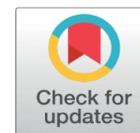


DETERMINATION OF RETAILER'S OPTIMAL POLICY IN A PRICE-SENSITIVE INVENTORY MODEL FACILITATING TWO TYPES OF PAYMENT SCHEME FOR THE CUSTOMERS



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ABSTRACT

This article investigates the effect of implementation of two types of payment scheme to the customer by a retailer in an inventory system. Our study is to determine optimal cycle time and selling price of a commodity when two types of payment scheme namely immediate payment scheme as well as trade credit policy are available to the customer. In reality, two trends of payment scheme are available in business world. Some people want to pay immediately after purchase while some favors some time delay in payment. From retailer's point of view, in many situations, retailer faces scarcity of capital to start his business. Now- a- day bank loan is available to the retailer at ease. We formulate a mathematical model keeping in mind all these things. Two tier credit policies have been considered. First of all, bank offers a delay period to repay loan. Secondly, retailer also offers a credit period to customer. Different cases possible owing to variable duration of credit periods. Our ultimate goal is to optimize retailer's profit for different cases arisen in the model. A numerical example has been posed and discussed in support of the model.

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Keywords: Inventory, Price, Bank Loan Facility, Two Types of Payment Scheme for Customers

1. INTRODUCTION

Mathematical modeling in inventory become very attractive day by day to the researchers and practitioners as problems associated with inventory has a close relation with business, economy, and finance. Harris (1913) was first to introduce inventory in mathematical modeling. There after many developments has been made. Determination of price of a commodity is very common problem in inventory. General perception is that decrease in selling price increases demand rate. Abad (1996) considered dynamic pricing together with lot-sizing in an inventory model. In another work Abad (2001). he assumed selling price dependent demand rate for items which decays instantaneously. After that many inventory model has been developed taking demand rate as a monotonic decreasing function of selling price. It should be noted that demand rate may be linear or non-linear function of selling price. As limitation of capital is a major challenge in today's business world, different types of credit policies have a great impact in that world. Studies based on the effect of inclusion of trade credit policy in an inventory system have become much attractive to the modelers and interpreters This work was started by Goyal (1985). He was the pioneer to include the effect of



introduction of single stage trade credit policy in the inventory system. [Huang \(2003\)](#) extended the concept between supplier and retailer. He assumed that retailer's offered credit period to the customer is less than the credit period offered by the supplier to the retailer. [Jaggi et al. \(2008\)](#) introduced credit linked demand rate in their model. [Chung \(2013\)](#) implemented the concept of two-level trade credit periods in economic production quantity model with storage capacity constraints. [Chung et al. \(2014\)](#) considered the facility of permissible delay in payment in economic production quantity model for deteriorating items. [Giri and Sharma \(2016\)](#) incorporated the concept of permissible delay in payment in a model where demand rate increases linearly with time. [Pal \(2018\)](#) examined the effect of trade credit policy with partial backlogging of shortages. [Giri and Sharma \(2019\)](#) developed inventory model with partial trade credit policy. [Tiwary et al. \(2018\)](#) introduced two level partial trade credit policy in a three-echelon supply chain associated with perishable items. [Mandal et al. \(2020\)](#) introduces reliability in a production inventory model with two tier credit policy. An inventory model with two stage deterioration under the atmosphere of permissible delay option was formulated by [Pal et al. \(2021\)](#).

2. NOTATIONS

Q	Ordering quantity of the retailer
W	Purchasing cost per unit item of the retailer
P	Selling price per unit item of the retailer
S	Set up cost
D	Demand rate
c_h	Holding cost per unit item
T	Cycle length
B	Credit period offered by the bank to the retailer
C	Credit period offered by retailer to the customer
I_o	Obtained interest rate
I_p	Paid interest rate.
η	A fraction portion of the total customer make immediate payment on purchase of goods
$I(t)$	Retailer's inventory level at time

3. BASIC ASSUMPTIONS

- A single item is considered
- Time horizon is infinite
- Retailer has some capital to arrange set-up cost. But he has to go for bank loan for ordering quantity Q . Banks provide a credit period. After that period bank will charge interest for remaining unpaid amount. Retailer can enjoy interest from sells revenue up to the time B .
- Obtained interest rate is less than paid interest rate.
- Retailer faces two types of customers. One category wants to pay immediately after purchase of a thing while another category wants to avail some time delay for making payment.
- Retailer offers a credit period C to the customer and sells all items by the time T . From the point of category of customer enjoying permissible delay in payment scheme, if buys item at time ξ ($0 < \xi \leq T$) arranges payment at time $\xi + C$.

