ISSN: 2454-1907
DOI: https://doi.org/10.29121/ijetmr.v6.i4.2019.365

IJETMR
International Journal of Engineering Technologies and Management Research
A Knowledge Repository

FULL CAR ACTIVE DAMPING SYSTEM FOR VIBRATION CONTROL

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Abstract:

Full car passive and active damping system mathematical model was developed. Computer simulation using MATLAB was performed and analyzed. Two different road profile were used to check the performance of the passive and active damping using Linear Quadratic Regulator controller (LQR)Road profile 1 has three bumps with amplitude of 0.05m, 0.025 m and 0.05 m. Road profile 2 has a bump with amplitude of 0.05 m and a hole of -0.025 m. For all the road profiles, there were 100% amplitude reduction in Wheel displacement, Wheel deflection, Suspension travel and body displacement, and 97.5% amplitude reduction in body acceleration for active damping with LQR controller as compared to the road profile and 54.0% amplitude reduction in body acceleration as compared to the passive damping system. For the two road profiles, the settling time for all the observed parameters was less than two (2) seconds. The present work gave faster settling time for mass displacement, body acceleration and wheel displacement.

Keywords: Damping; Vibration; Full Car; Amplitude; LQR Controller; Simulation.

Cite This Article: G. Yakubu, G. Sani, S. B. Abdulkadir, A. A. Jimoh, and M. Francis. (2019). "FULL CAR ACTIVE DAMPING SYSTEM FOR VIBRATION CONTROL." *International Journal of Engineering Technologies and Management Research,* 6(4), 1-17. DOI: https://doi.org/10.29121/ijetmr.v6.i4.2019.365.

1. Introduction

Effective control vibration usually requires several techniques, each of which contributes to a quieter environment [1]. For most applications, vibration can be controlled using four methods, namely Absorption, Use of barriers and enclosures, Structural damping and Vibration isolation/suspension [1].Although there is a certain degree of overlap in these classes, each method may yield a significant reduction in vibration and noise by proper analysis of the problem and application of the technique. The principles behind the use of absorption materials and heavy mass barrier layers are generally understood, so this thesis will focus on active damping/suspension, which deal with reducing vehicle vibration.

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

The implementation of control systems can improve a vehicle performance by reducing the vibration levels to acceptable values, defined for each case [2]. This can be achieved by several control devices that can be used to apply forces to the vehicle, calculated by a specific control algorithm.

[3] Defines vibration as a repetitive motion of an object in alternatively opposite direction from the equilibrium position when that equilibrium has been disturbed, which occurs in machines, structures, and dynamic systems leading to unwanted consequences. [3] describes vibration as a problem due to unwanted motion, noise and stresses that may lead to failure and /or fatigue of a machines or structures, losses in energy, reliability decreased and low performance below expectation.

According to [4] there are three types of damping system namely, passive damping, semi-active damping and active damping system.

'Active damping control' is realized by regarding the piezos as either sensors or actuators, to be used within a control loop, Contrary to passive vibration control. For active vibration control, two situations are often distinguished: collocation and non-collocation.

2. Vehicle Model Review

A Quarter-car model (Figure 1) is usually used for the analysis of suspension, because of it simplicity and can take the significant features of the full car model [5] & [6].According to [7] the equation for the model motions are found by adding upwafd and downward (vertical) forces on the car body (sprung mass) and the wheel and axle (unsprung mass).According to [7] the car body or sprung mass is represented by M1 while the tyre and axle or unsprung mass is represented byM2, what make up the damping system are the spring, damper, and a force-generating element that is variable and is positioned between the car body and the tyre and axle. Generally, the roll and pitch angle can be obtained from the connection between the car body and the wheel and axle masses.

From [7], the quarter car and full car model are the same, just that there is other additional consideration that most b added when dealing with full car. These are; Rolling, Pitching and Bouncing which are represented in the X, Y and Z axis respectively.

Figure 1 to figure 3 as described by [7], [8] and [9] shows the quarter, half and full car model respectively.

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

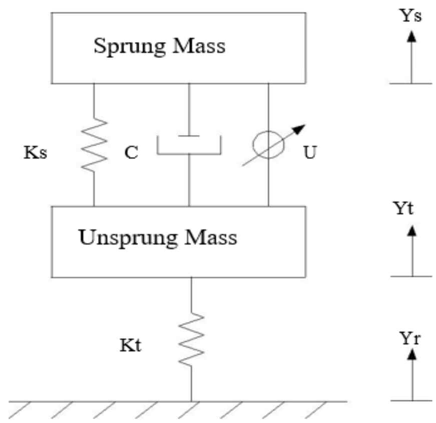

Figure 1: Quarter Car Model [10]

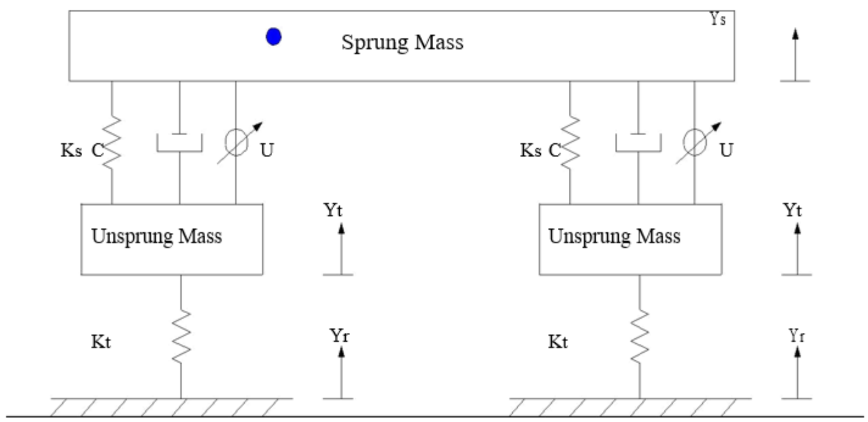

Figure 2: Half Car Model [11]

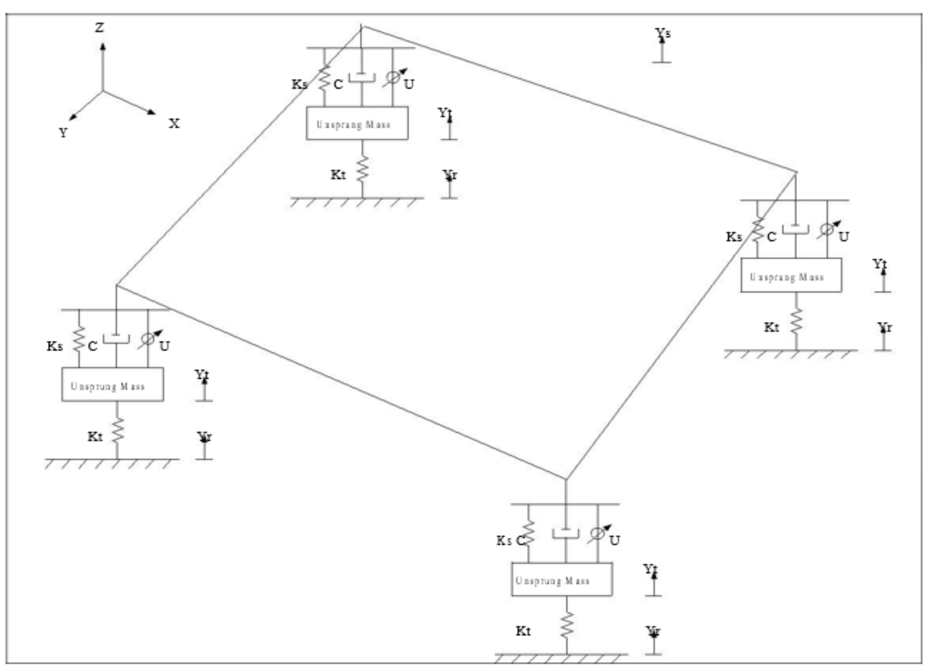

Figure 3: Full Car Model[12]

