ISSN: 2454-1907
DOI: https://doi.org/10.29121/ijetmr.v4.i12.2017.131

IJETMR
International Journal of Engineering Technologies and Management Research
A Knowledge Repository

MEAN SUM SQUARE PRIME LABELING OF SOME CYCLE RELATED GRAPHS

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Abstract:
Mean sum square prime labeling of a graph is the labeling of the vertices with {0,1,2---,p-1} and the edges with mean of the square of the sum of the labels of the incident vertices or mean of the square of the sum of the labels of the incident vertices and one, depending on the sum is even or odd. The greatest common incidence number of a vertex (gcin) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits mean sum square prime labeling. Here we identify some cycle related graphs for mean sum square prime labeling.

Keywords: *Graph Labeling; Sum Square; Greatest Common Incidence Number; Prime Labeling; Cycles.*

Cite This Article: Sunoj B S, and Mathew Varkey T K. (2017). "MEAN SUM SQUARE PRIME LABELING OF SOME CYCLE RELATED GRAPHS." *International Journal of Engineering Technologies and Management Research,* 4(12), 21-27. DOIhttps://doi.org/10.29121/ijetmr.v4.i12.2017.131.

1. Introduction

All graphs in this paper are simple, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. In [5], we introduced the concept of sum square prime labeling and proved the result for some cycle related graphs. In [6], [7], [8], [9], we proved the result for some path related graphs, some snake related graphs, some tree graphs, triangular belt, jelly fish graph, some star related graphs. In this paper we introduced mean sum square prime labeling using the concept greatest common incidence number of a vertex. We proved that some cycle related graphs admit mean sum square prime labeling.

 ISSN: 2454-1907
DOI: 10.5281/zenodo.1133510

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2. Main Results

Definition 2.1 Let G = (V, E) be a graph with p vertices and q edges. Define a bijection
$f : V(G) \to \{0,1,2,3,---,p-1\}$ by $f(v_i) = i-1$, $1\leq i\leq p$ and define a 1-1 mapping
f^*_{mssp} : E(G) → set of natural numbers N by
$f^*_{mssp}(uv) = \frac{\{f(u)+f(v)\}^2}{2}$, when f(u)+f(v) is even.
$f^*_{mssp}(uv) = \frac{\{f(u)+f(v)\}^2+1}{2}$, when f(u)+f(v) is odd. The induced function f^*_{mssp} is said to be a mean sum square prime labeling, if the ***gcin*** of each vertex of degree at least 2, is 1.

Definition 2.2 A graph which admits mean sum square prime labeling is called a mean sum square prime graph.

Theorem 2.1 Cycle C_n admits mean sum square prime labeling, if n is odd.
Proof: Let G = C_n and let $v_1,v_2,---,v_n$ are the vertices of G
Here |V(G)| = n and |E(G)| = n
Define a function $f : V \to \{0,1,2,3,---,n-1\}$ by

$$f(v_i) = i-1 , \ i = 1,2,---,n$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

$f^*_{mssp}(v_i\, v_{i+1}) = 2i^2 - 2i + 1$, $\quad i = 1,2,---,n-1$

$f^*_{mssp}(v_1\, v_n) = \frac{(n-1)^2}{2}$

Clearly f^*_{mssp} is an injection.

gcin of (v_1) = gcd of $\{f^*_{mssp}(v_1\, v_2) , f^*_{mssp}(v_1\, v_n)\}$
= gcd of $\{1, \frac{(n-1)^2}{2}\} = 1$.

gcin of (v_{i+1}) = gcd of $\{f^*_{mssp}(v_i\, v_{i+1}) , f^*_{mssp}(v_{i+1}\, v_{i+2})\}$
= gcd of $\{2i^2-2i+1, 2i^2+2i+1\}$
= gcd of $\{4i, 2i^2-2i+1\}$,
= gcd of $\{i, 2i^2-2i+1\} = 1$, $\quad i = 1,2,---,n-2$

gcin of (v_n) = gcd of $\{f^*_{mssp}(v_{n-1}\, v_n) , f^*_{mssp}(v_1\, v_n)\}$,
= gcd of $\{2n^2-6n+5, \frac{(n-1)^2}{2}\}$, put n = 2k-1
= gcd of $\{8k^2-20k+13, 2(k-1)^2\}$
= gcd of $\{(k-1), (k-1)(8k-12)+1\} = 1$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence C_n , admits mean sum square prime labeling. ■

