STEEL SEMIRIGID STRUCTURES ENERGY STATE UNDER SEISMIC ACTIONS

Moldovan Silviu-Marius 1

1 PhD Student, Civil Engineering Department, Faculty of Constructions, Technical University of Cluj-Napoca, Cluj-Napoca, Romania

ABSTRACT

Intended contribution proposes an energy-based approach to the assessment of capability of semirigid multi-storey steel structures to dissipate seismically induced energy via semirigidly connecting zones of the structure. The energy state of multi-storey structure is defined in terms of the energy balance equation. Total amount of seismically induced energy is divided into its classic components: kinetic energy, strain energy and dissipated energy. Dissipated energy is - in its turn - split into the amount dissipated by the structure itself ($E_{ds}$) and the amount dissipated by the semirigidly connected zones ($E_{dc}$). The last is computed as the equivalent of work performed by the bending moments associated to the semirigid connections through the relative rotations of the connections. Proposed procedure is further illustrated by several dynamic / seismical analyses of one multi-storey steel structure subjected to two reference earthquakes. Beam to column semirigid connections is of top - and seat - angle with double web-angles (TSDW) make up in several degrees of initial stiffness. The bending moment - relative rotation of semirigid connections are governed by Kishi - Chen relation. Obtained numerical results are presented into a comparative graphical manner. Short comments and conclusions end the contribution.

1. INTRODUCTION

Since the semirigidity of skeleton structures has been formally accepted as a new beam-to-column connecting state European Committee For Standardization CEN. (1992), American Institute of Steel Construction, Inc. (1989). their response to seismic action focused on traditional components as storey lateral displacements, their static state - mainly expressed by the bending moment diagrams and the real behaviour of their semirigid connections – expressed via the $M - \theta_r$ relation Frye and Morris (1975), Kishi and Chen (1990), Chen and Lui (1991). Little has been done in regarding the semirigid connections not just in terms of their semirigid behaviour but, as structural zones where induced seismic energy could be dissipated. Traditional seismic analysis of structures still focuses on their mechanical state
viewed as made up of the static-equilibrium state and kinematic compatibility state. Recently classical mechanical state has been enlarged by including energy state as a new component. While the mechanical parameters belonging to static and kinematic states (forces, displacements, strains, stresses) are vectorial entities, the parameters defining energy states are scalar entities which express synthetically the structural behaviour under seismic loads. It has to be emphasized that important mechanical features of a Civil Engineering type structure can only be involved in seismic analysis via energy. Such is the damping phenomenon that cannot be separated by its dual partner – the vibratory motion seismically induced.

The energy includes all structural aspects (seismic action, elastic state, inertia state, damping state) and associates them via mathematical relations that allow to track in time and space the development and evolution of the mechanical state of the structure. By the virtue of its scalar nature, the energy (either through its induced energy component $E_i$ or through its dissipated energy component $E_d$) is a cumulative parameter capable to express not just the present (that can, also, be expressed by the traditional mechanical state) but, also, the past of the mechanical state. The energy state of a structure acted upon by an earthquake is not necessarily a totally new concept. The concept of energy state as a component of (classical) mechanical state has been - in the last decades - associated to the seismic action and response Akbas et al. (2001), Ordaz et al. (2003). Inclusion in the structural analyses - allowed for by energy formulation - of masses set in motion by the dynamic action of earthquake, of their induced velocities and accelerations, widens beneficially the frame of structural analysis and, in the same time, allows for a direct assessment of dynamic response un-affected by substitutive / corrective coefficients.

The intended contribution focuses on energy state of semirigid multi-storey steel structures acted upon by earthquakes. Energy state is defined by its components: seismically induced energy $E_i$, kinetic energy $E_k$, strain energy $E_s$ and damping energy $E_d$. Similarly, to equilibrium equations associated to static state, to compatibility equations associated to kinematic state, energy state is governed by the energy balance equation:

$$E_i = E_k + E_s + E_d$$  \( 1 \)

The proposed contribution focuses on the contribution to dissipated energy component $E_d$ of semirigid connections of the structures in several cases of connecting solutions.