- Retailer's offered credit period is less than his cycle length ($C < T$)
- Demand rate D is a monotonic function which decreases linearly with selling price p . Exact form of demand rate is $D = D(p) = a - b p$ with $a > 0, b > 0$.

4. MODEL FORMULATION WITH JUSTIFICATION

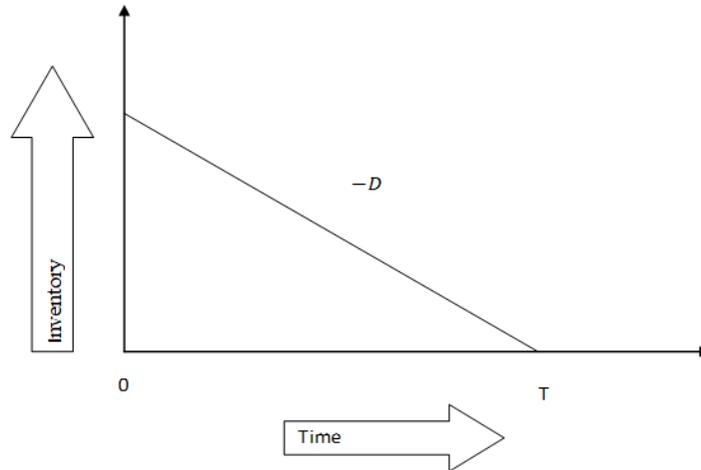


Figure 1 Retailer's time weighted inventory

Retailer ordering quantity is Q . This amount depletes at the rate $-D$.

The differential equation which governs inventory level of the retailer is given below.

$$\frac{dI}{dt} = -D \text{ subject to initial condition } I(0) = Q \quad (1)$$

Solution is given by

$$I(t) = Q - D T \quad (2)$$

$$\text{Terminal condition } I(T) = 0 \text{ gives, } Q = DT \quad (3)$$

Total holding cost is given as

$$c_h \int_0^T I(t) dt = \frac{c_h}{2} D T^2 \text{ [using (2) and (3)]} \quad (4)$$

Retailer goes for bank loan for purchasing amount Q . So, bank loan amount is $Q w = DT w$. Two cases come up.

Either Case1. $B \leq C$ or Case2. $C < B$. Now, Case 2 can be subdivided into three Sub Cases on the basis of the fact that retailer collects his final payment from the customer at time $T + C$. Possible three Sub Cases are given below.

- Sub Case2.1 $B < T$
- Sub Case2.2 $T < B < T + C$
- Sub Case 2.3 $T + C < B$

Now we will discuss all situations with figures.