Definition 2.3 The tadpole graph is the graph obtained by joining a cycle C_n to a path P_m and is denoted by $C_n(P_m)$

ISSN: 2454-1907
DOI: 10.5281/zenodo.1133510

Theorem 2.2 Tadpole graph $C_n(P_m)$ admits mean sum square prime labeling, if n is odd.
Proof: Let G = $C_n(P_m)$ and let $v_1, v_2, \ldots, v_{n+m-1}$ are the vertices of G
Here |V(G)| = n+m-1 and |E(G)| = n+m-1
Define a function f : V → {0,1,2,3,---,n+m-2 } by

$$f(v_i) = i-1 \,,\ i = 1,2,\ldots,n+m-1$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

$$f^*_{mssp}(v_i\, v_{i+1}) = 2i^2 - 2i + 1, \qquad i = 1,2,\ldots,n+m-2$$

$$f^*_{mssp}(v_1\, v_n) = \frac{(n-1)^2}{2}$$

Clearly f^*_{mssp} is an injection.

gcin of (v_1) = gcd of $\{f^*_{mssp}(v_1\, v_2)\,,\ f^*_{mssp}(v_1\, v_n)\}$= 1.

gcin of (v_{i+1}) = gcd of $\{f^*_{mssp}(v_i\, v_{i+1})\,,\ f^*_{mssp}(v_{i+1}\, v_{i+2})\}$
= 1, $\qquad i = 1,2,\ldots,n+m-3$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $C_n(P_m)$, admits mean sum square prime labeling. ■

Definition 2.4 Let G be the graph obtained by joining two copies of path P_m to two consecutive vertices of cycle C_n . G is denoted by $C_n(2P_m)$.

Theorem 2.3 The graph $C_n(2P_m)$ admits mean sum square prime labeling, if m and n are odd.
Proof: Let G = $C_n(2P_m)$ and let $v_1, v_2, \ldots, v_{n+2m-2}$ are the vertices of G
Here |V(G)| = n+2m-2 and |E(G)| = n+2m-2
Define a function f : V → {0,1,2,3,---,n+2m-3 } by

$$f(v_i) = i-1 \,,\ i = 1,2,\ldots,n+2m-2$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

$$f^*_{mssp}(v_i\, v_{i+1}) = 2i^2 - 2i + 1, \qquad i = 1,2,\ldots,n+2m-3$$

$$f^*_{mssp}(v_m\, v_{m+n-1}) = \frac{(2m+n-3)^2}{2}$$

Clearly f^*_{mssp} is an injection.

gcin of (v_{i+1}) = gcd of $\{f^*_{mssp}(v_i\, v_{i+1})\,,\ f^*_{mssp}(v_{i+1}\, v_{i+2})\}$
= 1, $\qquad i = 1,2,\ldots,n+2m-4$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $C_n(2P_m)$, admits mean sum square prime labeling. ■

Definition 2.5 Duplication of a vertex v of graph G produces a new graph H by adding a vertex u such that N(v) = N(u). N(x) represents neighborhood of the vertex x.

Theorem 2.4 Let G be the graph obtained by duplicating a vertex in Cycle C_n. G admits mean sum square prime labeling, if n is odd and $(n+1) \not\equiv 0 \pmod{10}$.
Proof: Let G be the graph and let $v_1, v_2, \ldots, v_{n+1}$ are the vertices of G
Here |V(G)| = n+1 and |E(G)| = n+2
Define a function f : V → {0,1,2,3,---,n } by

$$f(v_i) = i-1 \,,\ i = 1,2,\ldots,n+1$$

ISSN: 2454-1907
DOI: 10.5281/zenodo.1133510

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{mssp}^* is defined as follows

$f_{mssp}^*(v_i\, v_{i+1}) = 2i^2 - 2i + 1, \quad i = 1,2,---,n$

$f_{mssp}^*(v_1\, v_n) = \frac{(n-1)^2}{2}$

$f_{mssp}^*(v_2\, v_{n+1}) = \frac{(n+1)^2}{2}$

Clearly f_{mssp}^* is an injection.

