# ESTIMATION OF PROCESS CAPABILITY IN BAYESIAN PARADIGM

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Received 20 May 2022 Accepted 29 June 2022 Published 19 July 2022

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DOI

#### 10.29121/ijetmr.v9.i7.2022.1193

**Funding:** This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

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# ABSTRACT

The process capability index (PCI), Cp examines the capability of control charts. Bayesian techniques to estimate Cp are desirable when prior information about a process characteristic is available. In this paper, an estimator of Cp under normality with process variance having conjugate prior in Bayesian scenario is proposed. Its performance is studied and compared with Bayesian estimator developed by Cheng and Spiring (1989) An illustrative example is provided.

**Keywords:** Bayesian Estimator, Conjugate Prior, Posterior Distribution, Process Capability Index, Process Variance

## **1. INTRODUCTION**

Quality is a prudent characteristic in manufacturing industries. Cp plays pivotal role in deciding about capability of a process preset to meet the quality requirements. It is given by

$$C_p = \frac{USL - LSL}{UCL - LCL} = \frac{USL - LSL}{6\sigma}$$

**Equation 1** 

where  $\sigma$  is process standard deviation (sd), UCL, LCL are upper, lower control limits and USL, LSL are upper, lower specification limits of a control chart. *Cp* is a unitless measure and a process is considered as capable if  $C_p > k$  where k is a positive constant  $\ge 1$ . When  $\sigma$  is estimated by sample sd 's', an estimator for *Cp* is

given by 
$$\hat{C}_p = \frac{USL - LSL}{6s}$$
. Equation 2

Kane (1986) compared various PCIs and Montgomery (1996) carried out a detailed discussion on PCIs along with their illustrations. Bayesian procedures for PCI use prior information about the process parameters involved. Cheng and Spiring (1989) using Bayesian approach propose an estimator  $\hat{C}_{pn}$ , under normal model when process sd has noninformative prior. The posterior distribution of  $\hat{C}_{pn}$  derived by them is

$$\pi(y|\mathcal{C}_p) = \left[\Gamma\left(\frac{n-1}{2}\right)\right]^{-1} 2^{1-\left(\frac{n-1}{2}\right)} ((n-1)\mathcal{C}_p^2)^{\frac{n-1}{2}} y^{-n} e^{-\frac{(n-1)\mathcal{C}_p^2}{2y^2}}, y > 0 \qquad \text{Equation 3}$$

where  $\hat{C}_{pn}$  is realized by y. They investigate the performance of their measure in terms of minimum value of  $\hat{C}_{pn}$  required to ensure the probability that process achieves the desired specifications along with an illustration.

Chan et al. (1988) proposed a measure to process capability which accounts both target value and process variation simultaneously. They examined sampling distribution of the proposed measure with its practical applications to industrial data. Spiring (1995) outlined assessment of process capability as a tool of management. Shiau et al. (1999) studied Bayesian procedure for process capability by assuming noninformative and gamma priors for  $C_p^2$ . Kotz and Johnson (2002) reviewed some articles on PCIs studied during 1992 - 2000 from widely

scattered sources and record their interpretations along with comments. Pearn and Wu (2005) studied estimation of  $C_p$  by Bayesian approach using multiple samples.

In this paper, we establish an estimator of  $C_p$  in Bayesian paradigm under normality when process variance has conjugate prior. In section 2, we propose  $\hat{C}_{pc}$ and derive its posterior distribution. We study about its performance in section 3, illustrate its performance in section 4 and record our conclusions in section 5. The computed values supporting performance of  $\hat{C}_{pc}$  are given in tables provided in appendix.

## 2. PROPOSED BAYESIAN ESTIMATOR OF $C_p$

In this section, we propose a Bayesian estimator  $\hat{C}_{pc}$  for  $C_p$  when process variance has a conjugate prior distribution. That is,  $\hat{C}_{pc}$  is given by Equation 2 with an assumption that,  $\sigma^2$  has a conjugate prior.

