EFFECT OF MASS AND HEAT TRANSFER IN OSCILLATORY TWO DIMENSIONAL FLOW OF A MICRO POLAR FLUID OVER AN INFINITE PERMEABLE PLATE IN A POROUS MEDIUM UNDER THE EXISTENCE OF MAGNETIC FIELD

Ashok Kumar #1, Jyoti Chawla *2
1 BSAITM, Faridabad, 121005, India
2 MVN University, Palwal, 121105, India

Abstract:
In this paper we study the effect of Mass and Heat transfer under the existence of transverse magnetic field for an oscillating two dimensional Micro-polar fluid flow through a moving infinite permeable plate in a porous medium. Investigate the Solutions for angular momentum, governing momentum, energy and concentration equations. Study the effect of chemical reaction, permeability parameters, velocity profiles, micro rotation profiles, wall stress coefficient and skin friction coefficient by using graph.

Keywords: Mass and Heat transfer; Chemical reaction; Micro Polar fluid; MHD; Porous Medium.


1. Introduction

The fluids which contain dilute suspension of micro-molecules with individual motion are micro-polar fluid. This fluid deals with a class of special fluid which gives microscopic effect arising from local structure of micro motion of the micro polar fluid elements. This fluid have wide practical application i.e. Liquid – crystal, analyzing the behavior of exotic lubricants, additive suspension, turbulent shear flow, polymeric fluid etc. also Micro polar fluid over a vertical plate passes through a porous media has many different practical applications in modern industries and applications such as foams, Porous rocks, foamed solids, alloys, polymer blends, nuclear waste, building thermal insulator, power plant etc.

Erigen [1] first introduced and formulated the theory of micro polar fluid. This theory shows the effect of couple stress and local rotary inertia. Micro polar fluid theory is expected to a mathematical model of non-Newtonian fluid behavior which is observed in certain fluid like as colloidal fluid, liquid crystal etc. also Erigen [2] developed the theory of thermo- micropolar fluids. The effect of permeability medium on thermal convective in micropolar fluids has been
studied by Sharma and Gupta [3]. Beg, Bhargava, Rawat, Takhar and Beg [4] have studied the mass transfer of chemical reacting and double –diffusive free convection of a micropolar fluid passes through porous regime along a vertical stretching plane. Jat, Saxena and Rajotia [5] studied about the steady laminar flow of electrically conducting incompressible micropolar fluid passes through a porous medium under the effect of transverse magnetic field and the effect of various parameters like as magnetic material, porosity , Prandlt number etc. for corresponding temperature field and velocity have been discussed through graphical representation in the continuation Olajuwon and Oahimire [6] studied the effect of effect of thermal radiation and thermal-diffusion on micropolar free convective MHD fluid between rotating semi-infinite porous plate under the existence of transverse magnetic field also the infinite plate is oscillate with constant frequency due to which solutions are oscillating type. Chaudhary, Singh and Jain [7] studied about effect of different parameters on free convection fluid flow for amagneto-polar fluid in the existence of uniform magnetic field and thermal radiation through a porous medium. In the continuation the combined effect of dufour and Soret on a mixed convection unsteady MHD mass and heat transfer in a porous medium for a micropolar fluid in the existence of heat generation , thermal radiation, chemical reaction and ohmic heating have been studied by Aurangzaib, Kasim, Mohammed and Shafic [8]. In the continuation Aurangzaib and Shafic [9] investigate the effect of injection or suction on unsteady flow under the effect of magnetic field with mass and heat transfer in a micropolar fluid near the forward stagnation point flow. Mass and heat transfer effect on an unsteady chemically reacting MHD flow of a micropolar fluid over an vertical infinite porous plate with thermal radiation is investigate by Reddy, Babu, Varma and Reddy [10]. Mohanty, Mishra and Pattanayak [11] studied about the mass and heat transfer characteristics of electrically conducting incompressible viscous micropolar fluid also the flow past over a stretching sheet which is passes through porous media under the presser of viscous dissipation. Gopal and Prasad [12] investigate the combined effect of Dufour and Soret effects on unsteady convective mass and heat transfer flow of a micropolar fluid passes through porous media under a permeable sheet also micro rotation, velocity, concentration, temperature have been discussed for different values and couple stress, skin friction and the ration of mass and heat transfer have been evaluated for different parametric variables. Recently Kumar and Lal [13] studied the effect of oscillatory motion of a visco-elastic dusty fluid under the existence of magnetic field which is passes through a porous medium and find approximate solutions of and velocity distribution and skin friction.

