ISSN: 2454-1907
DOI: https://doi.org/10.29121/ijetmr.v5.i4.2018.218

IJETMR
International Journal of Engineering Technologies and Management Research
A Knowledge Repository

PRODUCT CORDIAL LABELING OF DOUBLE PATH UNION OF C_3 RELATED GRAPHS

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Abstract:
In this paper we discuss The double path unions obtained on C_3 related graphs for product cordial labeling given by $P_m(2G')$ where $G' = C_3$, $FL(C_3)$, C_3^+, bull, tail$(C_3,2P_2)$ etc.

Keywords: *Labeling; Cordial; Product; Bull Graph; Crown; Flag Graph; Tail Graph; Double Path Union.*

Subject Classification: *05C78*

Cite This Article: Mukund V. Bapat. (2018). "PRODUCT CORDIAL LABELING OF DOUBLE PATH UNION OF C3 RELATED GRAPHS." *International Journal of Engineering Technologies and Management Research,* 5(4), 146-152. DOI: 10.29121/ijetmr.v5.i4.2018.218.

1. Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [9], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[11]. I.Cahit introduced the concept of cordial labeling [5]. There are variety of cordial labeling available in labeling of graphs. Sundaram, Ponraj, and Somasundaram [10] introduced the notion of **product cordial labeling**. A product cordial labeling of a graph G with vertex set V is a function f from V to {0,1} such that if each edge uv is assigned the label f(u)f(v), the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a product cordial labeling is called a product cordial graph. We use $v_f(0,1) = (a, b)$ to denote the number of vertices with label 1 are a in number and the number of vertices with label 0 are b in number. Similar notion on edges follows for $e_f(0,1) = (x, y)$.

A lot of work is done in this type of labeling so far. One interested in survey may refer Dynamic survey in Graph labeling by J. Gillian. We mention some part of it. Sundaram, Ponraj, and Somasundaram have shown that trees; unicyclic graphs of odd order; triangular snakes; dragons; helms; Pm∪Pn; Cm∪Pn; Pm∪K1,n; Wm∪Fn (Fn is the fan Pn+K1); K1,m∪K1,n; Wm∪ K1,n; Wm∪Pn; Wm∪Cn; the total graph of Pn (the total graph of Pn has vertex set V (Pn)∪E(Pn) with two vertices adjacent whenever they are neighbors in Pn); Cn if and only if n is odd; $C_n^{(t)}$, the one-point union of t copies of C_n, provided t is even or both t and n are even; K2+mK1 if and only if m is odd; $C_m \cup P_n$ if and only if m+n is odd; $K_{m,n} \cup P_s$ if s >mn; Cn+2∪K1,n; Kn∪Kn,(n−1)/2

ISSN: 2454-1907
DOI: 10.5281/zenodo.1251967

when n is odd; Kn∪Kn−1,n/2 when n is even; and P2 n if and only if n is odd. They also prove that $K_{m,n}$ (m,n> 2), $P_m \times P_n$ (m,n> 2) and wheels are not product cordial and if a (p,q)-graph is product cordial graph, then q 6 (p−1)(p + 1)/4 + 1.In this paper we show thatPm(2G') where G' = C_3, FL(C_3), C_3^+, bull, tail(C_3,2P_2) are product cordial graphs.

2. Preliminaries

2.1. Fusion of Vertex

Let G be a (p,q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges.If u∊G_1 and v∊G_2, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with the concept is well elaborated in John Clark,Holton[6]

2.2. Crown Graph

It is C_nOK_2.At each vertex of cycle a n edge was attached. We develop the concept further to obtain crown for any graph. Thus crown (G) is a graph G OK_2.It has a pendent edge attached to each of it's vertex. If G is a (p,q) graph then crown(G) has q+p edges and 2p vertices.

2.3. Flag of a Graph G denoted by FL(G) is obtained by taking a graph G=G(p,q).At suitable vertex of G attach a pendent edge.It has p+1 vertices and q+1 edges.

2.4. A bull Graph Bull (G) was initially defined for a C_3-bull. It has a copy of G with an pendent edge each fused with any two adjacent vertices of G. For G is a (p,q) graph, bull(G) has p+2 vertices and q+2 edges.

2.5. A Tail Graph (Also called as antenna graph) is obtained by fusing a path p_k to some vertex of G. This is denoted by tail(G, P_k). If there are t number of tails of equal length say (k-1) then it is denoted by tail(G, tp_k). If G is a (p,q) graph and a tail P_k is attached to it then tail(G, P_k) has p+k-1 vertices and q+k-1 edges4.

