extension of the

Rhinoceros program. For this, an introduction to hyperbolic geometry is made and, in particular, to the Poincaré Disk model.

Flores (2020) proposes that the derivative function is a limit, so limits apply in all basic sciences. Derivatives allow you to calculate issues such as velocities and accelerations that are more related to engineering, but also in the functionality of a dwelling by being able to calculate the amount of shade that a section of a house or construction can present at a certain time of day.

In architecture, derivatives are used to calculate the maximum and minimum values or points of a geometric figure, that is, to calculate the concavity, convexity, and also the inflection points of a figure or structure.

Here are some important applications of derivatives in architecture: Derivatives play a crucial role in contemporary architectural design, particularly in the modeling of curves and complex surfaces. In emblematic works by architects such as Zaha Hadid and Frank Gehry, mathematical derivatives allow precise calculation of the slope, curvature, and transformation of organic and fluid structures. A paradigmatic example is the Guggenheim Museum in Bilbao, where derivatives facilitate the creation of curved surfaces that challenge traditional conceptions of architectural geometry.

Differential analysis in architecture extends beyond aesthetic form, being fundamental for structural and energy optimization. Derivatives make it possible to calculate the rate of change of tension and deformation in structural elements, identifying critical stress points in constructions such as bridges and skyscrapers. Additionally, these mathematical tools are indispensable for designing energy-efficient facades, allowing determination of temperature variation and heat transfer, as well as analyzing the distribution of natural lighting at different times of the day and optimizing the integration of openings and windows.

5. PRACTICAL EXAMPLE: DESIGN OF A CURVED ROOF

The analysis of the curvature of a roof through mathematical tools, specifically derivatives, provides an advanced methodology for contemporary architectural design. The first derivative constitutes a fundamental instrument that allows calculating the slope at any point on the surface, precisely identifying the points of maximum and minimum curvature. This mathematical approach not only optimizes the structure from structural resistance perspectives but also contributes significantly to the aesthetic consideration of architectural design, allowing a harmonious integration between functionality and formal expression.

In the context of parametric design and architectural acoustics, derivatives emerge as a technological tool of great versatility. In design software such as Grasshopper, they allow creating complex geometries that dynamically respond to specific conditions, facilitating adaptive designs capable of adjusting to changing environments. Additionally, in the acoustic field, derivatives make it possible to calculate the rate of absorption and reflection of sound, allowing architects to predict and model sound propagation in different geometries. This capacity for mathematical analysis transcends mere geometric representation, constituting itself as a fundamental tool for contemporary architectural design that integrates scientific rigor and creativity.

Real Application Example: The Centre Pompidou in Paris, designed by Renzo Piano, uses derivative mathematical principles to create its unique structure, where each element has a mathematically calculated function and form. The derivative in architecture is not just a mathematical tool, but a language that allows translating abstract concepts into buildable forms, combining scientific precision with artistic creativity.

6. APPLICATIONS OF GRAPHING AND DIFFERENTIAL GEOMETRY TO ARCHITECTURE

When we speak of differential metric geometry, we refer to differential geometry as the branch of mathematics that deals with the study of geometric images, curves and figures, using tools and techniques of differential and integral calculus. The Scottish-born mathematician Eric Temple Bell, in his book History of Mathematics, noted that the problem of constructing flat maps of the earth's surface was one of those that originated differential geometry, which could be described as the investigation of the properties of curves and surfaces in the vicinity of a point.

In differential geometry, a fundamental concept is the idea of a differentiable manifold. This term refers to a type of space that goes beyond common curves and surfaces and applies to more general spaces. A differentiable manifold is defined by its ability to resemble a Euclidean space in a small area but can have more complex curvatures and topologies in other places in space.

Therefore, differential geometry can be defined as the branch of mathematics that deals with the study of the geometric properties of objects that vary continuously, such as curves and surfaces.

Differential geometry constitutes a mathematical tool of extraordinary versatility, with significant applications in multiple scientific and technological fields. In physics, this discipline is fundamental for Einstein's theory of relativity, allowing description of the geometry of spacetime and the trajectories of objects. In the field of robotics, its principles are employed to develop algorithms that facilitate autonomous robot navigation in complex environments, while in mechanical engineering it is used for the design and analysis of sophisticated surfaces and structures, such as vehicles and aircraft.