In the present paper we study the effect of Mass and Heat transfer under the existence of transverse magnetic field for an oscillating two dimensional Micro-polar fluid flow through an moving infinite permeable plate in a porous medium. Investigate the Solutions for angular momentum, governing momentum, energy and concentration equations. Study the effect of chemical reaction, permeability parameters, velocity profiles, micro rotation profiles, wall stress coefficient and skin friction coefficient by using graph.

2. Nomenclature

- \( \rho \) ----- Density
- \( v \) ------ Kinematic viscosity
- \( V_r \) ----- Kinematic rotational viscosity
- \( g \) ------ Acceleration due to gravity
3. Problem Formulation

Consider the two dimensional unsteady mixed convection flow of viscous, incompressible; electrically conduction micro polar fluid passes through a vertically infinite moving porous plate. The strength of magnetic field $B_0$ which is applied to the perpendicular to the surface and the effect of megalithic field is neglected. Along the plane surface in the upward direction we take $x$-axis and normal to it take $y$-axis. Flow variables are function of $y$ and time $t$ only due to infinite plane surface assumption. We assume initially the fluid as well as plate is at rest and after some time whole system allow to move with a constant velocity. Also at $t=0$, the plate temperature is suddenly increase to $T_w$ and remains constant thereafter.

The equations for such a motion are given by:

\[
\frac{\partial v}{\partial y} = 0 \quad \text{(Continuity equation)} \tag{1}
\]

\[
\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = (v + v_r) \frac{\partial^2 u}{\partial y^2} + 2v_r \frac{\partial \omega}{\partial y} + g \beta_T(T - T_{\infty}) + g \beta_C(C - C_{\infty}) - \frac{v + v_r}{\kappa} u - \frac{\sigma B_0^2}{\rho} u \tag{2}
\]

(Linear momentum)

\[
\rho j \left( \frac{\partial \omega}{\partial t} + v \frac{\partial \omega}{\partial y} \right) = \gamma \frac{\partial^2 \omega}{\partial y^2} \quad \text{(Angular momentum)} \tag{3}
\]

\[
\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad \text{(Energy equation)} \tag{4}
\]

\[
\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \gamma_1 (C - C_{\infty}) \quad \text{(Concentration equation)} \tag{5}
\]

At any point $(x,y)$ velocity component are $(u,v)$, $T$ is the temperature of the fluid, angular velocity normal to $xy$ plane is $\omega$ and mass concentration is $C$.

\[
u = u_p \quad , \quad \omega = -\eta \frac{\partial u}{\partial y} \quad , \quad T = T_{\infty} + \epsilon(T_{\omega} - T_{\infty})e^{nt} \quad , \quad C = C_{\infty} + \epsilon(C_{\omega} - C_{\infty})e^{nt} \quad \text{at} \quad y = 0
\]

\[
u \rightarrow 0 \quad , \quad \omega \rightarrow 0 \quad , \quad T \rightarrow T_{\infty} \quad , \quad C \rightarrow C_{\infty} \quad \text{as} \quad y \rightarrow \infty \tag{6}
\]

Integrating Equation (1)

\[
\frac{\partial v}{\partial y} = -V_0 \tag{7}
\]