gcin of (v_1) = gcd of $\{f_{mssp}^*(v_1\, v_2)\,,\, f_{mssp}^*(v_1\, v_n)\}$= 1.

gcin of (v_{i+1}) = gcd of $\{f_{mssp}^*(v_i\, v_{i+1})\,,\, f_{mssp}^*(v_{i+1}\, v_{i+2})\}$
= 1, $\quad i = 1,2,---,n-1$

gcin of (v_{n+1}) = gcd of $\{f_{mssp}^*(v_{n+1}\, v_n)\,,\, f_{mssp}^*(v_2\, v_{n+1})\}$
= gcd of $\{ 2n^2\text{-}2n+1, \frac{(n+1)^2}{2}\}$, put n = 2k-1
= gcd of $\{ 8k^2\text{-}12k+5, 2k^2\}$
= gcd of $\{k, 8k^2\text{-}12k+5\}$
=1, since $(n+1)\not\equiv 0(\text{mod}10)$, then $k \not\equiv 0(\text{mod}5)$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits mean sum square prime labeling. ■

Example 2.1 Vertex duplication in cycle C_5

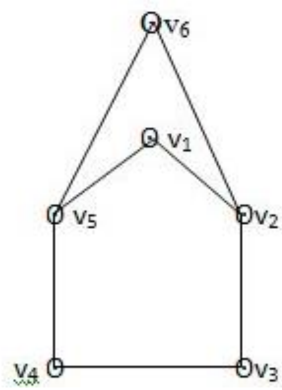

Figure 2.1:

Definition 2.6 Duplication of an edge e = ab of a graph G produces a new graph H by adding an edge f = xy such that N(x) = N(a) U {y}- {b} and N(y) = N(b) U {x}- {a}.

Theorem 2.5 Let G be the graph obtained by duplicating an edge in Cycle C_n . G admits mean sum square prime labeling, if n is odd and $(n+3) \not\equiv 0$ (mod 26).

Proof: Let G be the graph and let $v_1,v_2,---,v_{n+2}$ are the vertices of G

Here |V(G)| = n+2 and |E(G)| = n+3

Define a function f : V → {0,1,2,3,---,n+1 } by

$f(v_i) = i\text{-}1$, i = 1,2,---,n+2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{mssp}^* is defined as follows

$f_{mssp}^*(v_i\, v_{i+1}) = 2i^2 - 2i + 1, \quad i = 1,2,---,n+1$

ISSN: 2454-1907
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$f^*_{mssp}(v_1\ v_n) \qquad = \frac{(n-1)^2}{2}$

$f^*_{mssp}(v_3\ v_{n+2}) \qquad = \frac{(n+3)^2}{2}$

Clearly f^*_{mssp} is an injection.

gcin of (v_1) $\qquad$ = gcd of $\{f^*_{mssp}(v_1\ v_2)\ ,\ f^*_{mssp}(v_1\ v_n)\ \}$
= 1.

gcin of (v_{i+1}) $\qquad$ = gcd of $\{f^*_{mssp}(v_i\ v_{i+1})\ ,\ f^*_{mssp}(v_{i+1}\ v_{i+2})\ \}$
= 1, $\qquad$ i = 1,2,---,n.

gcin of (v_{n+2}) $\qquad$ = gcd of $\{f^*_{mssp}(v_{n+1}\ v_{n+2})\ ,\ f^*_{mssp}(v_3\ v_{n+2})\ \}$
= gcd of $\{\ 2n^2+2n+1,\ \frac{(n+3)^2}{2}\}$, put n = 2k-1
= gcd of $\{\ 8k^2-4k+1,\ 2(k+1)^2\}$
= gcd of $\{\ k+1,\ 8k^2-4k+1\}$
= gcd of {13, k+1}
= 1. Since $(n+3) \not\equiv 0 \pmod{26}$, then $(k+1) \not\equiv 0 \pmod{13}$.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence G, admits mean sum square prime labeling. ■

Example 2.2 Duplication of an edge in cycle C_5.