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

3. Mathematical Modeling; Passive Damping

According to [7], the vehicle body or sprung mass is free to bounce, roll and pitch. The damping system connects the vehicle body to the four wheel and axle masses, these are the front-right, front-left, rear right and rear left wheels. From [7] and [9], all the wheel and axle masses are free to bounce vertically with respect to the car body.

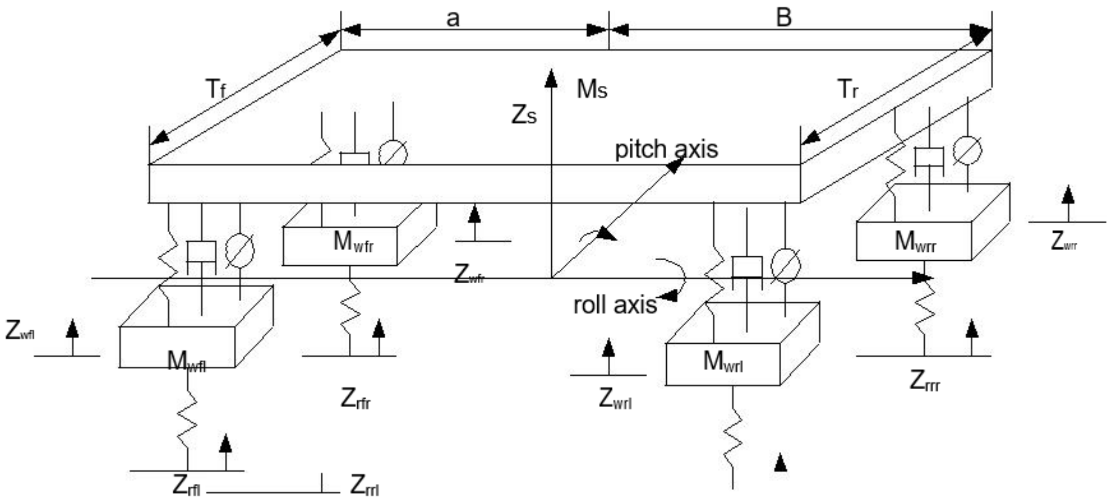

Figure 4: for Full Car Model for Passive and Active Damping

Rolling Motion; Sprung Mass

$$I_r\ddot{\varphi}_s = -d_f T_f[(T_f\dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfr}] + d_f T_f[(-T_f\dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfl}] - d_r T_r[(T_r\dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrr}] + d_r T_r[(-T_r\dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrl}] - k_f T_f[(T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfr}] + k_f T_f[(-T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfl}] - k_r T_r[(T_r\varphi_s - b\varphi_s + Z_s) - Z_{wrr}] + k_r T_r[(-T_r\varphi_s - b\varphi_s + Z_s) \text{------------} \quad (1)$$

Pitching Motion; Sprung Mass

$$I_p\ddot{\theta}_s = -d_f a[(T_f\dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfr}] - d_f a[(-T_f\dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfl}] + d_r b[(T_r\dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrr}] + d_r b[(-T_r\dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrl}] - k_f a[(T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfr}] - k_f a[(-T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfl}] + k_r b[(T_r\varphi_s - b\varphi_s + Z_s) - Z_{wrr}] + k_r b[(-T_r\varphi_s - b\varphi_s + Z_s) - Z_{wrl}] \text{--} \quad (2)$$

 ISSN: 2454-1907
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Bouncing; Sprung Mass

$$M_s\ddot{Z}_s = -d_f[(T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfr}] - d_f[(-T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfl}] - d_r[(T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrr}] - d_r[(-T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrl}] - k_f[(T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfr}] - k_f[(-T_f\ \varphi_s + a\varphi_s + Z_s) - Z_{wfl}] - k_r[(T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrr}] - k_r[(-T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrl}] \quad (3)$$

For each side of Wheel Motion (Vertical Direction)

$$M_{wfr}\ddot{Z}_{wfr} = d_f\ [(T_f\dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfr}] + K_f\ [(T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfr}] - K_{tf}Z_{wfr} + K_{tf}Z_{rfr} \quad (4)$$

$$M_{wfl}\ddot{Z}_{wfl} = d_f[(-T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfl}] + K_f[(-T_f\ \varphi_s + a\varphi_s + Z_s) - Z_{wfl}] - K_{tf}Z_{wfl} + K_{tf}Z_{rfl} \quad (5)$$

$$M_{wrr}\ddot{Z}_{wrr} = d_r\ [(T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrr}] + K_r\ [(T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrr}] - K_{tr}Z_{wrr} + K_{tr}Z_{rrr} \quad (6)$$

$$M_{wrl}\ddot{Z}_{wrl} = d_r\ [(-T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrl}] + K_r\ [(-T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrl}] - K_{tr}Z_{wrl} + K_{tr}Z_{rrl} \quad (7)$$

Where
M_s = mass of the car body or sprung mass (kg)
M_{uf} and M_{ur} = front and rear mass of the wheel or unsprung mass (kg)
I_p and I_r = pitch and roll of moment of inertia (kg m^2)
Z_s = car body displacement (m)
Z_{fr}, Z_{fl}, Z_{rr} and Z_{rl} = car body displacement for each corner (m)
Z_{wfr}, Z_{wfl}, Z_{wrr} and Z_{wrl} = wheel displacement (m)
T_f and T_r = treat (m)front and rear
a = distance from centre of car body mass to front wheel (m)
b = distance from centre of sprung mass to rear wheel (m)
d_f and d_r = front and rear damping (Nm/s)
k_f and k_r = stiffness of car body spring for front and rear(N/m)
k_{tf} and k_{tr} = stiffness of tire for front and rear (N/m)

 ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

The state variables of the system are shown in Table 1 and the definition of each state variables are given in Table 2.