Suppose, X<sub>1</sub>, X<sub>2</sub> ... X<sub>n</sub> is a random sample of size n from  $N(\mu, \sigma^2)$ , then the density function of X<sub>i</sub> is given by

$$f_{X_i}(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} - \infty < x, \mu < \infty , \sigma > 0 \qquad \text{Equation 4}$$

The likelihood function of the sample  $X = (X_1, X_2 \dots X_n)$  is given by

$$L(\mathbf{X}, \mu, \sigma^{2}) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^{n} e^{-\frac{1}{2}\sum_{i=1}^{n} \left(\frac{x_{i}-\mu}{\sigma}\right)^{2}}$$
  
Also,  $z = \frac{(n-1)s^{2}}{\sigma^{2}} \sim \chi^{2}_{(n-1)}$  where  $s^{2} = \frac{1}{n-1}\sum_{i=1}^{n} (x_{i} - \bar{x})^{2}$ 

We assume that  $\pi(\sigma^2) \sim IG(\eta, \delta), \eta > 0, \delta > 0$  where  $\eta$  is shape parameter and  $\delta$  is scale parameter. IG stands for inverse gamma which is a conjugate prior.

$$\pi(\sigma^2) = \frac{\delta^{\eta}}{\Gamma \eta} e^{-\frac{\delta}{\sigma^2}} \left(\frac{1}{\sigma^2}\right)^{\eta+1}, \sigma^2 > 0$$
 Equation 6

Using Equation 1 and Equation 2, z can be written as

$$z = \frac{(n-1)c_p^2}{\hat{c}_{pc}^2}$$
 Equation 7

Thus, realizing  $\hat{C}_{pc}$  by y, we have the posterior distribution of  $y^2 | X$  given by

$$\pi(y^2|\mathbf{X}) = \frac{\left(\frac{n-1}{2}C_p^2 + \delta\right)^{\frac{n-1}{2} + \eta}}{\Gamma\left(\frac{n-1}{2} + \eta\right)} e^{-\frac{1}{y^2}\left(\frac{n-1}{2}C_p^2 + \delta\right)} \left(\frac{1}{y^2}\right)^{\frac{n-1}{2} + \eta + 1}$$
Equation 8

Using appropriate transformation,  $\pi(y|X)$  is given by

$$\pi(y|\mathbf{X}) = \frac{2\left(\frac{n-1}{2}C_p^2 + \delta\right)^{\frac{n-1}{2} + \eta}}{\Gamma\left(\frac{n-1}{2} + \eta\right)} e^{-\frac{1}{y^2}\left(\frac{n-1}{2}C_p^2 + \delta\right)} \left(\frac{1}{y^2}\right)^{\frac{n-1}{2} + \eta + \frac{1}{2}} y > 0 \qquad \text{Equation 9}$$

When  $\eta = \delta = 0$ , Equation 9 reduces to Equation 3 indicating that the posterior distribution of  $\hat{C}_{pn}$  due to Cheng and Spiring (1989) is a particular case of posterior distribution of  $\hat{C}_{pc}$  given in Equation 9. From Bhat and Gokhale (2014) Bhat and Gokhale (2016) and Gokhale (2017) we observe that,

$$\pi(y^2|\mathbf{X}) = \pi(\sigma^2|\mathbf{X})$$

And

$$\pi(y|X) = \pi(\sigma|X)$$
 Equation 10

where in  $C_p^2$  in left hand side is replaced by  $s^2$  in right hand side.

# 3. PERFORMANCE OF $\hat{C}_{bc}$

In this section, we evaluate the performance of  $\hat{C}_{pc}$  by obtaining minimum value of  $\hat{C}_{pc}$  needed to assure  $P((C_p > k)|\mathbf{X})$ . That is,

 $\tau = \min \hat{C}_{pc} | p_c.$ Here,  $pc = p_c = P(process \ is \ capable| \ sample)$  $= P((C_n > k) | \mathbf{X})$ 

 $= P\left(\left(\frac{USL-LSL}{6\sigma} > k\right) | \mathbf{X}\right)$  $= P\left(\left(\frac{USL-LSL}{6k} > \sigma\right) | \mathbf{X}\right)$ Equation 11

which is equivalent to finding

$$p_{c} = \int_{0}^{\frac{USL-LSL}{6k}} \pi(y|\mathbf{X}) dy = \int_{0}^{\frac{USL-LSL}{6k}} \pi(\sigma|\mathbf{X}) d\sigma$$
$$= \int_{0}^{a} \frac{2\left(\frac{n-1}{2}s^{2}+\delta\right)^{\frac{n-1}{2}+\eta}}{\Gamma\left(\frac{n-1}{2}+\eta\right)} e^{-\frac{1}{\sigma^{2}}\left(\frac{n-1}{2}s^{2}+\delta\right)} \left(\frac{1}{\sigma^{2}}\right)^{\frac{n-1}{2}+\eta+\frac{1}{2}} d\sigma$$
Equation 12  
Where  $a = \frac{USL-LSL}{6k}$ .

