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CONTROL AND IDENTIFICATION OF CONTROLLED AUTO-REGRESSIVE MOVING AVERAGE (CARMA) FORM OF AN INTRODUCED SINGLE-INPUT SINGLE-OUTPUT TUMOR MODEL

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ABSTRACT

The article investigates the parameter estimation for controlled auto-regressive moving average models with gradient based iterative approach and two-stage gradient based iterative approach. Since deriving a new model for tumor model is substantial, introduced system identification algorithms are used in order to estimate parameters of a specific nonlinear tumor model. Besides, in order to estimate tumor model a collection of output and input data is taken from the nonlinear system. Apart from that, effectiveness of the identification algorithms such as convergence rate and estimation error is depicted through various tables and figures. Finally, it is shown that the two stage approach has higher identification efficacy.

Keywords: Gradient Based Iterative Algorithms, 2-STAGE Identification, System Identification, Parameter Estimation, Tumor Model

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1. INTRODUCTION

The iterative and recursive algorithms could be used to solve matrix equations Wang (2007), Ding (2005), Xie (2010), parameter estimation problems Li (2018), Li (2018), Liu (2010) and filtering issues Ma (2020). In parameter estimation approaches which are recursive, the estimation of parameters can to be calculated in an online framework Du (2017), Wei (2017). On the other hand, the primary notion of the hierarchical algorithms is to update estimation of the parameters by applying a set of data Ding (2018), Ding (2019), Sadeghi (2023). The hierarchical parameter estimation approaches make adequate use of all output and input Data Li (2020), Wang (2020), and could enhance the accuracy of estimation of parameters Li (2020), Ding (2020) and convergence rate of parameters Li (2021), Chen (2020).

Two-stage algorithms have an enormous usage in the realm of parameter identification Sadeghi (2023), Sadeghi (2023) developed a two-stage step-wise system identification approach for a class of nonlinear dynamic systems Li et al. (2006). In Raja (2015), two-stage least mean square adaptive methods relying on process of fractional signal were fostered regarding CARMA systems. A two-stage neural network algorithms related to ARMA model estimation by the use of a simple mean called extended sample autocorrelation function is presented Lee (1994). In Bin (2012), a two-stage method is introduced regarding the system identification of an ARMAX model which identifies ARX and MA part separately by bias-eliminated least squares method and another basic method respectively. Also in Ding (2020), a new two-stage algorithm for estimating parameter of system is brought up but in this article as a novelty, a CARMA system is discussed.

Having a suitable model for tumor system has become an integral issue since the death rate of cancer has become considerable. Accessing a suitable polynomial model for tumor can make the designing of a controller for system much easier. In Pillis (2020), a four population model is presented which contains tumor cells, host cells, drug interaction, immune cells and a controller based on optimization, which is used to satisfy the specific desire. In Sweilam & AL-Mekhlafi (2018), an updated nonlinear mathematical format of a general tumor beneath immune suppression is discussed. The brought up model in this paper is ruled by a fractional differential equations system. Lobato (2016) presented another model for tumor and in their works they aim to reach a protocol of optimization for injection of drug to sick individuals having cancer, by the making both of the cells having cancer and the drug concentration which has been prescribed minimum Lobato (2016). Tumor model presented in this last research is the basis of our study throughout the rest of the paper.

Controlling a CARMA or ARMAX model system has been the subject of a few papers and not much work has been done in this field. For instance, In Chen & Guo (1987), an optimal adaptive control for ARMAX systems using a quadratic loss function is introduced. In Li (2021), abrupt faults in ARMAX models have been taken into consideration and reliable control problem has been studied. Multivariable system control is discussed in Osorio-Arteaga (2020) where a robust adaptive control is applied to ARMA and ARMAX structures of an electric arc model. Furthermore, linear neural networks was set as a study tool for adpative control of CARMA systems Watanabe (1992).

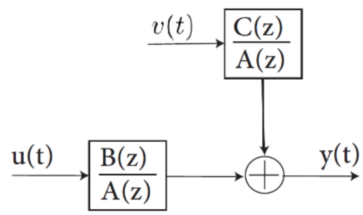
In the following section, a nuance characteristic of the system configuration regarding the CARMA configuration is brought up. Also, section section 3 includes the mathematics of two novel GI algorithm. Section 4 describes a specific tumor model. In section 5, all the necessary simulations for showing the effectiveness of new algorithms are illustrated by identifying a tumor model. Eventually, in the last section, all the outcomes were derived.

2. SYSTEM MODEL: CARMA SYSTEMS

Take the introduced below CARMA system into consideration:

$$A(q)y(t) = B(q)u(t) + C(q)v(t) \tag{1}$$

Here u(t) is the succession of input of the system, y(t) is the succession of output of the system and v(t) is a succession of white noise with zero mean and variance σ^2. Also A(q), B(q) and c(q) are multinomial in the monad backward variation agent [i.e. $q^{-1}u(t) = u(t-1)$]. For simplicity in the rest of the paper, we have the following notations: A =: X describes A is described as X; The indication I (I_n) is an identity matrix with suitable dimensions $(n \times n)$; 1_{n} indicates a vector of n-dimensional column which all components are 1. The superscript T indicates the transpose of a matrix; the matrix norm is described by $|X|^2 = tr(XX^T)$.