Where $V_0$ is a scale of suction velocity.
Now introducing the non-dimensional variables
\[ u = U_0 u^*, \quad v = V_0 v^*, \quad y = \frac{v^*}{V_0} y^*, \quad u_p = U_0 U_p, \quad \omega = \frac{v_0}{v^*} \omega^* \]
\[ t = \frac{v^*}{V_0} t^*, \quad T - T_\infty = (T_w - T_\infty) \theta^*, \quad C - C_\infty = (C_w - C_\infty) \Phi^*, \quad \eta = \frac{v_0^2}{v^*} \eta^*, \]
\[ j = \frac{v^2}{V_0^2} j^*, \quad P_r = \frac{v^*}{\alpha}, \quad S_c = \frac{v}{\rho}, \quad M = \frac{\sigma B_0^2 v^*}{\rho v_0^2}, \quad G_{rt} = \frac{v^* g(T_w - T_\infty)}{u_0 V_0^2}, \quad G_{rc} = \frac{v^* g(\beta - C_w)}{u_0 V_0^2}, \]
\[ \gamma = \left( \mu + \frac{\lambda}{2} \right), \quad j^* = \mu j^* (1 + \frac{\beta}{2}), \quad \beta = \frac{\lambda}{\mu} = \frac{v_r}{v^*}, \]
\[ K' = \frac{K^* u_0 V_0^2}{v^*}, \quad \eta = \frac{\mu j^*}{\gamma} = \frac{2}{2 + \beta}, \quad \gamma_1 = \frac{v r_1}{v V_0^2} \] (8)

Where \( U_0 \) is the free stream velocity and \( \beta \) denotes the dimensional viscosity ratio, \( \lambda \) is the coefficient of vortex viscosity.

By using equations (7) and (8), Equations (2) – (5) reduce and omitting * we get
\[ \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = (1 + \beta) \frac{\partial^2 u}{\partial y^2} + 2\beta \frac{\partial \omega}{\partial y} + G_{rt} \theta + G_{rc} \phi - M u - \frac{1 + \beta}{K} u \] (9)
\[ \frac{\partial \omega}{\partial t} - \frac{\partial \omega}{\partial y} = \frac{1}{\eta} \frac{\partial^2 \omega}{\partial y^2} \] (10)
\[ \frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\eta} \frac{\partial^2 \theta}{\partial y^2} \] (11)
\[ \frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial y} = \frac{1}{S_c} \frac{\partial^2 \phi}{\partial y^2} + \gamma_1 \phi \] (12)

With boundaries condition \( u = U_p, \quad \omega = -n \frac{\partial u}{\partial y}, \quad \theta = 1 + \epsilon \epsilon \epsilon nt, \quad \phi = 1 \epsilon \epsilon \epsilon nt \) at \( y = 0 \)
\[ u \to 0, \quad \omega \to 0, \quad \theta \to 0, \quad \phi \to 0, \] at \( y \to \infty \) (13)

Solution of Equations (9) – (12) by using Boundary conditions (13) we assume the solution for small value of \( \epsilon \)
\[ u(y, t) = u_0(y) + \epsilon \epsilon \epsilon nt u_1(y) + O(\epsilon^2) \]
\[ \omega(y, t) = \omega_0(y) + \epsilon \epsilon \epsilon nt \omega_1(y) + O(\epsilon^2) \]
\[ \theta(y, t) = \theta_0(y) + \epsilon \epsilon \epsilon nt \theta_1(y) + O(\epsilon^2) \]
\[ \phi(y, t) = \phi_0(y) + \epsilon \epsilon \epsilon nt \phi_1(y) + O(\epsilon^2) \] (14)

By putting the equations (14) into equations (9) –(13) , we get
\[ (1 + \beta) u_0'' + u_0' - \left( M - \frac{1 + \beta}{K} \right) u_0 = -G_{rt} \theta_0 - G_{rc} \phi_0 - 2\beta \omega_0' \] (15)
\[ (1 + \beta) u_1'' + u_1' + \left( n - M - \frac{1 + \beta}{K} \right) u_1 = -G_{rt} \theta_1 - G_{rc} \phi_1 - 2\beta \omega_1' \] (16)
\[ \omega_0'' + \eta \omega_0' = 0 \] (17)
\[ \omega_1'' + \eta \omega_1' + n \eta \omega_1 = 0 \] (18)
\[ \theta_0'' + P_r \theta_0' = 0 \] (19)
\[ \theta_1'' + P_r \theta_1' + n P_r \theta_1 = 0 \] (20)
\[ \phi_0'' + S_c \phi_0' + S_c \theta_1 \phi_0 = 0 \] (21)
\[ \phi_1'' + S_c \phi_1' + S_c(n + \gamma_1) \phi_1 = 0 \] (22)