By taking 
$$t = \frac{1}{\sigma^2} \left( \frac{n-1}{2} s^2 + \delta \right)$$
,  $b = \left( \frac{n-1}{2} \right) \frac{k^2}{\hat{c}_{pc}^2} + \frac{\delta}{a^2}$  and  $\xi = \left( \frac{n-1}{2} \right) + \eta$  and

proceeding on the lines of Chan et al. (1988), we express (12) as

$$p_c = \int_b^\infty \frac{1}{\Gamma\xi} t^{\xi-1} e^{-t} dt$$
 Equation 13

By using, Wilson-Hilferty (1931) transformation, (13) can be written as

$$p_c \simeq 1 - \Phi\left\{\frac{\left(\frac{2}{n}b\right)^{1/3} - \left(1 - \frac{1}{9\xi}\right)}{\frac{1}{3\xi^2}}\right\}$$
 Equation 14

Where  $\Phi(\cdot)$  is cumulative distribution function of standard normal variate. On simplifying Equation 14 we get

$$b = \frac{n}{2} \left( \frac{\Phi^{-1}(1-p_c)}{3\xi^{\frac{1}{2}}} + \left(1 - \frac{1}{9\xi}\right) \right)^3$$
 Equation 15

Therefore,

$$(n-1)\frac{k^2}{\hat{c}_{pc}^2} = n\left(\frac{\Phi^{-1}(1-p_c)}{3\xi^{\frac{1}{2}}} + \left(1 - \frac{1}{9\xi}\right)\right)^3 - \frac{2\delta}{a^2}$$

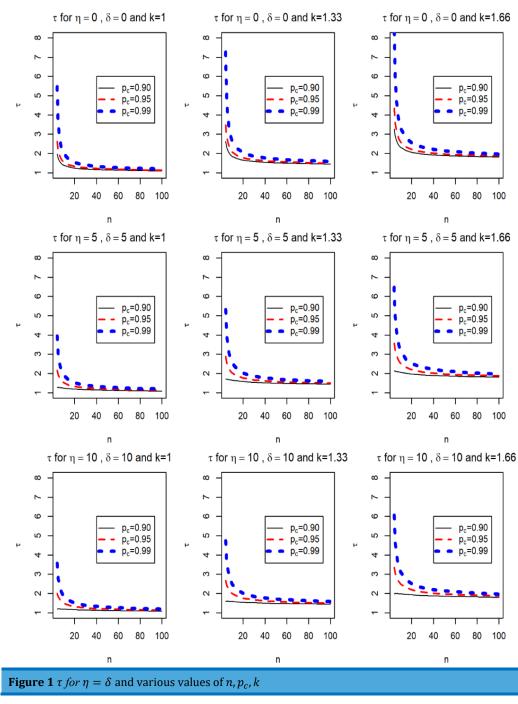
$$\Rightarrow \hat{C}_{pc} = k \left( \frac{n-1}{n \left\{ \frac{\Phi^{-1}(1-p_c)}{3\xi^{1/2}} + \left(1 - \frac{1}{9\xi}\right) \right\}^3 - \frac{2\delta}{a^2}} \right)^{1/2}$$

$$= k \left( \frac{n-1}{n \left\{ \frac{\Phi^{-1}(1-p_{c})}{3\xi^{1/2}} + \left(1 - \frac{1}{9\xi}\right) \right\}^{3} - \frac{72k^{2}\delta}{w^{2}}} \right)^{1/2}$$
 Equation 16

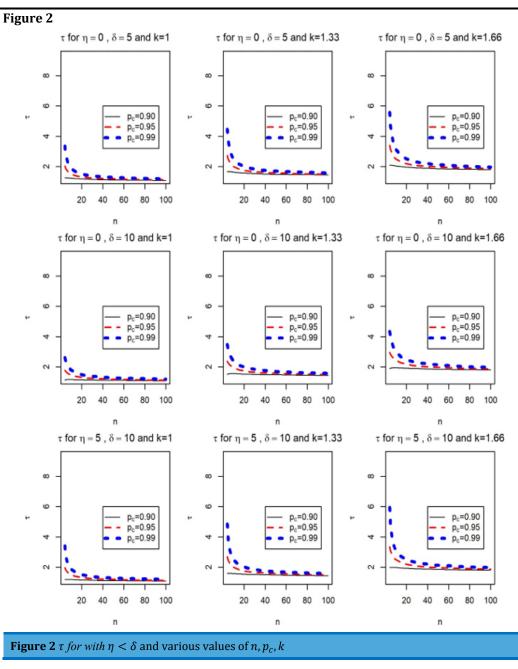
Where w = USL - LSL.