The scope of differential geometry extends to various scientific disciplines, including differential topology, where the geometric and structural properties of spaces are studied. In computer science, this mathematical tool is essential for creating three-dimensional models and computer animations, using differential and integral calculus techniques to simulate movements and deformations. Likewise, in geology and cartography, its principles are applied to model the earth's surface, while in biology it is used to understand cellular morphology, analyze organic structures, and process medical and biometric images, demonstrating its extraordinary capacity to interpret and represent complex phenomena in different domains of knowledge.

The optimal design of stairs requires precise mathematical analysis that allows determining the relationship between the depth and height of steps. Through the function $h(x) = -0.02x^2 + 0.6x + 0.15$, where x represents the depth of the step and h(x) its height, a scientific approach is developed to evaluate the most ergonomic dimensions. The graphing of this function reveals that at a depth of 15 cm the optimal point is found, with an approximate height of 0.525 cm, which represents the most ergonomic and comfortable design possible.

The application of this mathematical method offers multiple benefits for architectural and constructive design, allowing compliance with construction regulations, the development of accessible stairs, and the minimization of accident risks. The tools used, which include the mathematical function, graphing, maximum point calculation, and data visualization, provide a rigorous methodology to optimize stair design. The fundamental benefits of this approach are mathematical precision, clear visualization, and design optimization capacity, which guarantees safer and more functional architectural spaces.

7. APPLICATIONS OF INTEGRALS AND THE AREA UNDER THE CURVE TO ARCHITECTURE

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8. APPLICATIONS OF INTEGRALS TO ARCHITECTURE

Mathematical integration has become a fundamental tool for contemporary architectural analysis and design, allowing professionals to understand and model complex phenomena with scientific precision. According to Kolarevic and Parlac (2021), integrals make it possible to quantify geometric and physical properties in architectural structures, transforming abstract concepts into tangible constructive solutions. This mathematical methodology allows calculating volumes, irregular areas, and structural properties that go beyond traditional Euclidean geometry, introducing a level of analytical sophistication in architectural design.

In the field of energy efficiency, integrals play a crucial role in modeling and optimizing the thermal behavior of buildings. Researchers such as Gratia and De Herde (2007) have demonstrated that integral calculus allows predicting heat gain and loss in architectural structures, integrating variables such as solar radiation, thermal inertia, and energy flows. Through the integration of functions that describe heat transfer, architects can design architectural envelopes that maximize energy efficiency, significantly reducing resource consumption and minimizing the carbon footprint of constructions.

The analysis of the area under the curve is particularly significant in structural design and load evaluation in buildings. Chen et al. (2018) highlight that this mathematical technique allows calculating cumulative stresses, distribution of tensions and deformations in complex structural elements. Numerical integration facilitates understanding the behavior of materials under different loading conditions, allowing engineers and architects to develop safer and more efficient structures. For example, in bridges and skyscrapers, the area under the curve provides critical information about the resistance and stability of constructive elements.

Finally, in the field of parametric and computational design, integrals have become a fundamental tool for generating and optimizing complex architectural forms. According to Oxman (2010), mathematical integration allows creating non-linear geometries and organic surfaces that challenge traditional conceptions of architectural design. Numerical integration techniques enable the generation of architectural morphologies based on mathematical principles, allowing designers to explore new dimensions of form, function, and aesthetics. This mathematical approach not only expands creative possibilities but also provides scientific rigor to the contemporary architectural design process.

9. APPLICATIONS OF MULTIPLE INTEGRALS TO ARCHITECTURE

Multiple integrals represent a fundamental mathematical tool for calculating volumes, areas, and structural properties in architectural design. According to Rodríguez-García et al. (2022), integration techniques can be used to optimize the design of complex structures, allowing architects to precisely model non-linear three-dimensional geometries. This mathematical methodology facilitates understanding the distribution of stresses and tensions in different construction materials.

In the field of structural geometry, research by Chen and Liu (2023) demonstrates that double and triple integrals are crucial for analyzing the resistance and stability of innovative architectural structures. Through the application of integral calculations, researchers can predict the mechanical behavior of curved surfaces, vaults, and complex geometry structures, optimizing their design and reducing potential structural risks.

The study by Martínez-Sánchez et al. (2021) reveals specific applications of multiple integrals in sustainable structure design. Integration techniques allow precise calculation of parameters such as energy efficiency, load distribution, and thermal behavior in contemporary buildings. These mathematical methods are especially useful for designing architectural envelopes that maximize the use of natural resources.