Case 1. $B \leq C < T < T + C$

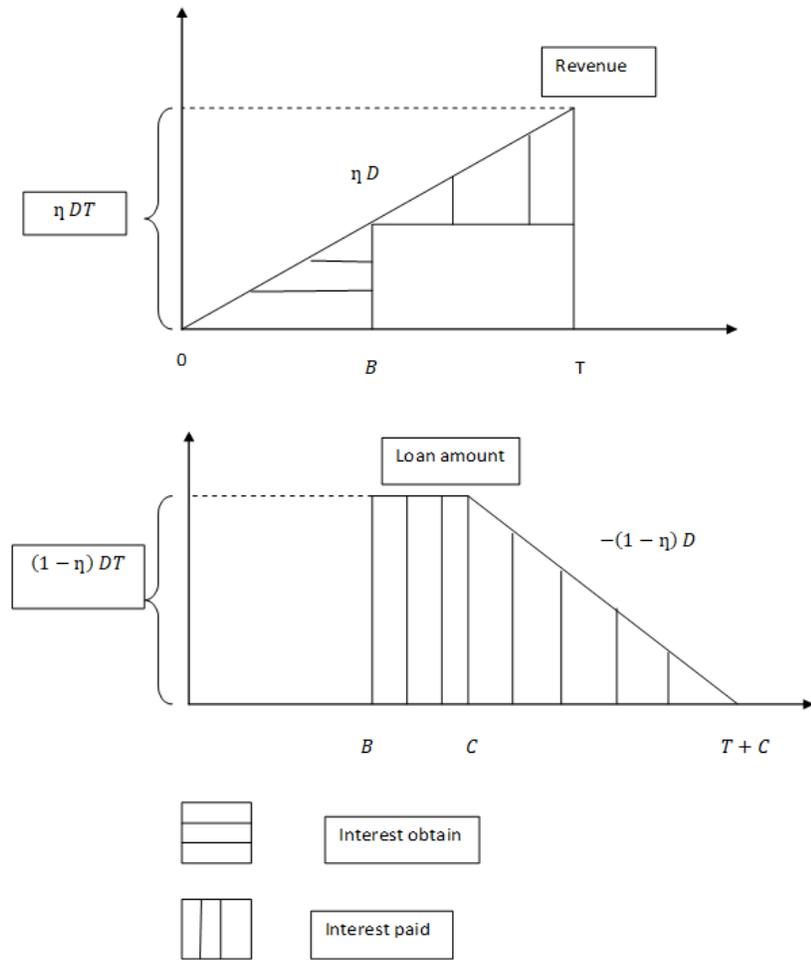


Figure 2 Immediate payment scheme (above) and trade credit situation (below) under Case 1

For immediate payment, retailer enjoys the interest from sales revenue for the period $[0, B]$. He has to pay interest for the period $[B, T]$.

$$\text{Interest obtained} = pI_o \frac{1}{2} \eta D B^2, \text{ Interest paid} = w I_p \frac{1}{2} \eta D (T - B)^2$$

For permissible delay in payment scheme, retailer does not enjoy any interest from sales revenue. Interest paid = $w I_p \{ (1 - \eta)DT (C - B) + \frac{1}{2} (1 - \eta)D T^2 \}$

Case 2.1: $C < B <$

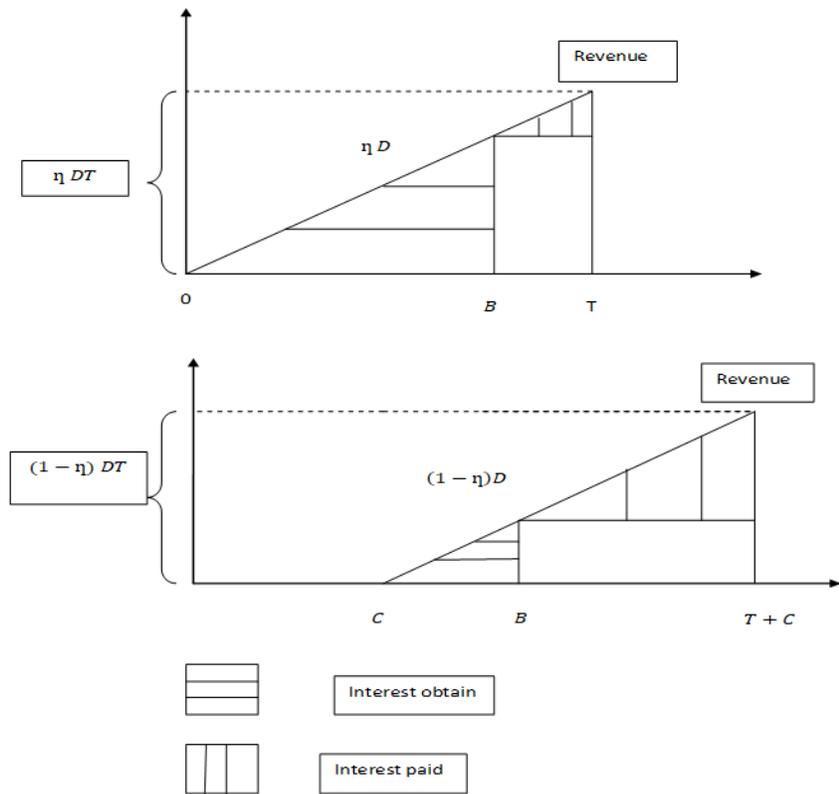


Figure 3 Immediate payment scheme (above) and trade credit situation (below) under Sub Case 2.1

For immediate payment, Interest obtained = $pI_o \frac{1}{2} \eta DB^2$, Interest paid = $w I_p \frac{1}{2} \eta D(T - B)^2$.