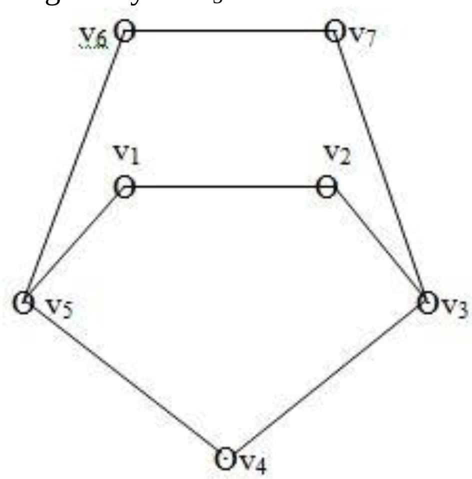

Figure 2.2:

Definition 2.7 Duplication of a vertex v by a new edge e = ab in a graph G produces a new graph H such that N(a) = {v,b} and N(b) = {v,a}.

Theorem 2.6 Let G be the graph obtained by duplicating a vertex by an edge in Cycle C_n . G admits mean sum square prime labeling, when n is odd and $(n+3) \not\equiv 0 \pmod{26}$.

Proof: Let G be the graph and let $v_1,v_2,\text{---},v_{n+2}$ are the vertices of G
Here |V(G)| = n+2 and |E(G)| = n+3
Define a function f : V → {0,1,2,3,---,n+1 } by

$$f(v_i) = i-1\ ,\ i = 1,2,\text{---},n+2$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows

 ISSN: 2454-1907
DOI: 10.5281/zenodo.1133510

$f^*_{mssp}(v_i\ v_{i+1})$ = $2i^2 - 2i + 1$, $i = 1,2,---,n+1$
$f^*_{mssp}(v_1\ v_3)$ = 2
$f^*_{mssp}(v_3\ v_{n+2})$ = $\frac{(n+3)^2}{2}$
Clearly f^*_{mssp} is an injection.
gcin of (v_1) = gcd of $\{f^*_{mssp}(v_1\ v_2)\ ,\ f^*_{mssp}(v_1\ v_3)\ \}$
= 1.
gcin of (v_{i+1}) = gcd of $\{f^*_{mssp}(v_i\ v_{i+1})\ ,\ f^*_{mssp}(v_{i+1}\ v_{i+2})\ \}$
= 1, $i = 1,2,---,n$.
gcin of (v_{n+2}) = gcd of $\{f^*_{mssp}(v_{n+1}\ v_{n+2})\ ,\ f^*_{mssp}(v_3\ v_{n+2})\ \}$
= gcd of $\{\ 2n^2+2n+1, \frac{(n+3)^2}{2}\}$= 1.
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence G, admits mean sum square prime labeling. ■

Theorem 2.7 Let G be the graph obtained by attaching pendant edges to each vertex of cycle C_n.G admits mean sum square prime labeling, if n is odd.
Proof: Let G be the grapg and let $v_1,v_2,---,v_{2n}$ are the vertices of G
Here |V(G)| = 2n and |E(G)| = 2n
Define a function f : V → {0,1,2,3,---,2n-1 } by
$f(v_i) = i-1$, $i = 1,2,---,2n$
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{mssp} is defined as follows
$f^*_{mssp}(v_{2i-1}\ v_{2i})$ = $8i^2 - 12i + 5$, $i = 1,2,---,n$.
$f^*_{mssp}(v_{2i-1}\ v_{2i+1})$ = $8i^2 - 8i + 2$, $i = 1,2,---,n-1$.
$f^*_{mssp}(v_1\ v_{2n-1})$ = $2n^2-4n+2$
Clearly f^*_{mssp} is an injection.
gcin of (v_{2i-1}) = gcd of $\{f^*_{mssp}(v_{2i-1}\ v_{2i})\ ,\ f^*_{mssp}(v_{2i-1}\ v_{2i+1})\ \}$
= gcd of $\{\ 8i^2 - 12i + 5,\ 8i^2 - 8i + 2\}$
= gcd of $\{4i-3,\ 8i^2 - 12i + 5\}$,
= gcd of $\{2i-1,\ 4i-3\}$
= gcd of $\{2i-1,\ 2i-2\} = 1$, $i = 1,2,---,n-2$
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence G , admits mean sum square prime labeling. ■

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