Table 1: Full Car Model State Variable

φ_s = X1	$\dot{\varphi}_s$ = X8
θ_s =X2	$\dot{\theta}_s$ = X9
Z_s = X3	$\dot{Z}_s$ = X10
Z_{wfr} = X4	$\dot{Z}_{wfr}$ = X11
Z_{wfl} = X5	$\dot{Z}_{wfl}$ = X12
Z_{wrr} = X6	$\dot{Z}_{wrr}$ =X13
Z_{wrl} = X7	$\dot{Z}_{wrl}$ = X14

Table 2: Variables Definitions

Variables	Definitions
φ_s	Roll angle
$\dot{\varphi}_s$	Roll rate
θ_s	Pitch angle
$\dot{\theta}_s$	Pitch rate
Z_s	Vertical displacement
$\dot{Z}_s$	Vertical velocity
Z_{wfr}	Vertical displacement of front right wheel
$\dot{Z}_{wfr}$	Vertical velocity of front right wheel
Z_{wfl}	Vertical displacement of front left wheel
$\dot{Z}_{wfl}$	Vertical velocity of front left wheel
Z_{wrr}	Vertical displacement of rear right wheel
$\dot{Z}_{wrr}$	Vertical velocity of rear right wheel
Z_{wrl}	Vertical displacement of rear left wheel
$\dot{Z}_{wrl}$	Vertical velocity of rear left wheel

Equations (1) ~ (7) is written instate space form as below

$\dot{\varphi}_s = \dot{X}_1 \approx X_8$
$\dot{\theta}_s = \dot{X}_2 \approx X_9$
$\dot{Z}_s = \dot{X}_3 \approx X_{10}$
$\dot{Z}_{wfr} = \dot{X}_4 \approx X_{11}$
$\dot{Z}_{wfl} = \dot{X}_5 \approx X_{12}$
$\dot{Z}_{wrr} = \dot{X}_6 \approx X_{13}$
$\dot{Z}_{wrl} = \dot{X}_7 \approx X_{14}$

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

$\dot{X}_8 = \ddot{\varphi}_s \approx [-d_f T_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + d_f T_f ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) - d_r T_r ((T_r X_8 - bX_9 + X_{10}) - X_{13}) + d_r T_r ((-T_r X_8 - bX_9 + X_{10}) - X_{14}) - K_f T_f ((T_f X_1 + aX_2 + X_3) - X_4) + K_f T_f ((-T_f X_1 + aX_2 + X_3) - X_5) - K_r T_r ((T_r X_1 - \mathrm{b}X_2 + X_3) - X_6) + K_r T_r ((-T_r X_1 - bX_2 + X_3) - X_7)]/\mathrm{I_r}$

$\dot{X}_9 = \ddot{\theta}_s \approx [-d_f a ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + d_f a ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) + d_r b ((T_r X_8 - bX_9 + X_{10}) - X_{13}) + d_r b ((-T_r X_8 - bX_9 + X_{10}) - X_{14}) - K_f a ((T_f X_1 + aX_2 + X_3) - X_4) - K_f a ((-T_f X_1 + aX_2 + X_3) - X_5) + K_r b ((T_r X_1 - \mathrm{b}X_2 + X_3) - X_6) + K_r b ((-T_r X_1 - bX_2 + X_3) - X_7)]/\mathrm{I_p}$

$\dot{X}_{10} = \ddot{Z}_s \approx [-d_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + d_f ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) - d_r ((T_r X_8 - bX_9 + X_{10}) - X_{13}) - d_r ((-T_r X_8 - bX_9 + X_{10}) - X_{14}) - K_f ((T_f X_1 + aX_2 + X_3) - X_4) - K_f ((-T_f X_1 + aX_2 + X_3) - X_5) - K_r ((T_r X_1 - \mathrm{b}X_2 + X_3) - X_6) - K_r ((-T_r X_1 - bX_2 + X_3) - X_7)]/\mathrm{M_s}$

$\dot{X}_{11} = \ddot{Z}_{wfr} \approx [d_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + K_f ((T_f X_1 + aX_2 + X_3) - X_4) - K_{tf} X_4 + K_{tf} Z_{rfr}]/M_{uf}$

$\dot{X}_{12} = \ddot{Z}_{wfl} \approx [d_f ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) + K_f ((-T_f X_1 + aX_2 + X_3) - X_5) - K_{tf} X_5 + K_{tf} Z_{rfl}]/M_{uf}$

$\dot{X}_{13} = \ddot{Z}_{wrr} \approx [d_f ((T_f X_8 - bX_9 + X_{10}) - X_{13}) + K_r ((T_r X_1 - bX_2 + X_3) - X_6) - K_{tr} X_6 + K_{tr} Z_{rrr}]/M_{ur}$

$\dot{X}_{14} = \ddot{Z}_{wrl} \approx [d_r ((-T_r X_8 - bX_9 + X_{10}) - X_{14}) + K_r ((-T_r X_1 - bX_2 + X_3) - X_7) - K_{tr} X_7 + K_{tr} Z_{rrr}]/M_{ur}$

Followed by, covert equation into the matrix yield

$$\dot{X}(\mathrm{t}) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} [\mathrm{x(t)}] + \mathrm{f}(t) \quad \text{---} \quad (8)$$

Where,

$\dot{X}(\mathrm{t}) = [\dot{X}_1\, \dot{X}_2\, \dot{X}_3\, \dot{X}_4\, \dot{X}_5\, \dot{X}_6\, \dot{X}_7 \dot{X}_8\, \dot{X}_9\, \dot{X}_{10}\, \dot{X}_{11}\, \dot{X}_{12}\, \dot{X}_{13}\, \dot{X}_{14}]^{\mathrm{T}}$

$\dot{X}(\mathrm{t}) \approx [\dot{\varphi}_s\, \dot{\theta}_s\, \dot{Z}_s\, \dot{Z}_{wfr}\, \dot{Z}_{wfl}\, \dot{Z}_{wrr}\, \dot{Z}_{wrl}\, \ddot{\varphi}_s\, \ddot{\theta}_s\, \ddot{Z}_s\, \ddot{Z}_{wfr}\, \ddot{Z}_{wfl}\, \ddot{Z}_{wrr} \ddot{Z}_{wrl}]^{\mathrm{T}}$

$\mathrm{X(t)} = [X_1\, X_2\, X_3\, X_4\, X_5\, X_6\, X_7\, X_8\, X_9\, X_{10}\, X_{11}\, X_{12}\, X_{13}\, X_{14}]^{\mathrm{T}}$

$\mathrm{X(t)} \approx [\varphi_s\, \theta_s\, Z_s\, Z_{wfr}\, Z_{wfl}\, Z_{wrr}\, Z_{wrl}\, \dot{\varphi}_s\, \dot{\theta}_s\, \dot{Z}_s \dot{Z}_{wfr}\, \dot{Z}_{wfl}\, \dot{Z}_{wrr}\, \dot{Z}_{wrl}]^{\mathrm{T}}$

3.1. Mathematical Modeling; Active Damping

Mathematical modeling of active damping is derived from Figure 4. [7] The equations of motion for full car model can be derived as follows.

 ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

equation (9) ~ equation (15) shows the equation of rolling, pitching and bouncing motion of the sprung mass and wheel motion.