For permissible delay in payment scheme,

Interest obtained = $pI_o \frac{1}{2} (1 - \eta) D(B - C)^2$,

Interest paid = $w I_p \frac{1}{2} (1 - \eta) D(T + C - B)^2$

Case 2.2: $C < T < B < T + C$

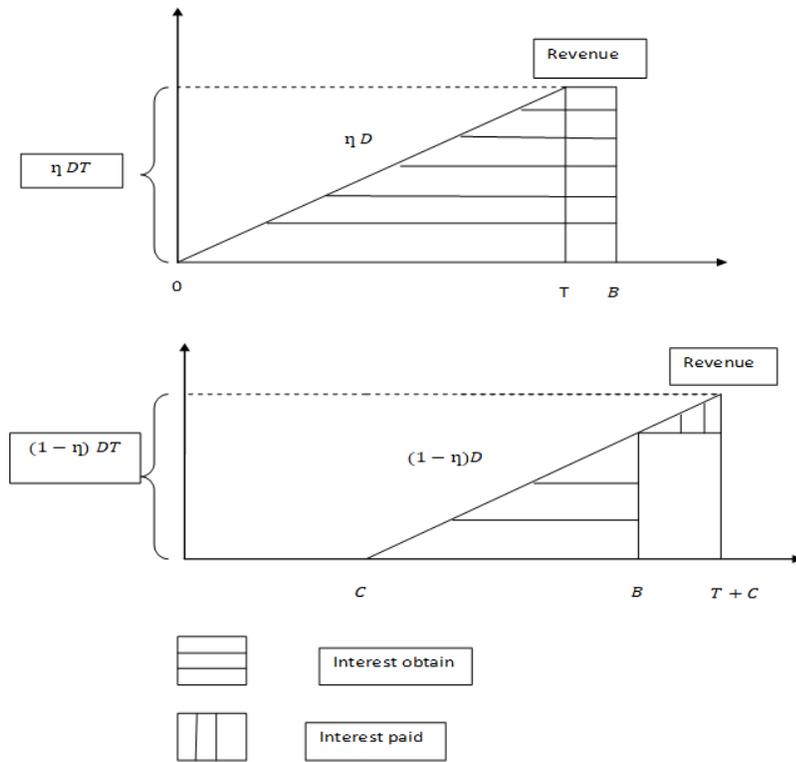


Figure 4 Immediate payment scheme (above) and trade credit situation (below) under Sub Case 2.2

For immediate payment, Interest obtained = $pI_o \left\{ \frac{1}{2} \eta DT^2 + \eta DT (B - T) \right\}$

Interest paid = 0.

For permissible delay in payment scheme,

Interest obtained = $pI_o \frac{1}{2} (1 - \eta) D(B - C)^2$

Interest paid = $w I_p \frac{1}{2} (1 - \eta) D(T + C - B)^2$

Case 2.3: $C < T < T + C < B$

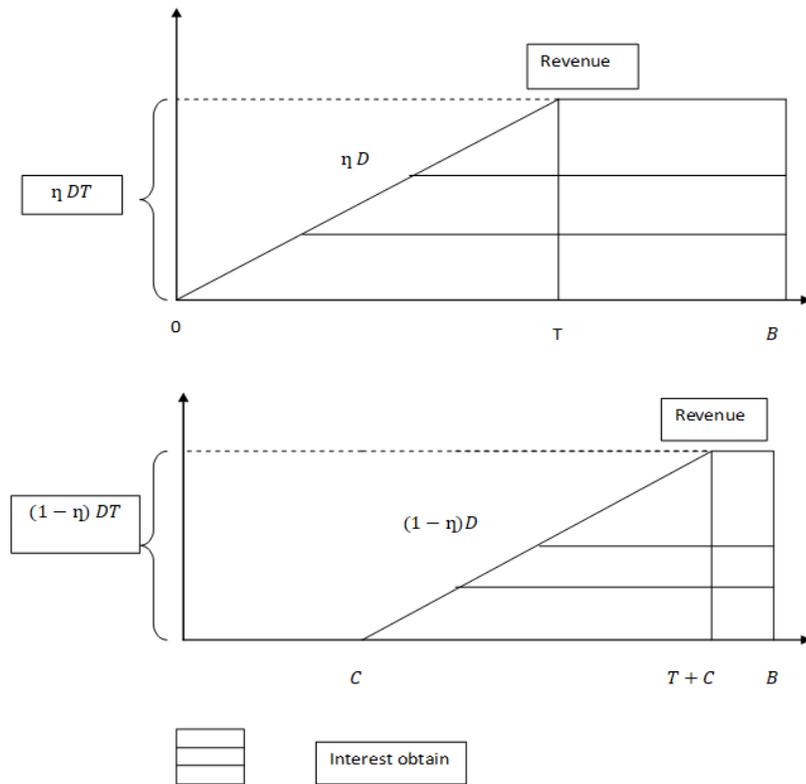


Figure 5 Immediate payment scheme (above) and trade credit situation (below) under Sub Case 2.3