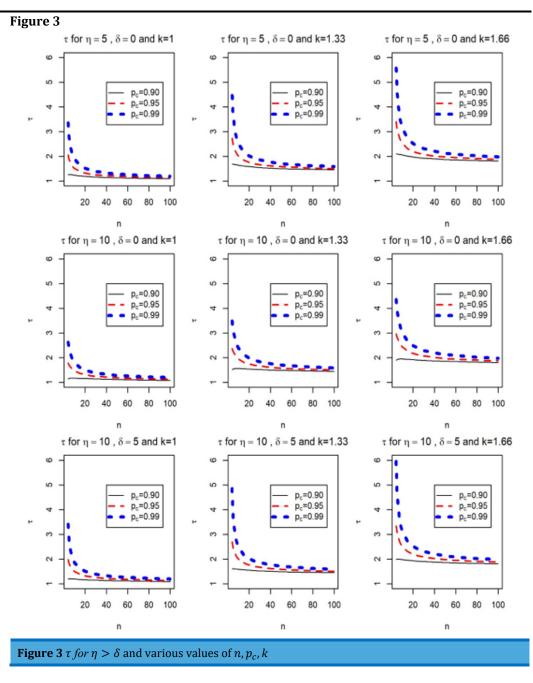
In order to evaluate  $\tau$ , one need to specify *w*, n, *pc*, k,  $\eta$  and  $\delta$ . To compute minimum *Cpc* obtained in Equation 16, the denominator has to be greater than zero.

upper integer greater than  $\gamma$ . In Table 2, we calculate  $\tau$  for higher values of w given in Table 1 n= 5, 15, 25, 50, 75, 100, pc = 0.90, 0.95, 0.99, k=1, 1.33,1.66,  $\eta = 0$ , 5, 10 and  $\delta = 0$ , 5, 10. Using Table 2 we plot  $\tau$  in Figure 1 Figure 2 and Figure 3 respectively for  $\eta = \delta$ ,  $\eta < \delta$  and  $\eta > \delta$ .









From Table 1 we observe that, *w* increases as *pc*, *k* increase and decreases as *n* increases. For fixed  $\eta$ , *w* increases as  $\delta$  increases, for fixed  $\delta$ , it decreases as  $\eta$  increases and for  $\eta = \delta$ , it increases for increasing values of  $\eta$  and  $\delta$ . From Figure 1 and Table 2 we observe that, for  $\eta = \delta$ ,  $\tau$  is higher for higher values of  $p_c$ . It is increasing for increasing value of k when n is small and remains nearly same when n is large. Also,  $\tau$  is smaller for higher values of  $\eta < \delta$  and as  $\eta$  increases for  $\eta > \delta$ . Also, from all the three figures it is observed that,  $\tau$  increases as k increases along with increase in  $p_c$ . Table 2 shows that, for fixed values of  $\eta = 0$ , there is no considerable change in values of  $\tau$  for various values of n, k and *pc* for increasing  $\delta$ .

## 4. ILLUSTRATION

In this section, we consider example given in Kane (1986) and discussed in Cheng and Spiring (1989)

#### Example 1

Example 1 For n=300, s=4.3,  $\hat{C}_{pn} = 1.5504$  and  $p_n = P((C_p > 1) | \hat{C}_{pn}) = 0.9999$ . Then for  $p_c = p_n$ , using (16), for different values of  $\eta$ ,  $\delta$  and n,  $\hat{C}_{pc}$  is given by

Срс											
η	δ	n=300	n=50	η	δ	n=300	n=50				
0	5	1.174	1.5479	10	0	1.167	1.4214				
0	10	1.1746	1.5565	5	5	1.1707	1.4782				
5	0	1.1701	1.4709	10	10	1.1682	1.4347				