2.6. Path union of G, i.e.P_m(G) is obtained by taking a path p_m and take m copies of graph G . Then fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges. Where G is a (p,q) graph.

2.7. Double path union of G,i.e.P_m(2G) is obtained by taking a path p_m and take 2m copies of graph G . Then fuse two copies of G at every vertex of path at given fixed vertex on G. It has 2mp vertices and 2mq +m-1 edges. Where G is a (p,q) graph.

3. Main Results

Theorem 3.1 Double path union of triangles given by G = Pm (2C_3) is product cordial for all m

ISSN: 2454-1907
DOI: 10.5281/zenodo.1251967

Proof: We define G as a graph with path P_m given by (v_1,e_1, v_2, e_2, v_3, e_3..,v_m).The one of the copies of C_3 fused at V_i is ($u_{i,1}$,$e_{i,1}$, $u_{i,2}$, $e_{i,2}$,$u_{i,3}$,$e_{i,3}$,$u_{i,1}$).The other copy of C_3 is given by ($w_{i,1}$,$c_{i,1}$, $w_{i,2}$, $c_{i,2}$,$w_{i,3}$,$c_{i,3}$,$w_{i,1}$).Note that $u_{i,1}$, v_i, $w_{i,1}$ are the same vertices as v_i; i = 1, 2, ..m. (being common vertex to both copies of C_3 and path vertex v_i).

Define a function f: V(G) → {0,1} as follows:

Case m = 2x ,

f(ui,j)= 0, j = 1, 2, 3; i = 1, 2, ..x.

f(wi,j) = 0, j = 1, 2, 3; i = 1, 2, ..x.

f(ui,j) = 1, j = 1, 2, 3; i = x+1,x+ 2, ..,2x.

f(wi,j) = 1, j = 1, 2, 3; i = x+1, x+2, ..,2x.

The label distribution is $v_f(0,1)$ = (5x,5x) and $e_f(0,1)$ =(7x, 7x-1) .

Case m = 2x+1

The f defined as above is followed for $P_{2x}(2C_3)$part of $P_{2x+1}(2C_3)$ and on the last vertex where i = 2x+1we have ,

f($u_{i,j}$)=1 fori = 2x+1 and j = 1,2,3.

f($w_{i,j}$)=0 for i = 2x+1 and j = 2,3.

The label distribution is $v_f(0,1)$ = (5x+2,5x+3) and $e_f(0,1)$ =(7x+3, 7x+3) .

Thus the graph is product cordial for all m. #.

Theorem 3.2 Let G' be FL (C_3) then P_m(2G') is product cordial .(All structures).

Proof: We define G as a graph with path P_m given by (v_1,e_1, v_2, e_2, v_3, e_3..,v_m).The one of the copies of C_3 fused at V_i is ($u_{i,1}$,$e_{i,1}$, $u_{i,2}$, $e_{i,2}$,$u_{i,3}$,$e_{i,3}$,$u_{i,1}$).The other copy of C_3 is given by ($w_{i,1}$,$c_{i,1}$, $w_{i,2}$, $c_{i,2}$,$w_{i,3}$,$c_{i,3}$,$w_{i,1}$).Note that $u_{i,1}$, v_i, $w_{i,1}$ are the same vertices as v_i; i = 1, 2, ..m. (being common vertex to both copies of C_3 and path vertex v_i). The pendent vertices u_i' and w_i' are are attached by pendent edges e_i' and e_i'' respectively to vertex $u_{i,2}$ and $w_{i,2}$.Thus G has 7m vertices and 9m-1 edges.

Define a function f: V(G) → {0,1} as follows:

Case m = 2x ,

f($u_{i,j}$) = 0, j = 1, 2, 3; i = 1, 2, ..x;

f($w_{i,j}$) = 0, j = 1, 2, 3; i = 1, 2, ..x;

f(u_i') = 0 for i = 1, 2, ..x;

f(w_i') = 0 for i = 1, 2, ..x

f($u_{i,j}$) = 1, j = 1, 2, 3; i = x+1,x+ 2, ..,2x.

f($w_{i,j}$) = 1, j = 1, 2, 3; i = x+1, x+2, ..,2x.

f(u_i') = 1 for i = x+1,x+2, ..,2x;

f(w_i') = 1 for i = x+1, x+2, ..,2x.

The label distribution is $v_f(0,1)$ = (8x,8x) and $e_f(0,1)$ =(9x, 9x-1) .

Case m = 2x +1,

The P_{2x}(2G') part of P_{2x+1}(2G') is labeled first as above.