2. METHOD

The computation of energy components of MDOF dynamic systems Figure 1 requires the introduction of well-known vectors and matrices of traditional structural analysis:
Figure 1

MDOF Dynamic System

- $u(nx1)$ - vector of DOF's (lateral storey displacements)
- $\dot{u}(nx1)$ - vector of velocities of masses
- $\ddot{u}(nx1)$ - vector of accelerations
- $M(nx\times n)$ - inertial matrix
- $C(nx\times n)$ - damping matrix
- $R(nx\times n)$ - stiffness matrix
- $\ddot{u}_g$ - ground acceleration

Computation of energy components follows the literature dealing with energy of seismically acted upon structures Uang and Bertero (1990), Uang and Bertero (1988), Manfredi (2001). Therefore, the relations stated below govern the computation of energy components and are assessed by numerical integration.

The relative seismic input energy:

$$E_{i/r} = -\int_0^t \dot{u}(t)^T M \ddot{u}_g(t) \, dt \quad (2)$$

The relative kinetic energy:

$$E_{k/r} = \frac{1}{2} \dot{u}(t)^T M \dot{u}(t) \quad (3)$$

The dissipated (via damping) energy:

$$E_d = \int_0^t \dot{u}(t)^T C \dot{u}(t) \, dt \quad (4)$$

The strain energy:

$$E_s = \int_0^t u(t)^T K u(t) \, dt \quad (5)$$

The specificity of semirigid structures is emphasized in the way the dissipated energy is generated. While in the case of rigidly connected structures, $E_d$ component is associated to the inherent damping properties of the structure itself, in the case of semirigidly connected structures the induce seismic energy is dissipated by both the structure and the semirigid connections. The contribution of semirigid connections to the dissipated energy component is – in fact – the main objective of
the contribution. Therefore, $E_d$ component is splitted into $E_{ds}$ – associated to the structure – and $E_{dc}$ – associated exclusively to the semirigid connections.

$$E_d = E_{ds} + E_{dc} \quad (6)$$

Regarding $E_{dc}$ component, it is generated during relative rotation component $\theta_r$ and its computation is based on the equivalence of elementary work $dL_j$ performed by bending moment $M_j$ of each $j$ connection through elementary relative rotation $d\theta_{r,j}$:

$$dL_j = M_j d\theta_{r,j} \quad (7)$$

Where

$$d\theta_{r,j} = \dot{\theta}_{r,j} dt \quad (8)$$

It leads to a total amount of dissipated energy via semirigid connections:

$$E_{dc} = \sum_{j=1}^{m} \left( \int_{0}^{t_{tc}} M_j \dot{\theta}_{r,j} dt \right) \quad (9)$$

Following a structural analysis program devoted to semirigid multi-storey steel structures, Seismostruct Seismosoft (2022). $M_j$ and $\theta_{r,j}$ are computed as functions of time. Integrating process (9) is computed by summing up the elementary quantities associated to an elementary time step $dt = 0.02$ sec.

**3. STRUCTURES, SEMIRIGID CONNECTIONS, SEISMIC ACTIONS**

In what follows, computation of dissipated energy $E_d$ – and implicitly of its $E_{ds}$ and $E_{dc}$ components – is associated to a 4 storey 3 span semirigid steel (of S355 class) structure **Figure 2**.

**Figure 2**

[Image: Four Storey Frame]
The semirigid connections are of top- and seat- angle with double web-angles (abbreviated as TSDW, from here on) type Figure 3.

**Figure 3**

![TSDW Semirigid Connection](image)

The Kishi-Chen (three parameter) power model \cite{kishi1990} of the M-\(\theta_r\) curve is adopted for the practical modelling of the connections, as described in \cite{chen1997}.

The mechanical and geometrical features of semirigid connections are presented in Table 1 and Table 2, respectively.

**Table 1**

<table>
<thead>
<tr>
<th>Connection</th>
<th>Node TSDW1</th>
<th>Node TSDW2</th>
<th>Node TSDW3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_i) (kNm/rad)</td>
<td>102200</td>
<td>205500</td>
<td>302200</td>
</tr>
<tr>
<td>(M_u) (kNm)</td>
<td>331.615</td>
<td>433.902</td>
<td>461.318</td>
</tr>
<tr>
<td>(n) (-)</td>
<td>1.151</td>
<td>0.891</td>
<td>0.827</td>
</tr>
</tbody>
</table>

- \(R_i\) - initial connection stiffness
- \(M_u\) - ultimate connection moment capacity
- \(n\) - shape parameter