#### Example 2

Example 2 For n=79, s=7.8, $\hat{C}_{pn} = 0.8547$ and $p_n = 0.0139$ . For $p_c = p_n$ , $\hat{C}_{pc}$ is given by											
<b>C</b> pc											
η	δ	n=79	n=5	η	δ	n=79	n=5				
0	5	0.8453	0.5091	10	0	0.8584	0.6783				
0	10	0.8462	0.5129	5	5	0.8528	0.6386				
5	0	0.8519	0.6314	10	10	0.8602	0.6966				

In Example 1, it is seen that for different values of  $\eta$ ,  $\delta$  and n=300,  $\hat{C}_{pc}$  is lesser than  $\hat{C}_{pn}$ , whereas for n=50,  $\hat{C}_{pc}$  is near to  $\hat{C}_{pn}$  when  $\eta = 0$  and is lesser than  $\hat{C}_{pn}$  for other values of  $\eta$  and  $\delta$ . In Example 2,  $\hat{C}_{pc}$  is near to  $\hat{C}_{pn}$  for n=79 when  $\delta = 0$ , whereas for n=5,  $\hat{C}_{pc}$  is lesser than  $\hat{C}_{pn}$  for various values of  $\eta$  and  $\delta$ . It is also observed that, sample sd is smaller in Example 1 when compared to sample sd in Example 2

## **5. CONCLUSIONS**

In this section, we furnish our conclusions based on our observations.

- Under Bayesian approach, the proposed estimator  $\hat{C}_{pc}$  includes  $\hat{C}_{pn}$  due to Cheng and Spiring (1989) as its particular case in the sense that, the posterior distribution of  $\hat{C}_{pc}$  reduces to that of  $\hat{C}_{pn}$  when hyper parameters  $\eta$  and  $\delta$  are zero.
- For all the values of  $\eta$  and  $\delta$  under consideration,  $\tau$  the minimum value of  $\hat{C}_{pc}$  needed to assure  $p_c$ , the probability that process is capable given the sample, increases along with increasing values of k and pc for smaller values of n.
- For  $\eta = \delta$ ,  $\tau$  is decreasing as  $\eta$  and  $\delta$  are increasing.
- For η < δ, τ decreases as δ increases and for η > δ, it decreases as η increases.
- $\hat{C}_{pc} < \hat{C}_{pn}$  when sample sd is small, n is large and also when sample sd is large, n is small.

# 6. APPENDIX

Table 1

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						-												
0.95       17       7       5       4       3       3       0.95       23       10       7       5       4       4         0.99       23       9       6       4       3       3       0.99       32       12       8       5       4       4         1.33       0.9       15       8       6       5       4       3       1.33       0.9       21       12       9       6       5       4         0.95       22       10       7       5       4       3       0.95       31       14       10       7       5       5         0.95       22       10       7       5       4       3       0.95       31       14       10       7       5       5         0.95       30       11       8       5       4       4       0.99       42       16       11       7       6       5         1.66       0.9       19       10       8       6       5       4       0.95       38       17       12       8       6       5         0.95       27       12       9       6 <t< th=""><th>105</th><th>1</th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th>10 10</th><th>1</th><th></th><th>-</th><th></th><th></th><th></th><th>-</th><th>-</th></t<>	105	1								10 10	1		-				-	-
0.99       23       9       6       4       3       3       0.99       32       12       8       5       4       4         1.33       0.9       15       8       6       5       4       3       1.33       0.9       21       12       8       5       4       4         0.95       22       10       7       5       4       3       0.95       31       14       10       7       5       5         0.95       22       10       7       5       4       3       0.95       31       14       10       7       5       5         0.99       30       11       8       5       4       4       0.95       42       16       11       7       6       5         1.66       0.9       19       10       8       6       5       4       1.66       0.9       26       14       11       8       6       5         0.95       27       12       9       6       5       4       0.95       38       17       12       8       6       6	10,5	1								10,10	1			-				
1.33       0.9       15       8       6       5       4       3       1.33       0.9       21       12       9       6       5       4         0.95       22       10       7       5       4       3       0.95       31       14       10       7       5       5         0.99       30       11       8       5       4       4       0.99       42       16       11       7       6       5         1.66       0.9       19       10       8       6       5       4       1.66       0.9       26       14       11       8       6       5         0.95       27       12       9       6       5       4       0.95       38       17       12       8       6       5						-											-	
0.95       22       10       7       5       4       3       0.95       31       14       10       7       5       5         0.99       30       11       8       5       4       4       0.99       42       16       11       7       6       5         1.66       0.9       19       10       8       6       5       4       1.66       0.9       26       14       11       8       6       5         0.95       27       12       9       6       5       4       0.95       38       17       12       8       6       6		1.33		-		-					1.33							
0.99       30       11       8       5       4       4       0.99       42       16       11       7       6       5         1.66       0.9       19       10       8       6       5       4       1.66       0.9       26       14       11       8       6       5         0.95       27       12       9       6       5       4       0.95       38       17       12       8       6       6																-		
1.66       0.9       19       10       8       6       5       4       1.66       0.9       26       14       11       8       6       5         0.95       27       12       9       6       5       4       0.95       38       17       12       8       6       6													-				-	
<b>0.95</b> 27 12 9 6 5 4 <b>0.95</b> 38 17 12 8 6 6		1.66		19		8		5	4		1.66					8		5
<b>0.99</b> 38 14 10 6 5 4 <b>0.99</b> 53 20 13 9 7 6				27	12	9	6		4				38			8	6	
			0.99	38	14	10	6	5	4			0.99	53	20	13	9	7	6

