ISSN: 2454-1907
DOI: https://doi.org/10.29121/ijetmr.v5.i3.2018.186

IJETMR
International Journal of Engineering Technologies and Management Research
A Knowledge Repository

WHEEL RELATED ONE POINT UNION OF VERTEX PRIME GRAPHS AND INVARIANCE

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Abstract:
We investigate one point unions of W4, and graphs obtained from W4 such as gear graph G4, each cycle edge of W4 replaced with Pm, each pokes of W4 replaced with Pm for vertex prime labeling. All different non isomorphic structures of these graphs obtained by taking one point union graphs are shown to be vertex prime. This property of graphs is called as invariance under vertex prime labeling.

Keywords: *Labeling; Vertex Prime; Wheel; One Point Union.*

Cite This Article: Mukund V. Bapat. (2018). "WHEEL RELATED ONE POINT UNION OF VERTEX PRIME GRAPHS AND INVARIANCE." *International Journal of Engineering Technologies and Management Research,* 5(3), 145-150.

1. Introduction

The graphs we consider are finite, connected, and simple and un- directed. We refer F.Harary [4], Dynamic survey of graph labeling [3] for definitions and terminology. Deretsky, Lee, Mitchem Proposed a labeling called as vertex prime labeling of graph.[2].A function f : E(G) → {1,2,..|E|} is such that for any vertex v the gcd of all labels on edges incident with v is 1. This is true to all vertices with degree at least 2.The graph that admits vertex prime labeling is called as vertex prime graph. They have shown that all forests, connected graphs, $5C_{2m,}$ graph with exactly two components one of which is not odd cycle etc are vertex prime. One should refer A Dynamic survey of graph labeling by Joe Gallian to find further work done in this type of labeling.

In this paper we discuss vertex prime labeling of one point union of k copies of graphs. If we change the vertex on G at which one point union is taken, we may will get non isomorphic structure. We have studied non isomorphic one point unions of $W_4$and graphs obtained from W_4 such as gear graph G_4, each cycle edge of W_4 replaced with P_m, each pokes of W_4 replaced with P_m, each cycle edge replaced with P_n and each pokes replaced with P_m for vertex prime labeling and have shown that they are vertex prime graphs. This is invariance under vertex prime labeling.

2. Preliminaries

1) A wheel graph W_n is obtained by taking a cycle C_n and a new vertex w outside of C_n. w is joined to each vertex of C_n by an edge each. It has 2n edges and n+1 vertices.

DOI: 10.5281/zenodo.1216849

2) $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If we change the vertex on graph G used to fuse on k copies we may will get a different structure of one point union. If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$ and $|E(G)|= k.q$
3) Fusion of vertex. Let G be a (p,q) graph. let $u \neq v$ be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges.[6]
4) A gear graph G_n is obtained from W_n by adding a new vertex on each edge of cycle C_n of W_n. It has 2n+1 vertices and 3n edges.

3. Theorems Proved

3.1. Theorem One point union of k copies of W_4 i.e. $G = (W_4)^{(K)}$ is vertex prime

Proof: G has 4k +1 vertices and 8k edges. The i^{th} copy of W_4 in $(W_4)^{(k)}$ be given ordinary labeling as follows : w as hub, and cycle vertices $w_{i,1}$, $w_{i,2}$, $w_{i,3}$, $w_{i,4}$. It's edge set is given by $E(G) = \{c_{i,j}=(w_iw_{i,j})\}$ and $\{e_{i,j}=(w_{i,j}w_{i,j+1})$, j= 1,2,3,4.j+1 taken (modulo 4)$\}$.

Define a function $f:E(G) \rightarrow \{1,2,\ldots,8k\}$ as follows.

$f(e_{i,j})=8(i-1)+j$ fori = 1,2, .. k and j = 1,2,3,4.

$f(c_{i,j})= f(e_{i,4})+j$.