Now boundary conditions are
\[ u_0 = U_p, \quad u_1 = 0, \quad \omega_0 = -n \epsilon \epsilon \epsilon nt, \quad \omega_1 = -n \epsilon \epsilon \epsilon nt \]
\[ \theta_0 = 1, \quad \theta_1 = 1, \quad \phi_0 = 1, \quad \phi_1 = 1 \] at \( y = 0 \)
Now solve the equations (15)-(22) by using equations (23) and putting the solution into equation (14), we get

\[
u = a_1 e^{-h_2 y} - a_2 e^{-P_r y} + a_3 e^{\eta y} + a_4 e^{-h_5 y} + \epsilon (b_1 e^{-h_1 y} + b_2 e^{-h_3 y} + b_3 e^{-h_4 y} + b_4 e^{-h_6 y}) e^{nt} \quad (24)
\]

\[
\omega = c_1 e^{-\eta y} + \epsilon (c_2 e^{-h_1 y}) e^{nt} \quad (25)
\]

\[
\theta = e^{-P_r y} + \epsilon (e^{-h_4 y}) e^{nt} \quad (26)
\]

\[
\phi = e^{-h_5 y} + \epsilon (e^{-h_6 y}) e^{nt} \quad (27)
\]

Where

\[
h_1 = \frac{\eta}{2} \left[ 1 + \sqrt{1 + 4n} \right]
\]

\[
h_2 = \frac{1}{2(1+\beta)} \left[ 1 + \sqrt{1 + 4(M + \frac{1+\beta}{K}) (1+\beta)} \right]
\]

\[
h_3 = \frac{1}{2(1+\beta)} \left[ 1 + \sqrt{1 + 4(n + M + \frac{1+\beta}{K}) (1+\beta)} \right]
\]

\[
h_4 = \frac{P_r}{2} \left[ 1 + \sqrt{1 + \frac{4n}{P_r}} \right]
\]

\[
h_5 = \frac{S_c}{2} \left[ 1 + \sqrt{1 - \frac{4y_1}{S_c}} \right]
\]

\[
h_6 = \frac{S_c}{2} \left[ 1 + \sqrt{1 - \frac{4(y_2-n)}{S_c}} \right]
\]

\[
a_1 = U_p - a_2 - a_3 - a_4
\]

\[
a_2 = - \frac{G_{RT}}{(1+\beta)P_t^2 - P_r - (M + \frac{1+\beta}{K})}
\]

\[
a_3 = \frac{2\beta \eta}{(1+\beta)\eta^2 - \eta - (M + \frac{1+\beta}{K})} c_1 = \lambda c_1
\]

\[
a_4 = - \frac{G_{cc}}{(1+\beta)h_5^2 - h_5 - (M + \frac{1+\beta}{K})}
\]

\[
b_1 = \frac{2\beta h_1}{(1+\beta)h_1^2 - h_1 - (n + M + \frac{1+\beta}{K})} c_2 = \xi c_2
\]

\[
b_2 = -(b_1 + b_3 + b_4)
\]
\[ b_3 = -\frac{G_r T}{(1+\beta)h_4^2 - h_4 - (n+M+1+\beta)} \]

\[ b_4 = -\frac{G_{rc}}{(1+\beta)h_6^2 - 6 - (n+M+1+\beta)} \]

\[ c_1 = \frac{n_1[h_2 u_p - h_2 a_2 - h_2 a_4 + P_r a_2 + h_5 a_4]}{1 + n_1 \lambda(h_2 - \eta)} \]

\[ c_2 = \frac{n_1 b_2(h_4 - h_3) + n_1 b_3(h_6 - h_3)}{1 + n_1 \xi(h_3 - h_1)} \]