Rolling Motion; Sprung Mass

$I_r\ddot{\varphi}_s = -d_fT_f[(T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfr}] + d_f\ T_f[(-T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfl}] - d_rT_r[(T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrr}]$
$+ d_rT_r[(-T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrl}] - k_fT_f[(T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfr}] + k_fT_f[(-T_f\ \varphi_s + a\varphi_s + Z_s) - Z_{wfl}]$
$- k_rT_r[(T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrr}] + k_rT_r[(-T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrl}] + T_fu_{fr} - T_fu_{fl} + T_ru_{rr} - T_fu_{rl}$ -- (9)

Pitching motion; Sprung Mass

$I_p\ddot{\theta}_s = -d_fa[(T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfr}] - d_f\ a[(-T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfl}] + d_rb[(T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrr}]$
$+ d_rb[(-T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrl}] - k_f\ a[(T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfr}] - k_f\ a[(-T_f\ \varphi_s + a\varphi_s + Z_s) - Z_{wfl}]$
$+ k_rb[(T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrr}] + k_rb[(-T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrl}] + au_{fr} + au_{fl} - bu_{rr} - bu_{rl}$ -- (10)

Bouncing; Sprung Mass

$M_s\ddot{Z}_s = -d_f[(T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfr}] - d_f[(-T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfl}] - d_r[(T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrr}]$
$- d_r[(-T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrl}] - k_f[(T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfr}] - k_f[(-T_f\ \varphi_s + a\varphi_s + Z_s) - Z_{wfl}] - k_r[(T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrr}] - k_r[(-T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrl}] + u_{fr} + u_{fl} + u_{rr} + u_{rl}$ ---------------- (11)

For each side of Wheel Motion (Vertical Direction)

$M_{uf}\ddot{Z}_{wfr} = d_f\ [(T_f\dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfr}] + K_f\ [(T_f\varphi_s + a\varphi_s + Z_s) - Z_{wfr}] - K_{tf}Z_{wfr} - u_{fr} + K_{tf}Z_{rfr}$ -- (12)

$M_{uf}\ddot{Z}_{wfl} = d_f[(-T_f\ \dot{\varphi}_s + a\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wfl}] + K_f[(-T_f\ \varphi_s + a\varphi_s + Z_s) - Z_{wfl}] - K_{tf}Z_{wfl} - u_{fl} + K_{tf}Z_{rfl}$ --- (13)

$M_{ur}\ddot{Z}_{wrr} = d_r\ [(T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrr}] + K_r\ [(T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrr}] - K_{tr}Z_{wrr} - u_{rr} + K_{tr}Z_{rrr}$ --- (14)

$M_{ur}\ddot{Z}_{wrl} = d_r\ [(-T_r\ \dot{\varphi}_s - b\dot{\varphi}_s + \dot{Z}_s) - \dot{Z}_{wrl}] + K_r\ [(-T_r\ \varphi_s - b\varphi_s + Z_s) - Z_{wrl}] - K_{tr}Z_{wrl} - u_{rl} + K_{tr}Z_{rrl}$ --- (15)

 ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

Where
M_s = mass of the car body or sprung mass (kg)
M_{uf} and M_{ur} = front and rear mass of the wheel or unsprung mass (kg)
I_p and I_r = pitch and roll of moment of inertia (kg m^2)
Z_s = car body displacement (m)
Z_{fr}, Z_{fl}, Z_{rr} and Z_{rl} = car body displacement for each corner (m)
Z_{wfr}, Z_{wfl}, Z_{wrr} and Z_{wrl} = wheel displacement (m)
T_f and T_r = front and rear treat (m)
a = distance from centre of sprung mass to front wheel (m)
b = distance from centre of sprung mass to rear wheel (m)
d_f and d_r = front and rear damping (Nm/s)
k_f and k_r = stiffness of car body spring for front and rear(N/m)
k_{tf} and k_{tr} = stiffness of tire for front and rear (N/m)
u_{fr} and u_{fl} = front right and left force actuators
u_{rr} and u_{rl} = rear right and left force actuators

Equations (9) ~equations (15) can be written instate space form as below
$\dot{\varphi}_s = \dot{X}_1 \approx X_8$
$\dot{\theta}_s = \dot{X}_2 \approx X_9$
$\dot{Z}_s = \dot{X}_3 \approx X_{10}$
$\dot{Z}_{wfr} = \dot{X}_4 \approx X_{11}$
$\dot{Z}_{wfl} = \dot{X}_5 \approx X_{12}$
$\dot{Z}_{wrr} = \dot{X}_6 \approx X_{13}$
$\dot{Z}_{wrl} = \dot{X}_7 \approx X_{14}$

$$\dot{X}_8 = \ddot{\varphi}_s \approx [-d_f T_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + d_f T_f ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) - d_r T_r ((T_r X_8 - bX_9 + X_{10}) - X_{13}) + d_r T_r ((-T_r X_8 - bX_9 + X_{10}) - X_{14}) - K_f T_f ((T_f X_1 + aX_2 + X_3) - X_4) + K_f T_f ((-T_f X_1 + aX_2 + X_3) - X_5) - K_r T_r ((T_r X_1 - bX_2 + X_3) - X_6) + K_r T_r \left((-T_r X_1 - bX_2 + X_3) - X_7\right) + T_f u_{fr} - T_f u_{fl} + T_r u_{rr} - T_f u_{rl}]/I_r$$

$$\dot{X}_9 = \ddot{\theta}_s \approx [-d_f a ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + d_f a ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) + d_r b ((T_r X_8 - bX_9 + X_{10}) - X_{13}) + d_r b ((-T_r X_8 - bX_9 + X_{10}) - X_{14}) - K_f a ((T_f X_1 + aX_2 + X_3) - X_4) - K_f a ((-T_f X_1 + aX_2 + X_3) - X_5) + K_r b ((T_r X_1 - bX_2 + X_3) - X_6) + K_r b \left((-T_r X_1 - bX_2 + X_3) - X_7\right) + au_{fr} + au_{fl} - bu_{rr} - bu_{rl}]/I_p$$

$$\dot{X}_{10} = \ddot{Z}_s \approx [-d_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + d_f ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) - d_r ((T_r X_8 - bX_9 + X_{10}) - X_{13}) - d_r ((-T_r X_8 - bX_9 + X_{10}) - X_{14}) - K_f ((T_f X_1 + aX_2 + X_3) - X_4) - K_f ((-T_f X_1 + aX_2 + X_3) - X_5)$$

 ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

$$-K_r((T_r X_1 - \mathrm{b}X_2 + X_3) - X_6) - K_r\big((-T_r X_1 - bX_2 + X_3) - X_7\big) + u_{fr} + u_{fl} + u_{rr} + u_{rl}]/\mathrm{M}_s$$

$\dot{X}_{11} = \ddot{Z}_{wfr} \approx [d_f((T_f X_8 + aX_9 + X_{10}) - X_{11}) + K_f((T_f X_1 + aX_2 + X_3) - X_4) - K_{tf}X_4 - u_{fr} + K_{tf}Z_{rfr}]/M_{uf}$

$\dot{X}_{12} = \ddot{Z}_{wfl} \approx [d_f((-T_f X_8 + aX_9 + X_{10}) - X_{12}) + K_f((-T_f X_1 + aX_2 + X_3) - X_5) - K_{tf}X_5 - u_{fl} + K_{tf}Z_{rfl}]/M_{uf}$

$\dot{X}_{13} = \ddot{Z}_{wrr} \approx [d_f((T_f X_8 - bX_9 + X_{10}) - X_{13}) + K_r((T_r X_1 - bX_2 + X_3) - X_6) - K_{tr}X_6 - u_{rr} + K_{tr}Z_{rrr}]/M_{ur}$

X ̇_14 = Z ̈_wrl≈ [d_r((-T_(r) X_8- 〖bX〗 _9 + X_10) -X_14) + K_r((-T_(r) X_1- 〖bX〗 _2 + X_3) -X_7)-K_tr X_7-u_rl+K_tr Z_rrr]/M_ur

.