Та	ble 2										
Table 2	Table 2 $\tau$ for various values of $\eta$ , $\delta$ , $k$ , $p_c$ and n										
η, δ	n		k=1			k=1.33			k=1.66		
		p <sub>c</sub> =0.9 0	p <sub>c</sub> =0.9 5	p <sub>c</sub> =0.9 9	p <sub>c</sub> =0.9 0	p <sub>c</sub> =0.9 5	p <sub>c</sub> =0.9 9	p <sub>c</sub> =0.9 0	p <sub>c</sub> =0.9 5	p <sub>c</sub> =0.9 9	
0,0	5	1.7372	2.1531	3.5876	2.3105	2.8636	4.7715	2.8838	3.5741	5.9555	
	15	1.2947	1.4108	1.6818	1.722	1.8764	2.2368	2.1493	2.342	2.7918	
	25	1.2128	1.29	1.4589	1.613	1.7157	1.9404	2.0132	2.1414	2.4218	
	50	1.142	1.1897	1.2886	1.5188	1.5822	1.7139	1.8957	1.9748	2.1391	
	75	1.1133	1.1502	1.2251	1.4807	1.5298	1.6294	1.8481	1.9093	2.0337	
	100	1.0969	1.1279	1.19	1.4589	1.5001	1.5827	1.8209	1.8723	1.9754	
0,5	5	1.7382	2.1549	3.596	2.3127	2.8678	4.7912	2.8881	3.5823	5.9938	
	15	1.2949	1.411	1.682	1.7223	1.8767	2.2373	2.1498	2.3426	2.7929	
	25	1.2129	1.2901	1.459	1.6131	1.7158	1.9406	2.0135	2.1417	2.4223	
	50	1.142	1.1897	1.2887	1.5189	1.5823	1.7139	1.8958	1.9749	2.1393	