Shear stress:

\[ \tau_w^* = (\mu + \Lambda) \left[ \frac{\partial u}{\partial y} \right]_{y=0} + [\Lambda \omega]_{y=0} \]

\[ = \rho U_0 V_0 [1 + (1 - n_1) \beta] u'(0) \quad (28) \]

Skin-friction factor:

\[ C_f = \frac{2\tau_w^*}{\rho U_0 V_0} = 2[1 + (1 - n_1) \beta] u'(0) \]

\[ = 2[1 + (1 - n) \beta] [-a_1 h_2 + a_2 P_r - a_4 h_5 - \eta a_3 - \epsilon e^{nt} (b_1 h_1 + b_2 h_3 + b_3 h_4 + b_4 h_6)] \]

Couple stress:

\[ M_w = \gamma \left[ \frac{\partial \omega}{\partial y} \right]_{y=0} \quad (30) \]

Couple stress coefficient:

\[ C_w' = \frac{M_w v^2}{\rho U_0 V_0^2} = \omega'(0) = -c_1 \eta + \epsilon e^{nt} (-h_1 c_2) \quad (31) \]

Nusselt number:

\[ Nu = x \left[ \frac{\partial T}{\partial y} \right]_{y=0} \quad (32) \]

\[ Nu R e_x^{-1} = -\phi'(0) = P_r + h_4 \epsilon e^{nt} \]

where \[ Re_x = \frac{x v}{v} \] (Reynolds number)

Local Sherwood number (Rate of mass transfer):

\[ Sh = x \left[ \frac{\partial C}{\partial y} \right]_{y=0} \]

\[ Sh R e_x^{-1} = -\phi'(0) = h_5 + h_6 \epsilon e^{nt} \quad (33) \]
4. Result and Discussion

Figure 1: (u with different M)
\[ t = 1, \varepsilon = 0.01, n_1 = 0.5, n = 0.1, \beta = 1, G_{rt} = 2, G_{rc} = 1, P_r = 1, S_c = 2, U_p = 0.5, K = 5, \gamma_1 = 0.1, \eta = 0.0 \]

Figure 2: (u with different Grt)
\[ t = 1, \varepsilon = 0.01, n_1 = 0.5, n = 0.1, \beta = 1, M = 5, G_{rc} = 1, P_r = 1, S_c = 2, U_p = 0.5, K = 5, \gamma_1 = 0.1, \eta = 0.01 \]

Figure 3: (u with different t)
\[ G_{rt} = 2, \varepsilon = 0.01, n_1 = 0.5, n = 0.1, \beta = 1, M = 5, G_{rc} = 1, P_r = 1, S_c = 2, U_p = 0.5, K = 5, \gamma_1 = 0.1, \eta = 0.01 \]
Figure 4: (u with different K)
\[ t = 1, \varepsilon = 0.01, n_1 = 0.5, n = 0.1, \beta = 1, M = 2, G_{rt} = 2, G_{rc} = 1, P_r = 1, S_c = 2, U_p = 0.5, \gamma_1 = 0.1, \eta = 0.01 \]

Figure 5: (\omega with different K)
\[ t = 1, \varepsilon = 0.01, n_1 = 0.5, n = 0.1, \beta = 1, M = 2, G_{rt} = 2, G_{rc} = 1, P_r = 1, S_c = 2, U_p = 0.5, K = 1, \gamma_1 = 0.1, \eta = 0.01 \]

Figure 6: (\omega with different \gamma_1)
\[ t = 1, \varepsilon = 0.01, n_1 = 0.5, n = 0.1, \beta = 1, M = 2, G_{rt} = 2, G_{rc} = 1, P_r = 1, S_c = 2, U_p = 0.5, K = 2, \eta = 0.01 \]
Figure 7: ($\omega$ with different M)

t = 1, \varepsilon = 0.01, n_1 = 0.5, n = 0.1, \beta = 1, M = 4, G_{rt} = 2, G_{rc} = 1, P_r = 1, S_c = 2, U_p = 0.5, K