All the consecutive cycle edges of C_4has label as consecutive numbers in addition to one edge with label 1.Therefore the gcd of labels incident on any vertex on cycle is one. The vertex w_i has at least four edges with label as consecutive integer. Label of hub w_i is 1.In the structure where one point union is taken at w_i as shown in figure the G has vertex prime label. Note that if we change the common vertex of one point union from hub to any vertex on cycle of W_4we will get another structure of $G^{(k)}$.These are the only two non-isomorphic structures possible on $G^{(k)}$. In both cases the same labeling function will work as vertex prime label function and the resultant graph is be vertex prime.

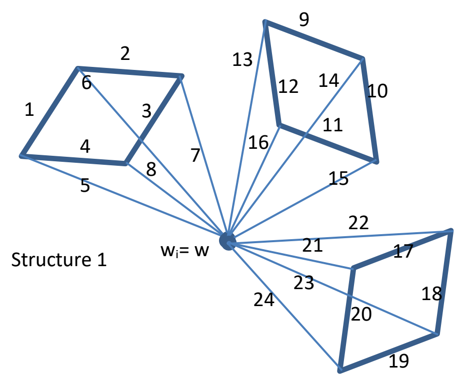
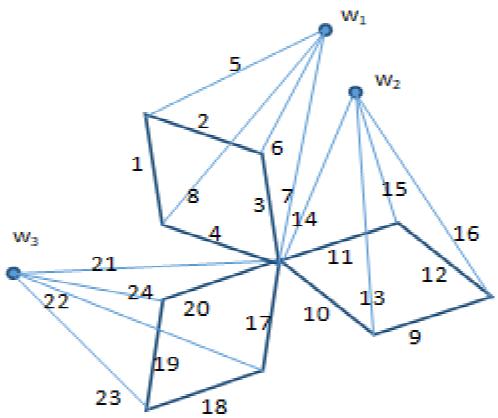


Fig 4.1: $W_4^{(3)}$ with hub as common point: edge labels are shown.

ISSN: 2454-1907
DOI: 10.5281/zenodo.1216849



Structure 2

Fig 4.2: $W_4^{(3)}$ with hub as common vertex: edge labels are shown.

Thus there are two non-isomorphic structures possible on G and both are vertex prime. Thus G is invariant under vertex prime labeling.

3.2. Theorem One Point Union of G_4 I.E. $G = (G_4)^{(K)}$ is Vertex Prime

Proof: G_4 has 9 vertices and 12 edges. $G= (G_4)^{(K)}$has 8k+1 vertices and 12k edges. The i^{th} copy of G_4 in $(G_4)^{(k)}$ be given ordinary labeling as: was hub, the vertices on cycle C_8 be $w_{i,1}$, $w_{i,2}$, …$w_{i,7}$, $w_{i,8}$. It's edge set is given by E(G) = { $c_{i,j}=(ww_{i,j})$ where j= 1,3,5,7 and $e_{i,j}=(w_{i,j}w_{i,j+1})$, j = 1,2 ..8;j +1 taken (modulo 8),for i = 1, 2, .., k}.

Define a function $f:E(G)\rightarrow \{1,2,\ldots,12k\}$ as follows.

$f(e_{i,j})=12(i-1)+j$ for i = 1,2, .. k and j = 1,2,3,4.

$f(c_{i,j})= f(e_{i,8})+j$.

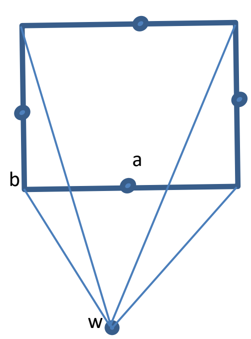


Fig. 4.3 Gear graph G_4

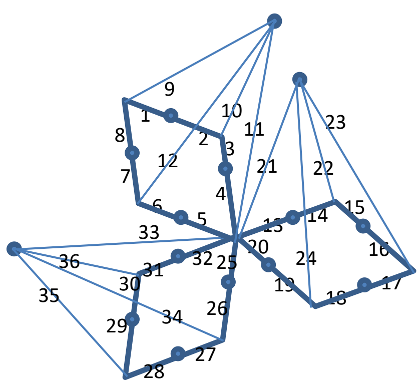


Fig 4.4: $G_4^{(3)}$ with 'b' as common vertex: edge labels are shown.