Followed by, covert equation into the matrix yield

$$\dot{X}(\mathrm{t}) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}[\mathrm{x(t)}] + \mathrm{Bu(t)} + f(t) \quad (16)$$

Where,

$\dot{X}(\mathrm{t}) = [\dot{X}_1\, \dot{X}_2\, \dot{X}_3\, \dot{X}_4\, \dot{X}_5\, \dot{X}_6\, \dot{X}_7 \dot{X}_8\, \dot{X}_9\, \dot{X}_{10}\, \dot{X}_{11}\, \dot{X}_{12}\, \dot{X}_{13}\, \dot{X}_{14}]^{\mathrm{T}}$

$\dot{X}(\mathrm{t}) \approx [\dot{\varphi}_s\, \dot{\theta}_s\, \dot{Z}_s\, \dot{Z}_{wfr}\, \dot{Z}_{wfl}\, \dot{Z}_{wrr}\, \dot{Z}_{wrl}\, \ddot{\varphi}_s\, \ddot{\theta}_s\, \ddot{Z}_s\, \ddot{Z}_{wfr}\, \ddot{Z}_{wfl}\, \ddot{Z}_{wrr} \ddot{Z}_{wrl}]^{\mathrm{T}}$

$\mathrm{X(t)} = [X_1\, X_2\, X_3\, X_4\, X_5\, X_6\, X_7\, X_8\, X_9\, X_{10}\, X_{11}\, X_{12}\, X_{13}\, X_{14}]^{\mathrm{T}}$

$\mathrm{X(t)} \approx [\varphi_s\, \theta_s\, Z_s\, Z_{wfr}\, Z_{wfl}\, Z_{wrr}\, Z_{wrl}\, \dot{\varphi}_s\, \dot{\theta}_s\, \dot{Z}_s \dot{Z}_{wfr}\, \dot{Z}_{wfl}\, \dot{Z}_{wrr}\, \dot{Z}_{wrl}]$

3.2. Parameters Used

The full car mathematical model was based on [10] and [15]. The parameters were obtained from [10] and [16]. From [16], spring stiffness is 55000 N/mand damping coefficient 4000 Nm/s, 1000 Nm/s respectively. The fixed parameters of full car model are;
Sprung mass = mass of vehicle body = 1200kg = M_s
Wheel mass of front right and left = 60kg = M_{uf}
Wheel mass of rear right and left = 60kg = M_{ur}
Spring stiffness of front right and left tyre = 30000N/m = k_{tf} and k_{tr}
Spring stiffness of rear right and left tyre = 30000N/m = k_{tf} and k_{tr}
Pitch moment of inertia = 4000kg-m^2 = I_p
Roll moment of inertia = 952kg-m^2 = I_r
Distance from CG to front and rear wheel = 1.5m = a
Distance from CG to left and right wheel =1.0m = b
Front and rear treat = T_f and T_r = 0.507m and 0.559m
The fixed parameters for fullcar model are; [3]
Unsprung mass = mass of the wheel/tyre = 59kg = M_1
Sprung mass = mass of vehicle body = 290kg = M_2
Stiffness of the wheel/tyre = 190000N/m

 ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

3.3. Linear Quadratic Regulator (LQR) Controller Design

The LQR control approaches in controlling a linear active damping system was presented by [16]. [7] concluded that the LQR control approach will give a better performance in terms of ride comfort.

This study considered the following state variable feedback regulator.

$u = -kx$ -- (17)

Where k is the state feedback gain matrix.

Optimization of control system consists of determining the control input u, which minimizes the performance index (J), which represents the performance characteristics requirement as well as controller input limitations [16].
The performance index

$J = \frac{1}{2}\int_0^t (x'Qx + u'Ru)dt$ --- (18)

Where $u =$ actuator force (N)
Q and R are positive definite weighting matrices [17].

Linear optimal control theory provides the solution of equation (18) in the form of equation (17)

$k = R^{-1}B'\mathrm{P}$ -- (19)

Where matrix P must satisfy the reduced – matrix Riccati equation

$A'P + \mathrm{PA} - \mathrm{PB}R^{-1}\mathrm{B}'\mathrm{P} + Q = 0$ --- (20)

Then the feedback regulator u is given by

$u(t) = -(R^{-1}B')(\mathrm{t})$

$= -kx(t)$

4. Result Analysis

The computer simulation work based on equation (1) ~ (16) has been performed. Comparison between passive and active suspension for full car model was observed. For the LQR controller, the weighing matrix Q and weighing matrix R are set to obtain suitable feedback gain k.