$$\text{Interest obtained} = pI_o \left\{ \frac{1}{2} \eta DT^2 + \eta DT (B - T) \right\}$$

$$\text{Interest paid} = 0$$

For permissible delay in payment scheme,

$$\text{Interest obtained} = pI_o \left\{ \frac{1}{2} (1 - \eta)DT^2 + (1 - \eta)DT (B - T - C) \right\}$$

$$\text{Interest paid} = 0$$

Retailer’s average profit = (Selling price – Bank loan- Set up cost- Holding cost + Interest obtained from sales revenue – Interest paid to the bank) / Cycle length.

Average profit function of the retailer for different situations is given in the following table.

Table 1		
Case / Sub Case	Profit function	Expression
1	π_1	$\left[Qp - Qw - S - \frac{c_h}{2} DT^2 + pI_o \frac{1}{2} \eta DB^2 - wI_p \frac{1}{2} \eta D(T - B)^2 - wI_p \left\{ (1 - \eta)DT (C - B) + \frac{1}{2} (1 - \eta)DT^2 \right\} \right] / T$
2.1	π_2	$\left[Qp - Qw - S - \frac{c_h}{2} DT^2 + pI_o \frac{1}{2} \eta DB^2 + pI_o \frac{1}{2} (1 - \eta) D(B - C)^2 - wI_p \frac{1}{2} \eta D(T - B)^2 - wI_p \frac{1}{2} (1 - \eta) D(T + C - B)^2 \right] / T$

2.2	π_3	$[Q p - Q w - S - \frac{c_h}{2} D T^2 + p I_o \{ \frac{1}{2} \eta D T^2 + \eta D T (B - T) \} + p I_o \frac{1}{2} (1 - \eta) D (B - C)^2 - w I_p \frac{1}{2} (1 - \eta) D (T + C - B)^2] / T$
2.3	π_4	$[Q p - Q w - S - \frac{c_h}{2} D T^2 + p I_o \{ \frac{1}{2} \eta D T^2 + \eta D T (B - T) \} + p I_o \{ \frac{1}{2} (1 - \eta) D T^2 + (1 - \eta) D T (B - T - C) \}] / T$

5. SOLUTION METHODOLOGY

We first solve the following partial differential equations

$$\frac{\partial \pi_i}{\partial p} = 0, \frac{\partial \pi_i}{\partial T} = 0 \text{ for } i = 1,2,3,4$$

Then we get solutions for cycle length and selling price as $= p^*, T = T^*$. These solutions are optimal if the eigen values of the following Hessian matrix at (p^*, T^*) are negative.

$$H = \begin{bmatrix} \frac{\partial^2 \pi_i}{\partial p^2} & \frac{\partial^2 \pi_i}{\partial p \partial T} \\ \frac{\partial^2 \pi_i}{\partial p \partial T} & \frac{\partial^2 \pi_i}{\partial T^2} \end{bmatrix}$$

6. NUMERICAL EXAMPLE

Values of parameters for different situations are given below.

Common parameters to all situations.

S	\$500 per set up
c_h	\$1.5 per unit
I_o	\$0.07 per annum
I_p	\$0.09 per annum
η	0.3
w	\$6 per unit
a	100
b	4

Parameter varies along with situations.

Case 1	Sub Case 2.1	Sub Case 2.2	Sub Case 2.3
B = 2	B = 2	B = 4	B = 6
C = 3	C = 1	C = 2	C = 2

Optimal results obtained

	Case 1	Sub Case 2.1	Sub Case 2.2	Sub Case 2.3
p^*	\$17.56	\$17.11	\$16.58	\$15.99

T^*	4.01	3.86	3.54	3.25
Average profit	\$99.07	\$124.10	\$163.01	\$238.36

Conditions of optimality

Eigen values of the hessian matrix H	Case 1	Sub Case 2.1	Sub Case 2.2	Sub Case 2.3
	-17.01, -6.20	-18.35, -6.49	-22.44, -7.37	-29.91, -8.88

Now we show three-dimensional plotting of retailer’s average profit function with respect to two decision variables selling price and cycle length for Case1, Sub Case 2.1, Sub Case 2.2, Sub Case 2.3 respectively.