Now look at the CARMA system shown in Figure \ref{fig.1}. We define A(q), B(q) and C(q) as polynomials of known orders (n_a, n_b, n_c) as

follows:

$$A(q) \coloneqq 1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_{n_a} q^{n_a},$$
$$B(q) \coloneqq b_1 q^{-1} + b_2 q^{-2} + \cdots + b_{n_b} q^{n_b},$$
$$C(q) \coloneqq 1 + c_1 q^{-1} + c_2 q^{-2} + \cdots + c_{n_c} q^{n_c}.$$

In a generic way, it is presumed that y(t) = 0, u(t) = 0 and v(t).= 0 for $t \ll 0$. Take $n \coloneqq n_a + n_b + n_c$, Consider the system parameter vectors:

$$\Theta \coloneqq \begin{bmatrix} \theta \\ \vartheta \end{bmatrix} \in R^n,$$
$$\theta \coloneqq \left[a_1, a_2, \cdots, a_{n_a}, b_1, b_2, \cdots, b_{n_b}\right]^T \in R^{n_a+n_b},$$
$$\vartheta \coloneqq \left[c_1, c_2, \cdots, c_{n_c}\right]^T \in R^{n_c},$$

and the corresponding information vectors:

Type equation here.

$$\varphi(t) \coloneqq \begin{bmatrix} \phi(t) \\ \psi(t) \end{bmatrix} \in R^n,$$
$$\phi(t) \coloneqq [-y(t-1), -y(t-2), \cdots, -y(t-n_a), u(t-1), u(t-2), \cdots, u(t-n_b)]^T \in R^{n_a+n_b},$$
$$\psi(t) \coloneqq [v(t-1), v(t-2), \cdots, v(t-n_c)]^T \in R^{n_c}.$$

Based on the above definitions and equation (\ref{eq.1}), we attain the the below parameter estimation configuration:

$y(t) = [1 - A(q)]y(t) + B(q)u(t) + C(q)v(t)$=$[1 - 1 - a_1q^{-1} - a_2q^{-2} - \cdots - a_{n_a}q^{n_a}]y(t) + [b_1q^{-1} + b_2q^{-2} + \cdots + b_{n_b}q^{n_b}]u(t) + 1 + [c_1q^{-1} + c_2q^{-2} + \cdots + c_{n_c}q^{n_c}]\nu(t) = [-y(t-1) - y(t-2) - \ldots - y(t-n_a) + u(t-1) + u(t-2) + \ldots + u(t-n_b)]\theta + [\nu(t-1) + \nu(t-2) + \nu(t-n_c)]\vartheta$,

$$y(t)= \phi(t)\theta + \psi(t)\vartheta + \nu(t), \quad (2)$$

$$y(t)= \varphi(t)\Theta + \nu(t), \quad (3)$$

3. THEORY OF IDENTIFICATION AND CONTROL ALGORITHMS

3.1. GRADIENT BASED ITERATIVE ALGORITHMS(GI)

We consider k=1,2,3,... as an hierarchical variable $\widehat{\Theta}_k := \begin{bmatrix} \hat{\theta}_k \\ \hat{\vartheta}_k \end{bmatrix} \in R^n$, and $\Theta := \begin{bmatrix} \theta \\ \vartheta \end{bmatrix}$ as the hierarchical identification of and while k iteration has established. Beyond that $\lambda_{max}[X]$ is the biggest eigenvalue of the matrix of symmetric format X.

Now we take an array of data with length L which works with the model introduced in. Here, we consider the vector of stacked output data Y(L) and matrix of the stacked data $\Phi(L)$ like:

$$Y(L):=\begin{bmatrix} y(1) \\ y(2) \\ . \\ y(L) \end{bmatrix} \in R^n,$$

$$\Phi:=\begin{bmatrix} \phi(1) \\ \phi(2) \\ . \\ \phi(L) \end{bmatrix} \in R^{L\times n},$$

Now we define the static criterion function as follows:

$$J_1(\Theta) = \frac{1}{2}\sum_{t=1}^{L}[y(t) - \varphi^T\Theta]^2,$$

which can be equally described as:

$$J_1(\Theta) = \frac{1}{2}|y(L) - \Phi(L)\Theta|^2.$$

By taking advantage of negative gradient probe, calculating the partial derivative of $J_1(\Theta)$ regarding Θ, we attain this iterative relation:

$$\widehat{\Theta}_k = \widehat{\Theta}_{k-1} - \mu grad J_1(\widehat{\Theta}_{k-1})$$
$$= \widehat{\Theta}_k + \mu\Phi^T(L)[Y(L) - \Phi(L)]\widehat{\Theta}_{k-1}$$
$$= [I_n - \mu\Phi^T(L)\Phi(L)]\widehat{\Theta_{k-1}} + \mu\Phi^T(L)Y(L),$$