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

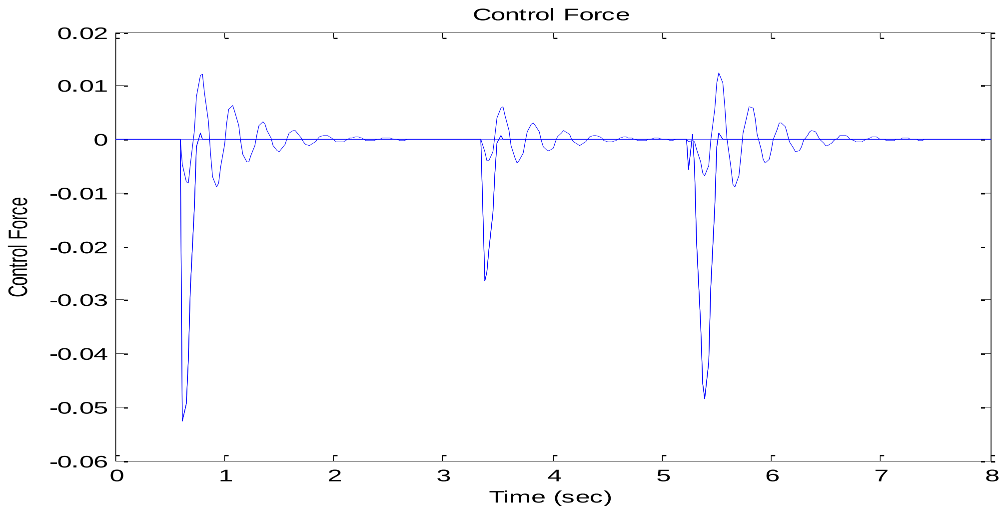

Figure 5: Force Generated using LQR Controller with Road Profile 1

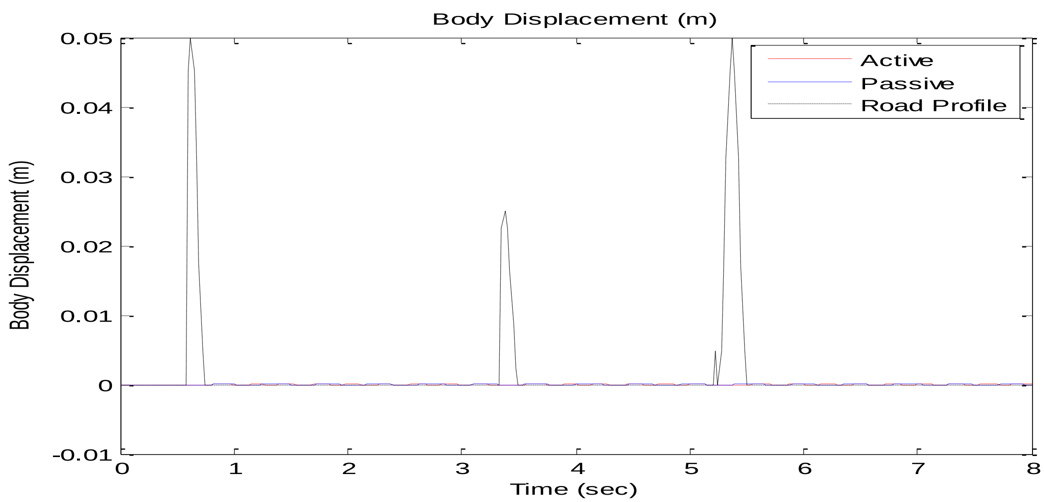

Figure 6: Body Displacement with Road Profile 1

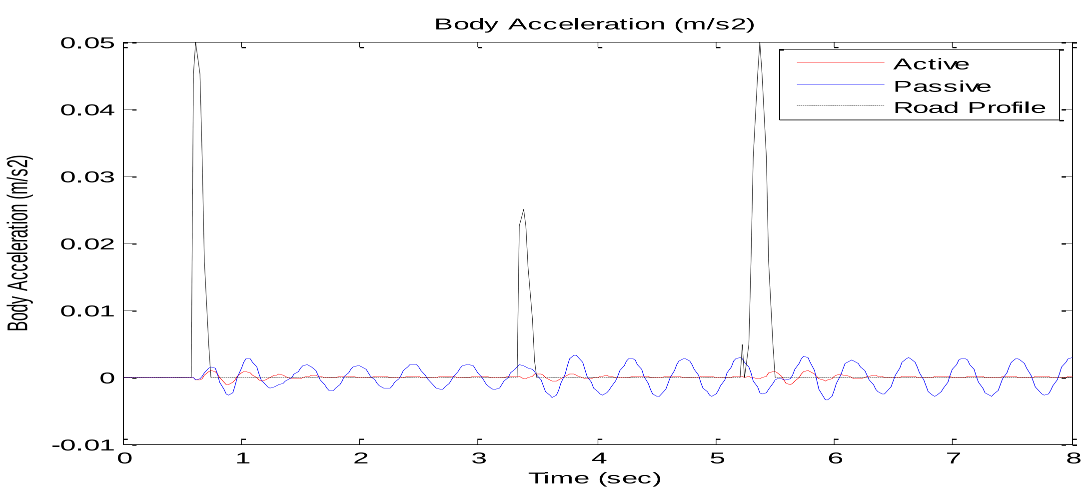

Figure 7: Body Acceleration with Road Profile 1

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

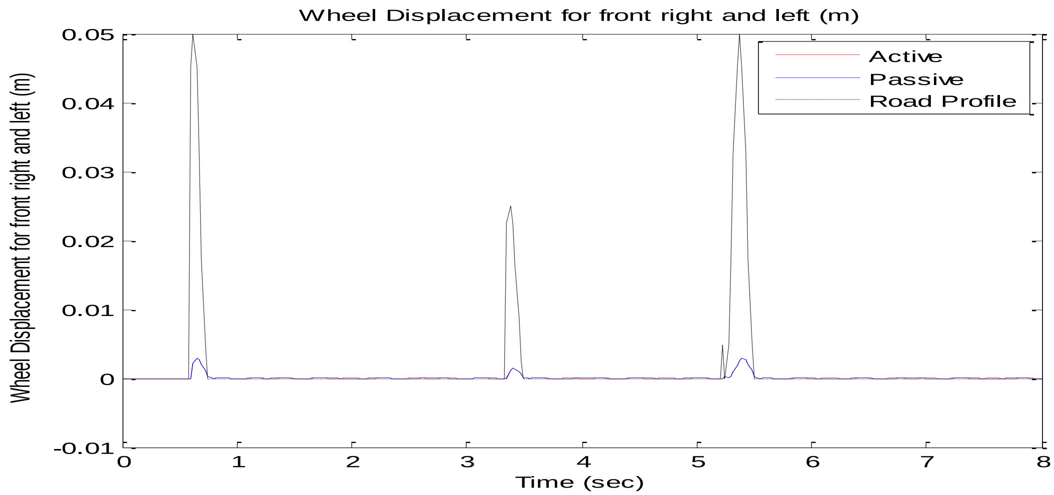

Figure 8: Wheel Displacement for Front/Rear Right and Left with Road Profile 1

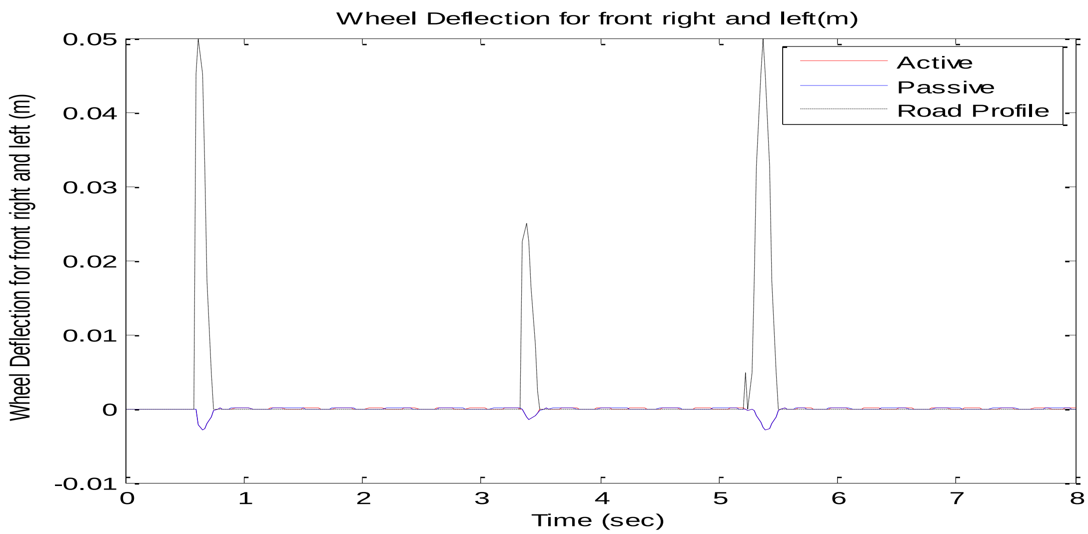

Figure 9: Wheel Deflection for Front/Rear Right and Left with Road Profile 1

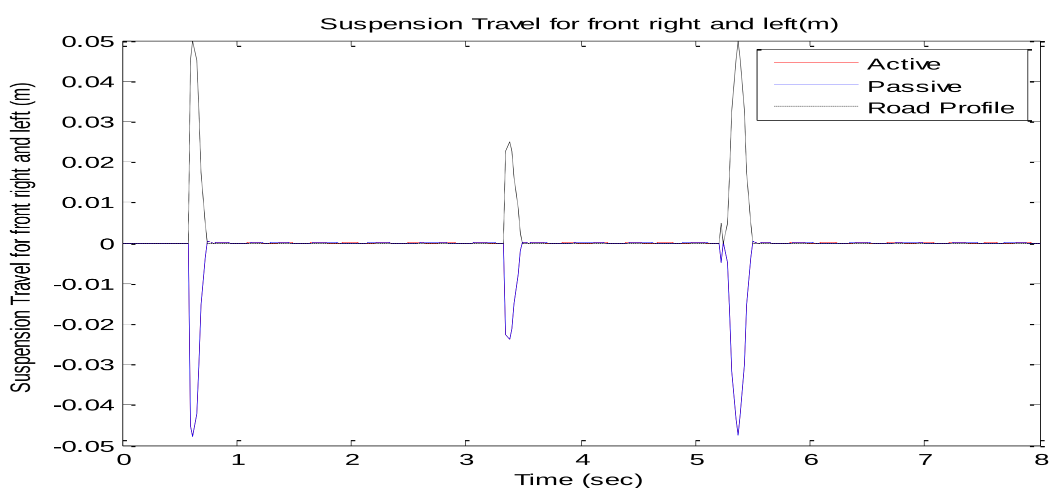

Figure 10: Suspension Travel for Front/Rear Right and Left with Road Profile 1

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

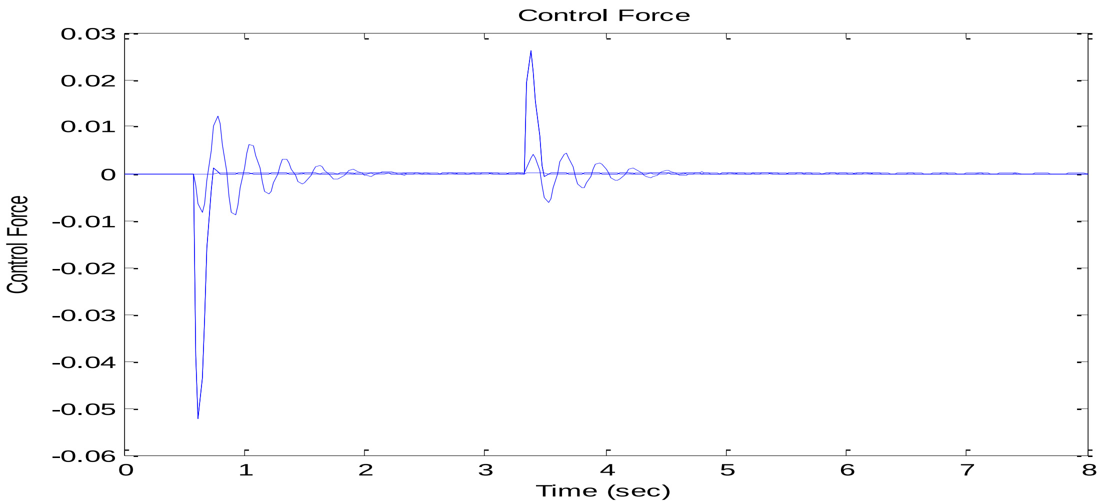

Figure 11: Force generated using LQR Controller with Road Profile 2

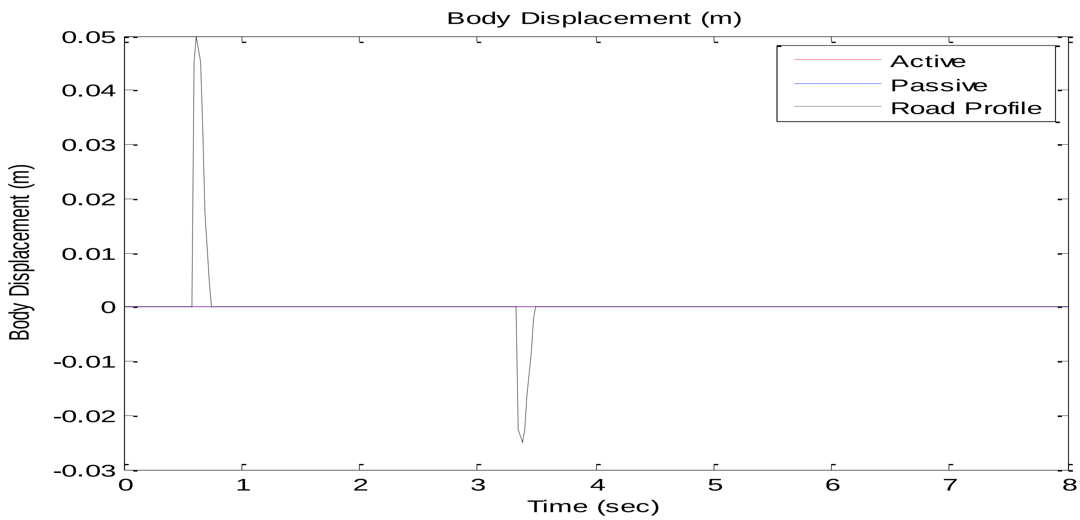

Figure 12: Body Displacement with Road Profile 2

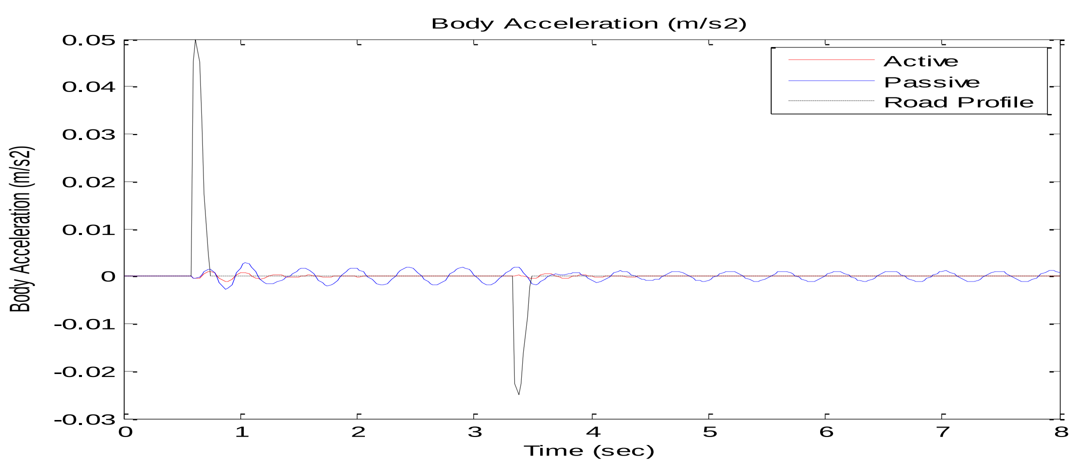

Figure 13: Body Acceleration with Road Profile 2

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

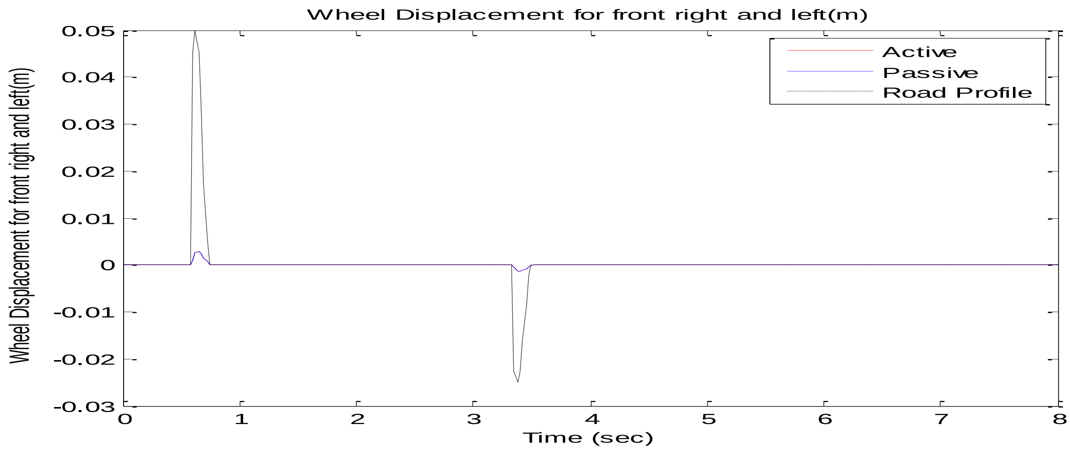

Figure 14: Wheel Displacement for Front/Rear Right and Left with Road Profile 2

Figure 15: Wheel Deflection for Front/Rear Right and Left with Road Profile 2

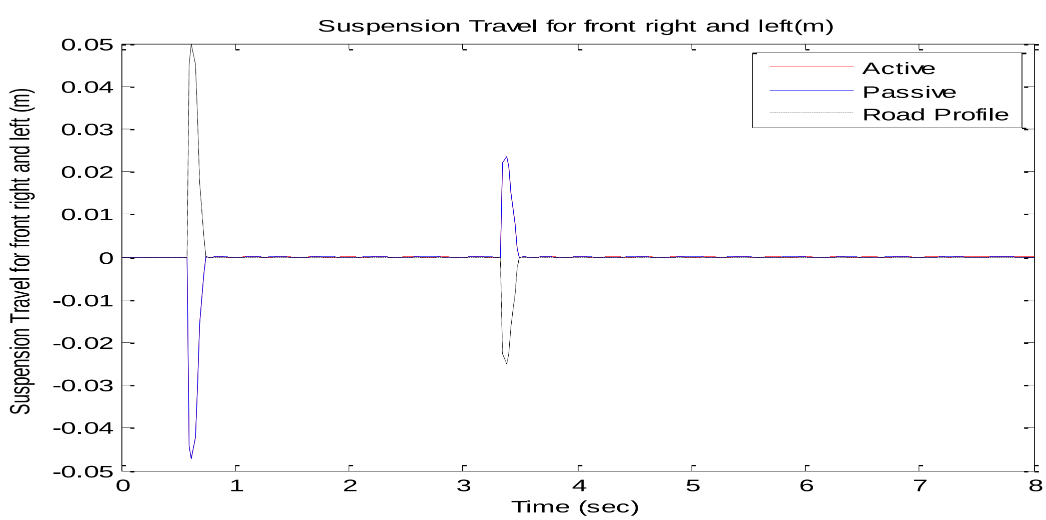

Figure 16: Suspension Travel for Front/Rear Right and Left with Road Profile 2

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

4.1. Discussion of Result