= 5, \gamma_1 = 1, \eta = 0.01

Figure 8: ($\theta$ with different t)

\varepsilon = 0.1, n = 0.1, P_r = 1, \gamma_1 = 0.2

Figure 9: ($\theta$ with different Pr)

\varepsilon = 0.01, n = 0.1, t = 10, \gamma_1 = 0.2
Figure 10: (Φ with different Sc)

\[ t = 10, \varepsilon = 0.01, n = 0.1, \quad \gamma_1 = 0.3 \]

Figure 11: (Φ with different t)

\[ S_c = 0.40, \varepsilon = 0.01, n = 0.1, \quad \gamma_1 = 0.2 \]

Figure 12: (Φ with different \( \gamma_1 \))

\[ t = 10, S_c = 1, \varepsilon = 0.01, n = 0.1 \]
Figure 13: (Cf with different K)

\[ \varepsilon = 0.01, n_1 = 0.5, n = 0.1, \beta = 1, M = 5, G_{rt} = 2, G_{rc} = 1, P_r = 1, S_c = 2, U_p = 0.5, \gamma_1 = 0.1, \eta = 0.01 \]

Figure 14: (Cf with different M)

\[ \varepsilon = 0.01, n_1 = 0.5, n = 0.1, \beta = 1, G_{rt} = 2, G_{rc} = 1, P_r = 1, S_c = 2, U_p = 0.5, K = 5, \gamma_1 = 0.1, \eta = 0.01 \]

Figure 15: (Nu with different Pr)

\[ \varepsilon = 0.01, n = 0.1 \]
Figure-1 shows that velocity component $u$ is increasing for the decreasing value of magnetic field parameter $M$ but in figure 2, 3, 4 shows that velocity component increasing with increasing the value of $G_{rt}$, time $t$, and permeability parameter $K$ also in figure 5 to figure 7 shows that Rotational velocity $\omega$ increasing with decreasing the value of permeability parameter $K$, Chemical reaction parameter $\gamma_1$, magnetic field parameter $M$. In figure 8 temperature $\theta$ increasing with increasing the value of time $t$.Figure 9 shows that value of temperature $\theta$ increasing with decreasing the value of prandlt number $Pr$. Figure 10 and 12 shows that value of concentration $\varphi$ increasing with decreasing the value of $Sc$ and Chemical reaction parameter $\gamma_1$. Figure 11 shows that value of concentration $\varphi$ increasing with increasing the value time $t$.Figure 13 and 14 shows that value of local friction $C_f$ increasing with increasing the value of permeability parameter $K$ and decreasing with increasing the value of magnetic field parameter $M$. Figure 15 shows that Nusselt number increasing with increasing the value of prandlt number $Pr$.

5. Conclusions

We investigate the effect of Mass and Heat transfer under the existence of transverse magnetic field for an oscillating two dimensional Micro-polar fluid flow through an moving infinite permeable plate in a porous medium. Investigate the Solutions for angular momentum, governing momentum, energy and concentration equations were solved by using small perturbation approximation method. The results are studied through graph for different values of fluid flow parameters. The conclusion are:

- Velocity component $u$ increasing with increasing the value of $G_{rt}$, time $t$, permeability parameter $K$ and decreasing with increasing the value of magnetic field parameter $M$.

- Rotational velocity $\omega$ increasing with decreasing the value of magnetic field parameter $M$, Chemical reaction parameter $\gamma_1$ and decreasing with the value of permeability parameter $K$.

- Temperature $\theta$ decreasing with the increasing the value of prandlt number $Pr$ and increasing with increasing the value of time $t$.

- Concentration $\varphi$ increasing with increasing the value of time $t$ and decreasing with increasing the value of $Sc$ and Chemical reaction parameter $\gamma_1$.

- Local friction $C_f$ increasing with increasing the value of permeability parameter $K$ and decreasing with increasing the value of magnetic field parameter $M$.

- Nusselt number increasing with increasing the value of prandlt number $Pr$.

References


*Corresponding author.

E-mail address: Ashok2bahin@gmail.com/jyotichawla@mvneducation.com