On gear graph G_4 we can take non-isomorphic one point union at points 'a' or 'b' or 'c' only. Thus there are only three non-isomorphic structures possible on G.For all these structures the labeling function f as above gives vertex prime labeling. All structures are vertex prime graphs.

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3.3. Theorem Let G'_4 be a graph obtained from W_4 by replacing each cycle edge (these edges are other than pokes) by a path P_m. Then G'_4 and one point union of k copies of G'_4 i.e. $G = (G'_4)^{(K)}$ are vertex prime.

Proof: G'_4 has 4m-3 vertices and 4m edges. G has (4m-5)k+1 vertices and 4mk edges.The i^{th} copy of G'_4 in $(G'_4)^{(k)}$ be given ordinary labeling as w_i as hub, the vertices on cycle C_{4m-4} be $w_{i,1}$, $w_{i,2}$, …$w_{i,7}$, $w_{i,4m-4}$. It's edge set is given by $E(G) = \{c_{i,j}=(w_i w_{i,j})$ where j= 1,m,2m-1,3m-2; for i = 1, 2, .., k;$\}U\{e_{i,j}=(w_{i,j}w_{i,j+1})$, j = 1,2 ..4m-4; j +1 taken (modulo 4m-4)); for i = 1, 2, .. , k$\}$.

Define a function $f:E(G) \rightarrow \{1, 2, \ldots, 4mk\}$ as follows.

$f(e_{i,j})=4m(i-1)+j$ for i = 1, 2, .. k and j = 1, 2, .., 4m-4.

$f(c_{i,j})= f(e_{i,4m-4})+j$; j=1,2,3,4

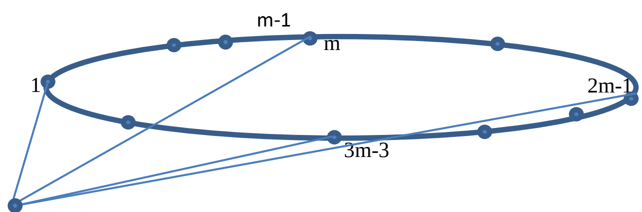

Fig 4.5 $G'_4 = W_4$ with each cycle edge replaced

We can take one point union at hub or at any of degree - 3 vertices on cycle (there are four such vertices and all four will produce isomorphic structure.) or at any degree two vertex on C_n. There are 4m-8 degree -2 vertices of which m-2vertices between v_1 to v_m will produce non-isomorphic structures. Thus there can be m non-isomorphic structures possible on one point union of k copies of G'_4.The above function f will work for all structures and produce vertex prime labeling. Thus the graph is vertex prime.

3.4. Theorem Let G'_4 be a graph obtained from W_4 by replacing each pokes edge (these edges are incident with hub) by a path P_m. Then G'_4 and one point union of k copies of G'_4 i.e. $G = (G'_4)^{(K)}$ are vertex prime

Proof: G' has 4m-3 vertices and has 4m edges.$G=(G'_4)^{(k)}$ has 4km edges and 4km-4k+1 vertices. The ith copy of G be given ordinary labeling as $\{(v_{i1}e_{i1}v_{i2}e_{i2}v_{i3}e_{i3}v_{i4}e_{i4})$ on cycle $C_4\}$ and on four paths for j = 1..4 be given by $(w_i v_{ij})=\{w_i c^j_{i1} u_{i1} c^j_{i2} u_{i2} \ldots u_{i(m-2)} c^j_{i(m-1)} v_{ij}\}$.

Define a function $f:E(G) \rightarrow \{1, 2, \ldots, 4mk\}$ as follows.

$f(e_{i,j})=4m(i-1)+j$ for i = 1, 2, .. k and j = 1, 2, .., 4.