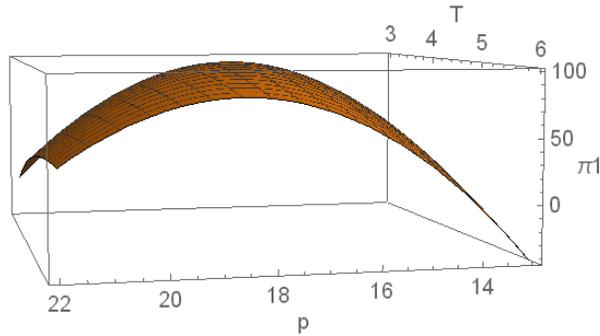


Figure 6 Profit function for Case 1 with respect to decision variables selling price (p) and cycle length (T)

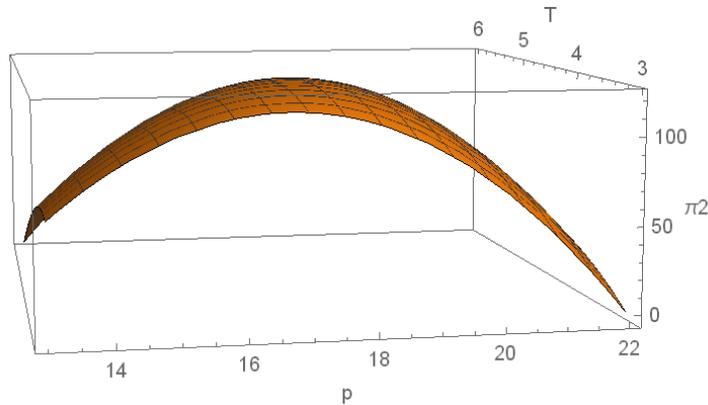


Figure 7 Profit function for Sub Case 2.1 with respect to decision variables

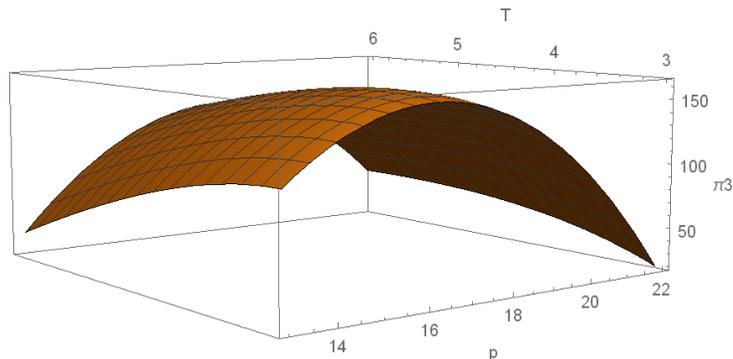


Figure 8 Profit function for Sub Case 2.2 with respect to decision variables

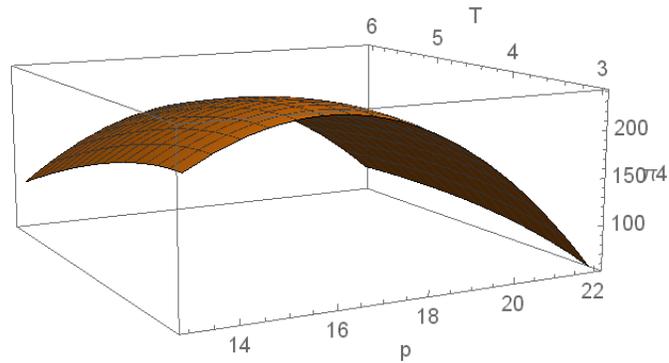


Figure 9 Profit function for Sub Case 2.3 with respect to decision variables

All these diagrams ensure maximization of retailer's profit function. This section has been performed with the help of MATHEMATICA SOFTWARE.

7. CONCLUSION

In this work we have presented a mathematical model which maximizes retailer average profit depending on decision variables namely cycle time and selling price. Different situations associated with two stage trade credit have been considered. It has been observed from numerical results that retailer profit increases as length of the credit period offered by the bank increases. Here we take demand pattern based on selling price linearly. This work may be extended by taking non-linear demand pattern. Also credit linked as well as selling price dependent demand pattern may be a possible extension of this modelling frame work.

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