**Table 2**

<table>
<thead>
<tr>
<th>Connection</th>
<th>Node TSDW1</th>
<th>Node TSDW2</th>
<th>Node TSDW3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_{sup};t_{inf}) (mm)</td>
<td>14</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>(t_{in}) (mm)</td>
<td>9</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>(l_{sup};l_{inf}) (mm)</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>(l_{in}) (mm)</td>
<td>400</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Bolt size</td>
<td>M20</td>
<td>M20</td>
<td>M24</td>
</tr>
<tr>
<td>(g_{sup}) (mm)</td>
<td>68</td>
<td>65</td>
<td>63</td>
</tr>
</tbody>
</table>
Seismic actions are introduced via recorded ground accelerations of El Centro 1940 NS Figure 4 and Vrancea 1977 NS Figure 5 earthquakes, scaled down to peak values of 0.20g and 0.25g, respectively.

### Figure 4

![El Centro 1940 NS - Ground Acceleration](image)

### Figure 5

![Vrancea 1977 NS - Ground Acceleration](image)

4. RESULTS AND DISCUSSION

Computed results include seismically induced energy $E_i$, dissipated energy by the structure itself $E_{ds}$, dissipated energy by the semirigid connections $E_{dc}$ and the total dissipated energy $E_d$. The results are presented graphically in a comparative...
manner Figure 6, Figure 7, Figure 8, Figure 9, Figure 10, Figure 11, Figure 12, Figure 13, Figure 14, Figure 15, Figure 16, Figure 17, Figure 18, Figure 19, Figure 20. The fraction of critical damping is considered $\zeta = 5\%$. The presented results are excerpts from a larger study regarding the capability of semirigid steel structures to dissipate seismically induced energy.

**Figure 6**

![Energy Input Figure 6](image)

**Figure 6** Input Energy $E_I$ – El Centro 1940 NS

**Figure 7**

![Energy Input Figure 7](image)

**Figure 7** Input Energy $E_I$ – Vrancea 1977 NS

**Figure 8**

![Energy Input Figure 8](image)

**Figure 8** Dissipated energy $E_{dc}$ – El Centro 40 NS
Figure 9: Dissipated Energy $E_{dc}$ – Vrancea 1977 NS

Figure 10: Total Dissipated Energy $E_d$ – El Centro 40 NS

Figure 11: Total Dissipated Energy $E_d$ – Vrancea 1977 NS
Figure 12 Energy Components – Vrancea 1977 NS – TSDW1

Figure 13 Energy Components $E_{dc}$ And $E_d$ – Vrancea 1977 NS – TSDW1

Figure 14 Contribution of $E_{dc}$ To $E_d$ – Vrancea 1977 NS – TSDW1
Figure 15

Energy Components – Vrancea 1977 NS – TSDW2

Figure 16

Energy Components $E_{dc}$ And $E_d$ – Vrancea 1977 NS – TSDW2

Figure 17

Contribution of $E_{dc}$ To $E_d$ – Vrancea 1977 NS – TSDW2
Figure 18
Energy Components – El Centro 40 NS – TSDW3

Figure 19
Energy Components $E_{dc}$ and $E_d$ – El Centro 40 NS – TSDW3

Figure 20
Contribution of $E_{dc}$ to $E_d$ – El Centro 40 NS – TSDW3
The two components of the dissipated energy are separately emphasized for each case of mechanical make-up semirigid beam-to-column connections. Clear remarks regarding the amount of dissipated energy by the structure itself and by the semirigid connections are possible from the comparative graphical manner the results are presented.

As it can be seen from the results, the dissipated energy via the semirigid connections $E_{dc}$ accounts for an important percent (50-70%) of the total dissipated energy $E_d$.

5. CONCLUSIONS
The proposed objective has been accomplished by computed energy parameters associated to dissipation capability of semirigid connections. The first conclusion to presented results is that semirigidity of beam-to-column connections is capable of dissipating seismically induced energy.

A second conclusion can be drawn referring to the amount of dissipated energy by semirigid connections $E_{dc}$ as part of the total dissipated energy $E_d$. It may be concluded that this part depends to a large extent on both seismic action and rigidity of the connection. Therefore, in the design activity of semirigid multi-storey steel structures, the specificity of semirigid connections should be closely associated to the specificity of seismic actions.

CONFLICT OF INTERESTS
None.

ACKNOWLEDGMENTS
None.

REFERENCES