ISSN: 2454-1907
DOI: 10.5281/zenodo.1251967

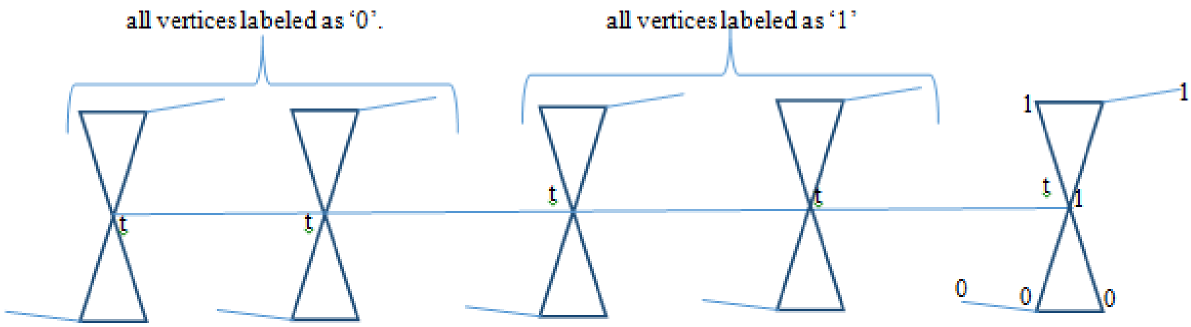

Figure 3.1: Labeled copy of $P_5(2FL(C_3))v_f(0,1) = (19,20)$; $e_f(0,1) = (22,22)$; Structure 1

for i = 2x+1 we have,
$f(u_{i,j}) = 1$, *for all j = 1,2, 3.*
$f(w_{i,j}) = 1$, *for all j = 2, 3.*
$f(u_i') = 1$;
$f(w_i') = 0$.

The label distribution is $v_f(0,1) = (8x+3,8x+4)$ and $e_f(0,1) =(9x+4, 9x+4)$ .In above labeling the label of pendent vertex is done separately in the sense that it is independent of which vertex on C_3 the pendent edge is fused with. Hence the same function works for all structures to produce product cordiallabeling.

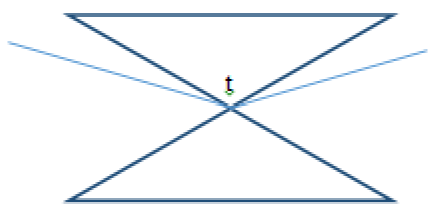

Figure 3.2: Structure 2: path union can be taken at vertex 't'. Any other structure is out of scope

Thus the graph is product cordial for all m. #.

Theorem 3.3 Double path union on bull graph given by $P_m(2bull)$ is product cordial.

Proof: We define G as a graph with path P_m given by $(v_1,e_1, v_2, e_2, v_3, e_3..,v_m)$.The one of the copies of C_3 fused at v_i is $(u_{i,1}, e_{i,1}, u_{i,2}, e_{i,2}, u_{i,3}, e_{i,3}, u_{i,1})$.The other copy of C_3 is given by $(w_{i,1},c_{i,1}, w_{i,2}, c_{i,2},w_{i,3},c_{i,3},w_{i,1})$.Note that $u_{i,1}$, v_i, $w_{i,1}$ are the same vertices as v_i; i = 1, 2, ..m. (being common vertex to both copies of C_3 and path vertex v_i). The pendent verticesarebu$_i$',u$_i$" and w$_i$',w$_i$" with corresponding pendent edges $(u_{i,1}u_i')$,$(u_{i,1}u_i'')$, $(w_{i,1}u_i')$,$(w_{i,2}u_i'')$respectively. Thus G has has9m vertices and 11m-1 edges.

Define a function f: V (G) → {0, 1} as follows:
Case m = 2x,

 ISSN: 2454-1907
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$f(u_{i,j}) = 0, j = 1, 2, 3; i = 1, 2, ..x;$
$f(w_{i,j}) = 0, j = 1, 2, 3; i = 1, 2, ..x;$
$f(u_i') = 0$ for i = 1, 2, ..x;
$f(u_i'') = 0$ for i = 1, 2, ..x;
$f(w_i') = 0$ for i = 1, 2, ..x;
$f(w_i'') = 0$ for i = 1, 2, ..x;
$f(u_{i,j}) = 1, j = 1, 2, 3; i = x+1, x+2, ..,2x.$
$f(w_{i,j}) = 1, j = 1, 2, 3; i = x+1, x+2, ..,2x.$
$f(u_i') = 1$ for i = x+1,x+2, ..,2x;
$f(w_i') = 1$ for i = x+1, x+2, ..,2x.
$f(u_i'') = 1$ for i = x+1,x+2, ..,2x;
$f(w_i'') = 1$ for i = x+1, x+2, ..,2x.
The label distribution is $v_f(0,1) = (9x,9x)$ and $e_f(0,1) = (11x, 11x-1)$.