The control force generated by the actuator for road profiles 1 and 2 are shown in Figure 5, and Figure 14. The body displacement which represents the ride quality for both Passive and Active damping with LQR have very low amplitude with very fast settling time for road profiles 1 and 2 as can be seen in Figure 6 and Figure 12. The Suspension Travel for the two road profiles (Figure 10 and Figure 16), the active damping system shows very lower amplitude and very fast settling time. Wheel Displacement with road profiles 1 and 2 (Figure 9 and Figure 14), also shows a balance in the amplitude, that is, no amplitude rises. The wheel displacement represents the car handling performance. For the Body Acceleration, for active damping, there is amplitude that makes few oscillations (Figure 7 and Figure 13) but, it settles faster with a settling time of less than 2 seconds. While that of the passive damping generates a continuous harmonic. Therefore, more modification is required on the passive damping system in regard to Body Acceleration. The Body Displacement which represents the ride quality shows a balance using active damping with LQR controller for the three road profiles. The Active damping with LQR controller gives lower amplitude and fast settling time for all the parameters compared to passive damping system. The output performance is the same because each pairs of wheel set have the same output performance due to the mathematical modeling; this shows that there is relationship between these wheels that is, front wheel right and left and rear wheel right and left receives same types of disturbance.

The table below shows the amplitude reduction in all parameters by passive and active damping as compared to the road profile with amplitude of 0.05m. The amplitude reduction for the active is given by:

$$\frac{\text{peak value of passive} - \text{peak value of active}}{\text{peak value of passive}} \text{ X 100 for all parameters.}$$

Table 3: Percentage Reduction in Amplitude Value for Passive and Active Damping for Full Car Model

Parameters	Maximum Overshoot			Amplitude Reduction (%)		
	Profile	Passive	Active	Profile Vs Passive	Profile Vs Active	Passive Vs Active
Wheel displacement for front right and left (m)	0.05	0.00296	0.000	94.1	100	100
Wheel displacement for rear right and left (m)	0.05	0.00296	0.000	94.1	100	100
Wheel deflection for front right and left (m)	0.05	0.002867	0.000	94.3	100	100
Wheel deflection for front right and left (m)	0.05	0.002867	0.000	94.3	100	100
Suspension travel for front right and left (m)	0.05	0.04782	0.000	4.40	100	100
Suspension travel for rear right and left (m)	0.05	0.04782	0.000	4.40	100	100
Body displacement (m)	0.05	0.0000	0.000	100	100	100
Body Acceleration (m/s^2)	0.05	0.002773	0.001277	94.5	97.5	54.0

ISSN: 2454-1907
DOI: 10.5281/zenodo.2653079

5. Conclusion

Mathematical model for Passive and Active damping system for a full car were derived and validated. Simulation of the Passive and Active damping system with LQR controller was performed and comparison were made between the Passive and Active damping system. The Active damping with LQR gives lower amplitude and faster settling time of less than 2 seconds as compared to the Passive damping and other controllers (e.g. PID) that have been used in Active damping system.

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