International Journal of Engineering Technologies and Management Research

Estimation	of Process	Capability in	Ravesian	Paradigm
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		75	1.1134	1.1502	1.2251	1.4808	1.5298	1.6294	1.8482	1.9094	2.0337
		100	1.0969	1.1279	1.19	1.4589	1.5001	1.5827	1.821	1.8723	1.9754
	0,10	5	1.7382	2.1549	3.5961	2.3128	2.8679	4.7916	2.8882	3.5825	5.9946
		15	1.2949	1.411	1.682	1.7223	1.8768	2.2374	2.1498	2.3427	2.7929
		25	1.2129	1.2901	1.459	1.6132	1.7158	1.9406	2.0135	2.1417	2.4223
		50	1.142	1.1897	1.2887	1.5189	1.5823	1.7139	1.8958	1.975	2.1393
		75	1.1134	1.1502	1.2251	1.4808	1.5298	1.6294	1.8482	1.9094	2.0337
		100	1.0969	1.1279	1.19	1.4589	1.5001	1.5827	1.821	1.8723	1.9754
	5,0	5	1.2617	1.8611	2.7631	1.678	2.4753	3.6749	2.0944	3.0895	4.5868
		15	1.2072	1.3868	1.6384	1.6056	1.8445	2.1791	2.0039	2.3022	2.7197
		25	1.1718	1.2809	1.4436	1.5585	1.7035	1.92	1.9452	2.1262	2.3964
		50	1.1275	1.187	1.2844	1.4996	1.5787	1.7082	1.8717	1.9704	2.1321
		75	1.1055	1.1488	1.223	1.4704	1.528	1.6266	1.8352	1.9071	2.0302
		100	1.0919	1.127	1.1887	1.4522	1.4989	1.581	1.8126	1.8709	1.9732
	5,5	5	1.2805	1.9233	2.9799	1.7049	2.5641	3.9862	2.1237	3.1859	4.9218
		15	1.2118	1.3939	1.65	1.6122	1.8545	2.1957	2.0112	2.3131	2.7379
		25	1.1743	1.2841	1.4482	1.562	1.7081	1.9266	1.949	2.1313	2.4036
		50	1.1286	1.1882	1.286	1.5011	1.5805	1.7105	1.8733	1.9723	2.1345
		75	1.1062	1.1496	1.2239	1.4713	1.529	1.6279	1.8362	1.9083	2.0316
		100	1.0924	1.1276	1.1893	1.4529	1.4997	1.5818	1.8133	1.8717	1.9742
	5,10	5	1.2801	1.922	2.9751	1.7001	2.548	3.9265	2.1296	3.2062	4.9975
		15	1.2117	1.3938	1.6498	1.611	1.8528	2.1928	2.0126	2.3153	2.7415
		25	1.1742	1.284	1.4481	1.5614	1.7073	1.9254	1.9498	2.1323	2.405
		50	1.1286	1.1882	1.2859	1.5009	1.5801	1.7101	1.8737	1.9727	2.135
		75	1.1062	1.1496	1.2239	1.4712	1.5289	1.6277	1.8364	1.9085	2.0319
		100	1.0924	1.1275	1.1893	1.4528	1.4996	1.5817	1.8135	1.8719	1.9744
	10,0	5	1.1598	1.6933	2.3537	1.5425	2.2521	3.1304	1.9252	2.8109	3.9071
		15	1.1649	1.366	1.6012	1.5494	1.8168	2.1296	1.9338	2.2676	2.658
		25	1.1467	1.2724	1.4295	1.5251	1.6923	1.9013	1.9035	2.1122	2.373
		50	1.1165	1.1844	1.2803	1.4849	1.5752	1.7028	1.8534	1.9661	2.1253
		75	1.0991	1.1475	1.221	1.4618	1.5262	1.6239	1.8245	1.9049	2.0268
		100	1.0875	1.1262	1.1874	1.4464	1.4978	1.5793	1.8053	1.8695	1.9711
	10,5	5	1.1961	1.8128	2.7109	1.6002	2.4441	3.7191	1.9943	3.0402	4.6058
		15	1.1751	1.3825	1.6279	1.5654	1.8429	2.1719	1.9531	2.2989	2.7087
		25	1.1523	1.2801	1.4405	1.534	1.7045	1.9186	1.9142	2.1268	2.3938
		50	1.119	1.1874	1.2841	1.4889	1.58	1.7088	1.8582	1.9718	2.1325
		75	1.1007	1.1493	1.2232	1.4643	1.5291	1.6273	1.8275	1.9083	2.031
		100	1.0887	1.1275	1.1889	1.4483	1.4999	1.5817	1.8075	1.8719	1.974
	10,10	5	1.2009	1.8295	2.768	1.5969	2.4323	3.678	1.9974	3.0509	4.6435
		15	1.1764	1.3846	1.6314	1.5645	1.8414	2.1695	1.9539	2.3002	2.7109
		25	1.153	1.2811	1.4419	1.5335	1.7038	1.9176	1.9146	2.1274	2.3946
		50	1.1193	1.1878	1.2846	1.4887	1.5797	1.7085	1.8584	1.972	2.1328
		75	1.1009	1.1496	1.2234	1.4641	1.5289	1.6272	1.8276	1.9085	2.0311
		100	1.0888	1.1276	1.1891	1.4482	1.4998	1.5815	1.8076	1.872	1.9741

## **CONFLICT OF INTERESTS**

None.

## ACKNOWLEDGMENTS

None.

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