The P_{2x}(2bull) part of P_{2x+1}(2bull) is labeled first as above,
for i = 2x+1 we have,
$f(u_{i,j}) = 1$, *for all j = 1,2, 3.*
$f(w_{i,j}) = 0$, *for all j = 2, 3.*
$f(u_i') = 1;$
$f(w_i') = 0.$
$f(u_i'') = 1;$
$f(w_i'') = 0.$

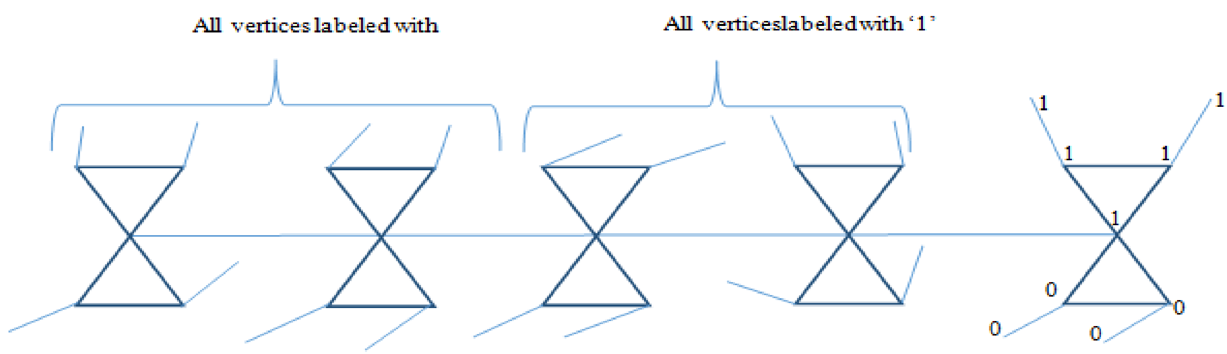

Figure 3.3: Pm (2bull), labeled copy, numbers stands for vertex labels

The label distribution is $v_f(0,1) = (9x+4,9x+5)$ and $e_f(0,1) = (11x+5, 11x+5)$.

Thus the graph is product cordial for all m. #.

Theorem 3.4 Double path union on crown C_3^+ given by G $=P_m(2C_3^+)$ is product cordial.

Proof: We define G as a graph with path P_m given by (v_1,e_1, v_2, e_2, v_3, e_3..,v_m).The one of the copies of C_3 fused at v_i is ($u_{i,1}$, $e_{i,1}$, $u_{i,2}$, $e_{i,2}$, $u_{i,3}$, $e_{i,3}$, $u_{i,1}$).The other copy of C_3 is given by ($w_{i,1}$, $c_{i,1}$, $w_{i,2}$, $c_{i,2}$, $w_{i,3}$, $c_{i,3}$, $w_{i,1}$). Note that $u_{i,1}$, v_i, $w_{i,1}$ are the same vertices as v_i; i = 1, 2, ..m. (being common vertex to both copies of C_3 and path vertex v_i). At each $u_{i,j}$ a pendent vertex is attached is

ISSN: 2454-1907
DOI: 10.5281/zenodo.1251967

$u_{i,j}$', j = 1, 2, 3.At each $w_{i,j}$ a pendent vertex is attached is $w_{j,j}$', j = 1, 2, 3.Thus G has |V(G)|=11m and |E(G)|= 13m-1.

Define a function f: V(G) → {0,1} as follows:

Case m = 2x ,

f($u_{i,j}$) = 0, j = 1, 2, 3; i = 1, 2, ..x;

f($w_{i,j}$) = 0, j = 1, 2, 3; i = 1, 2, ..x;

f($u_{i,j}$') = 0 for i = 1, 2, ..x; j= 1,2, 3.

f($w_{i,j}$') = 0 for i = 1, 2, ..x; *j = 1, 2, 3.*

f($u_{i,j}$) = 1, j = 1, 2, 3; i = x+1,x+ 2, ..,2x.

f($w_{i,j}$) = 1, j = 1, 2, 3; i = x+1, x+2, ..,2x.

f($u_{i,j}$') = 1 for i = x+1,x+2, ..,2x;j = 1, 2, 3.

f($w_{i,j}$') = 1 for i = x+1, x+2, ..,2x; j = 1, 2, 3.