$f(c^j_{i,t})= f(e_{i,4})+(j-1)(m-1)+t$; t=1,2,3,..(m-1), j = 1, 2, 3, 4; i= 1, 2, ..k

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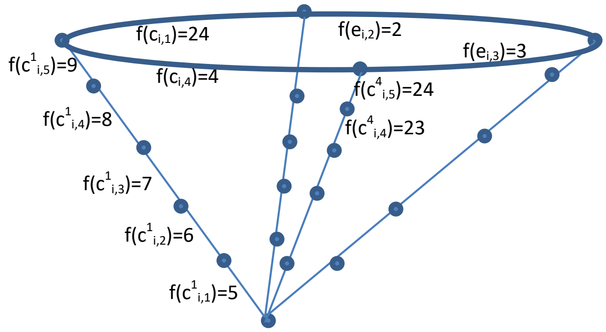

Fig 4.6 $G'_4 = W_4$ with each pokes replaced with P_6 with labels

3.5. Theorem Let G''_4 be a graph obtained from W_4 by replacing each cycle edge (these edges are other than pokes) by a path P_m and each pokes is replaced by a path p_n. Then G''_4 and one point union of k copies of G''_4 i.e. $G = (G''_4)^{(K)}$ with all structures are vertex prime

Proof: In G we define ordinary labeling of i^{th} copy of G" as on cycle of i^{th} copy we have $\{v_{i1}e_{i1}v_{i2}e_{i2}..v_{i(n-1)}\ e_{im}v\}$. The four pokes are $C^J = \{(w_i v^j_i), j = 1,2,3,4\}$.These are given by $C^j = \{(w_i, c^j_1, u^j_{i1}, c^j_2, .. c^j_{i(n-1)}, u^j_{i(n-1)}, \ c^j_n, u^j_{in}\ / j=1,2,3,4;\ i = 1,2,..k\}$. Note that $u^j_{in} = v^j_i$ for j =1,2,3,4 and i= 1,2,..k. G" has p= 4m+4n-7 vertices and q = 4m+4n-8 edges. G has kp-k+1 vertices and qk edges.

Define a function $f:E(G) \rightarrow \{1, 2, \ldots, 4mk\}$ as follows.

$f(e_{i,j}) = q(i-1)+j$ for i = 1, 2, .. k and j = 1, 2, .., q.

$f(c^j_{i,t}) = f(e_{i,q})+(=j-1)(n-1)+t$; t=1,2,3,..(n-1), j = 1, 2, 3, 4; i= 1, 2, ..k.

Different pair wise non isomorphic structures can be obtained are n+m-2 .These are obtained by choosing a common point as a point on poke P_n and m-2 points between 3-degree consecutive vertices on cycle C_{4m-4}.The above function f will serve as vertex prime labeling for all these structures. Thus the graph G" and $G=(G''_4)^{(K)}$ with all n+m-2 non isomorphic structures of one point union of k copies of G" are vertex prime. That establishes the invariance under vertex prime labeling of $G = (G''_4)^{(K)}$.

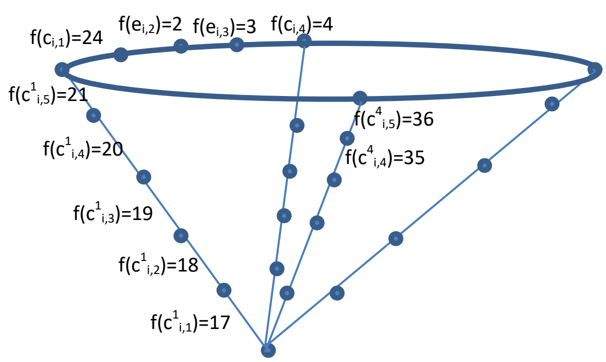

Fig 4.7 $G''_4 = W_4$ with each cycle edge replaced with p_5 and pokes replaced with P_6.

ISSN: 2454-1907
DOI: 10.5281/zenodo.1216849

4. Conclusions

We have considered W_4 and all of it's related graphs such as gear graph G_4 and each cycle edge of W_4 replaced with P_m, each pokes of W_4 replaced with P_m are shown to be vertex prime graphs. All different non isomorphic structures of these graphs obtained by taking one point union graphs are shown to be vertex prime. This property of graphs is called as invariance under vertex prime labeling. It is necessary to investigate these graphs further.

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