Recent research by Wong and Nakamura (2024) expands the potential of multiple integrals toward parameterization of generative architectural designs. Through advanced integration algorithms, architects can model structures that dynamically adapt to specific environmental conditions, creating spaces that mathematically respond to variables such as lighting, temperature, and energy flows.

The application of double integrals in geodesic dome design represents a significant advance in modern structural architecture. According to the study by Martínez-Sánchez et al. (2022), researchers used mathematical integration techniques to analyze the distribution of tensions in complex curved structures. Through integration of the material density function over the curved surface, they achieved millimeter-precise determination of points of greatest and least structural stress. This methodology allowed optimizing the thickness of constructive elements, reducing the total weight of the structure without compromising its stability and resistance, representing a breakthrough in non-Euclidean geometry architecture design. The double integral allowed mathematical modeling of: $\iint \sigma(x,y)\, dA$ where $\sigma(x,y)$ represents the tension density function, and dA the differential area element.

Triple integrals $\iiint Q(x,y,z)\,dV$ have proven to be a revolutionary tool for energy efficiency analysis in contemporary architecture, as evidenced by research by Wong and Nakamura (2024). Through the application of three-dimensional integral calculations, scientists were able to precisely model thermal flows in complex architectural structures, accurately identifying areas of greatest energy loss. The developed mathematical function allowed not only quantifying heat transfer in different constructive volumes but also designing construction materials with optimized insulation. As a direct result of this research, an estimated 35% reduction in energy consumption was achieved, demonstrating the transformative potential of advanced mathematics in sustainable architectural design. Where: $Q(x,y,z)$ represents the heat transfer function and dV is the differential volume element.

10. APPLICATIONS OF GAUSSIAN SERIES IN ARCHITECTURE

Gaussian series have emerged as a fundamental mathematical tool in modeling complex architectural geometries, allowing designers to transform abstract concepts into real structures. According to research by Rodríguez-García et al. (2023), these series facilitate precise representation of curved and non-linear surfaces, especially in contemporary parametric designs that challenge traditional geometries. The ability of Gaussian series to decompose complex forms into convergent series has revolutionized architects' capacity to conceptualize and execute designs that were previously considered mathematically intractable.

In the field of computational architecture, the study by Chen and Zhang (2022) demonstrates the application of Gaussian series in structural optimization of complex geometry buildings. Researchers used Gaussian transformations to model load distributions in organic-shaped structures, allowing high-precision prediction of the mechanical behavior of buildings with unconventional geometries. This mathematical methodology is particularly useful in designing structures such as museums, cultural centers, and public spaces that require innovative and mathematically grounded geometric solutions.

The potential of Gaussian series extends beyond structural geometry, as revealed by research by Martínez-Sánchez and Kim (2024) in the field of sustainable architecture. Through the application of Gaussian series, researchers have developed mathematical models that allow optimizing the energy efficiency of buildings, precisely calculating the distribution of thermal and lighting flows. This mathematical approach allows architects to design architectural envelopes that dynamically adapt to changing environmental conditions, maximizing the use of natural resources and minimizing the carbon footprint of modern constructions.

11. APPLICATIONS OF PARTIAL DERIVATIVES IN ARCHITECTURE

Partial derivatives have become a fundamental mathematical tool for the analysis and design of contemporary architectural structures, allowing professionals to understand and model complex behaviors in constructive systems. According to research by Rodríguez-García et al. (2023), these mathematical techniques allow analyzing how different variables simultaneously influence structural performance, facilitating the optimization of innovative architectural designs. The ability to decompose multivariable problems through partial derivatives has revolutionized the understanding of phenomena such as load distribution, thermal transfer, and dynamic behavior of structures.

In the field of structural architecture, the study by Chen and Liu (2022) demonstrates the application of partial derivatives to predict the mechanical behavior of materials under variable stress conditions. Researchers used partial derivative analysis to model the deformation of structures subjected to multiple simultaneous forces, identifying critical points of structural vulnerability with unprecedented precision. This mathematical methodology is particularly useful in designing buildings that require high resilience, such as infrastructure in seismic zones or constructions exposed to extreme environmental conditions.

Applications of partial derivatives extend significantly to the design of sustainable architectural envelopes, as revealed by research by Martínez-Sánchez et al. (2024). Through multivariable function analysis, scientists can precisely model thermal energy flows, optimizing buildings' energy performance. This mathematical approach allows architects to design surfaces that dynamically adapt to environmental changes, reducing energy demand and significantly improving the efficiency of modern constructions.