The label distribution is vf(0,1) = (11x,11x) and e_f(0,1) =(13x, 13x-1) .

The P_{2x} (2bull) part of $P2x_{+1}(C_3^+$ is labeled first as above. For i = 2x+1 we have,

f($u_{i,j}$) = 1, j = 1, 2, 3;

f($w_{i,j}$) = 0, j = 2, 3;

f($u_{i,j}$') = 1 for j =1, 2, 3,

f($w_{i,j}$') = 0fo j= 1, 2, 3,

The label distribution is vf(0,1) = (11x+5,11x + 6) and e_f(0,1) =(13x+6, 13x+6) .

Thus the graph is product cordial for all m. #.

Theorem 3.4 Double path union on G' = tail (C_3,2P_2) given by P_m(2G') is product cordial.

Proof: We define G as a graph with path P_m given by (v_1,e_1, v_2, e_2, v_3, e_3..,v_m).The one of the copies of C_3 fused at v_i is ($u_{i,1}$, $e_{i,1}$, $u_{i,2}$, $e_{i,2}$, $u_{i,3}$, $e_{i,3}$, $u_{i,1}$).The other copy of C_3 is given by ($w_{i,1}$, $c_{i,1}$, $w_{i,2}$, $c_{i,2}$, $w_{i,3}$, $c_{i,3}$, $w_{i,1}$). Note that $u_{i,1}$, v_i, $w_{i,1}$ are the same vertices as v_i; i = 1, 2, ...m. (being common vertex to both copies of C_3 and path vertex v_i). At $u_{i,1}$ two pendent vertices are fused $u_{i,1}$',$u_{i,1}$''with edges given by ($u_{i,1}u_{i,1'}$) and ($u_{i,1}u_{i,1''}$). At $w_{i,1}$ two pendent vertices are fused $w_{i,1}$',$w_{i,1}$''with edges given by ($w_{i,1}w_{i,1'}$) and ($w_{i,1}w_{i,1''}$). Thus, G has |V(G)|= 9m and |E(G)|= 11m-1.

Define a function f: V (G) → {0, 1} as follows:

Case m = 2x,

f($u_{i,j}$) = 0, j = 1, 2, 3; i = 1, 2, ..x;

f($w_{i,j}$) = 0, j = 1, 2, 3; i = 1, 2, ..x;

f($u_{i,1}$') = 0 for i = 1, 2, ..x;

f($w_{i,1}$') = 0 for i = 1, 2, ..x;

f($u_{i,1}$'') = 0 for i = 1, 2, ..x;

f($w_{i,1}$'') = 0 for i = 1, 2, ..x;

f($u_{i,j}$) = 1, j = 1, 2, 3; i = x+1,x+ 2, ..,2x.

f($w_{i,j}$) = 1, j = 1, 2, 3; i = x+1, x+2, ..,2x.

f($u_{i,j}$') = 1 for i = x+1,x+2, ..,2x;

f($w_{i,j}$') = 1 for i = x+1, x+2, ..,2x.

ISSN: 2454-1907
DOI: 10.5281/zenodo.1251967

f($u_{i,1}$”) = 1 for i = 1, 2, ..x;
f($w_{i,1}$”) = 0 for i = 1, 2, ..x;
The label distribution is $v_f(0,1) = (9x,9x)$ and $e_f(0,1) =(11x, 11x-1)$.
Case m = 2x+1
The P_{2x} (2G’) part of $P2x_{+1}$(2G’) is labeled first as above. For i = 2x+1 we have,
f($u_{i,j}$) = 1, j = 1, 2, 3;
f($w_{i,j}$) = 0, j = 2, 3;
f($u_{i,1}$’) = 1 ,
f($w_{i,1}$’) = 0 ,
f($u_{i,1}$”) = 1 ,
f($w_{i,1}$”) = 0 ,
The label distribution is $v_f(0,1) = (9x+4,9x + 5)$ and $e_f(0,1) =(11x+5, 11x+5)$.

Thus the graph is product cordial for all m. #.

4. Conclusions

We have obtained double path union G = P_m(2G’) by fusing two copies of same graph G’ at every vertex of a path P_m. We have shown that G is product cordial when G’ is 1) triangle C_3, 2) Fl(C_3), 3) bull(C_3), 4) C_3^+, 5) tail(C_3,$2P_2$). We conjecture that G = P_m(G’)is product cordial when G’ is 1)cycle C_n, 2) Fl(C_n), 3) bull(C_n), 4) C_n^+, 5) tail(C_n,$2P_2$) etc.

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