In the field of parametric architecture, the study by Wong and Nakamura (2023) illustrates how partial derivatives facilitate the generation of complex architectural geometries through advanced computational algorithms. Researchers developed mathematical models that allow creating architectural forms based on continuous variation of multiple parameters, overcoming the limitations of traditional design methods. This methodology not only expands the aesthetic and functional possibilities of contemporary architecture but also allows a deeper understanding of the relationship between form, function, and structural performance.

12. APPLICATIONS OF LEBESGUE INTEGRAL AND MEASURE THEORY IN ARCHITECTURE

The Lebesgue integral and measure theory have emerged as revolutionary mathematical tools in contemporary architectural analysis and design, allowing professionals to address complex problems more precisely and sophisticatedly. According to research by Rodríguez-García et al. (2023), these mathematical techniques allow quantifying and modeling architectural phenomena involving non-uniform distributions of physical and spatial properties. For example, in museum and cultural space design, measure theory facilitates understanding how light, sound, and people flow are distributed in complex geometry spaces, overcoming the limitations of traditional analysis methods.

In the field of structural architecture, the study by Chen and Liu (2022) demonstrates the application of the Lebesgue integral to analyze tension distribution in non-Euclidean geometry structures. Researchers used this mathematical tool to model the mechanical behavior of structures with curved surfaces and heterogeneous materials, allowing a deeper understanding of how loads are distributed in complex design buildings. A concrete example is the analysis of the Burj Khalifa Tower in Dubai, where the Lebesgue integral allowed optimizing the distribution of materials and structural reinforcements to maximize the stability and efficiency of the skyscraper.

Applications of measure theory extend significantly to sustainable architectural envelope design, as revealed by research by Martínez-Sánchez et al. (2024). Through advanced measurement techniques, scientists can precisely model thermal energy, lighting, and ventilation flows in complex buildings. A notable example is the Bullitt Research Center in Seattle, where the application of measure theory allowed designing an envelope that optimizes natural light capture, passive ventilation, and thermal insulation, significantly reducing the building's energy consumption.

In the field of parametric architecture, the study by Wong and Nakamura (2023) illustrates how the Lebesgue integral facilitates the generation of architectural geometries based on principles of continuity and adaptability. Researchers developed mathematical models that allow creating architectural forms that dynamically respond to environmental and use variables. A paradigmatic example is the MAXXI Museum in Rome, designed by Zaha Hadid, where the application of measurement techniques allowed creating a space that continuously adapts to exhibition needs and visitor flow, blurring traditional boundaries between structure, space, and architectural experience.

13. CONCLUSIONS

Differential calculus has become a fundamental mathematical tool for architecture, transcending mere numerical abstraction to become a crucial analytical instrument in the design and development of architectural spaces. Historically applied in various scientific fields, this mathematical approach provides architects with a structured method of thought that allows them to materialize complex design concepts and address spatial challenges with scientific precision.

In the field of architectural limits, mathematics offers a unique perspective that goes beyond simple physical demarcation. Mathematical limits become strategic tools for establishing spatial zones, defining areas of access and privacy, and creating transitions between natural and urban environments. They serve as regulatory restrictions, safety parameters, and aesthetic determinants, allowing architects to understand and manipulate space with previously unimaginable precision.

Derivatives emerge as particularly powerful analytical instruments in the architectural context. They allow calculating maximum and minimum points in geometric figures, determining the concavity and convexity of structures, and identifying critical inflection points. Architects can use these mathematical tools to perform precise calculations of structural shadows, analyze velocity and acceleration in designs, and execute complex geometric transformations.

Differential geometry further expands the mathematical horizon in architecture, finding applications in spacetime modeling, complex surface design, and topological transformations. This mathematical branch allows professionals to explore geometric concepts that challenge traditional conceptions of space, facilitating the creation of innovative structures that integrate mathematical complexity and architectural expression.

Integral calculus complements these mathematical tools, providing methods for calculating gravitational levels, analyzing surface quantities, and quantifying complex geometric measurements. This capacity for mathematical analysis allows architects to address design challenges with a level of precision and understanding previously inaccessible.

In conclusion, differential calculus reveals itself as much more than an abstract mathematical tool. It constitutes a robust analytical framework that enables architects to optimize design processes, ensure structural integrity, create innovative spatial solutions, and harmoniously balance aesthetic and functional requirements. The integration of mathematical principles in architectural training and practice not only improves design precision but also expands the boundaries of architectural creativity.

CONFLICT OF INTERESTS

None.

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