Here, $\mu > 0$ is a convergence factor or an iterative step-size. To make sure about convergence of $\widehat{\Theta}_k$, all the eigenvalues of $I_n - \mu\Phi^T(L)\Phi(L)$ should be in the monad circle, so $-I_n \leq I_n - \mu\Phi^T(L)\Phi(L) \leq I_n \; or \; 0 \leq I_n - \mu\Phi^T(L)\Phi(L) \leq 2I_n$ therefore as suitable conservative form of μ we have:

$$\mu \leq \frac{2}{\lambda_{\max}[\Phi^T(L)\Phi(L)]} = 2\lambda_{max}^{-1}[\Phi^T(L)\Phi(L)].$$

As to eschew calculating the intricate eigenvalues of a matrix which is square and to decrease evaluation expense, the trace of matrix is taken advantage of and capitalized on a different manner for picking up the convergence rate:

$$\mu \leq \frac{2}{|\Phi(L)|^2} = 2|\Phi(L)|^{-2}.$$

Now it is possible to attain the gradient based iterative method for CARMA system presented in equation (1) with the following set of equations:

$$\widehat{\Theta}_k = \widehat{\Theta}_{k-1} + \mu\Phi^T(L)\big[Y(L) - \Phi(L)\widehat{\Theta}_{k-1}\big], k = 1,2,3,\dots \quad (4)$$

$$\mu = 2|\Phi(L)|^{-2}, \quad (5)$$

$$Y(L) = [y(1), y(2), \dots, y(L)]^T \quad (6)$$

$$\Phi(L) = [\varphi(1), \varphi(2), \cdots, \varphi(L)]^T \quad (7)$$

$$\varphi(t) := \begin{bmatrix}\phi(t)\\ \psi(t)\end{bmatrix}, \qquad t = 1,2,\dots, L \quad (8)$$

$$\phi(t) = [-y(t-1), -y(t-2), \cdots, -y(t-n_a), u(t-1), u(t-2), \cdots, u(t-n_b)]^T, \quad (9)$$

$$\psi(t) = [v(t-1), v(t-2), \cdots, v(t-n_c)]^T. \quad (10)$$

The steps of calculating $\widehat{\Theta}_k$ from equation (4)-(10) summarized as below:

1) Regarding $t \leq 0$ set every variable to zero. Assume k = 1, take the data length L (L≫ n) and take the primary amounts, $\widehat{\Theta}_k = \frac{1_n}{p_0}, p_0 = 10^6$ and the system identification precision ε.

2) Gather all the input u(t) and output y(t) for t=1,2,…,L.
3) Attain the vectors of information ϕ by equation (9), ψ by equation (10) and ϑ by equation (8).
4) Form the vector of stacked output Y(L) regarding equation (6) and the matrix of stacked information $\Phi(L)$ regarding equation (7), also pick up a large μ based on equation (5).
5) Upgrade the parameter estimation vector $\hat{\Theta}{k}$ by equation (\ref{eq.4}).
6) Contrast $\widehat{\Theta}_k$ with $\widehat{\Theta}_{k-1}$. If $||\widehat{\Theta}_k - \widehat{\Theta}_{k-1}|| < \varepsilon$extend k in unit order and start from step 5. In all other respects, attain iteration k and the system identification vector $\widehat{\Theta}_k$.

3.2. TWO-STAGE GRADIENT BASED ITERATIVE ALGORITHMS (2S-GI)

Consider the CARMA model described in equation (\ref{eq.2}).

First, we define these two imaginary output variables:

$$y_1(t) := y(t) - \psi^T(t)\vartheta \in R$$
$$y_2(t) := y(t) - \phi^T(t)\theta \in R$$

Afterwards by these definitions we have:

$$y_1(t) = \phi^T(t)\theta + \nu \tag{11}$$

$$y_2(t) = \psi^T(t)\vartheta + \nu \tag{12}$$

Take L as data length. According to equation (11) and (12), we define these two static criterion functions:

$$J_1(\theta) = \tfrac{1}{2}\textstyle\sum_{t=1}^{L}[y_1(t) - \phi^T(t)\theta]^2, \tag{13}$$

$$J_2(\vartheta) = \frac{1}{2}\sum_{t=1}^{L}[y_2(t) - \psi^T(t)\vartheta]^2. \tag{14}$$

Consider the vector of stacked output Y(L), vectors of the stacked imaginary outputs $Y_1(L)$ and $Y_2(L)$, and the matrices of stacked information $\phi(L)$ and $\psi(L)$ are as follows:

$$Y(L) := [y(1), y(2), \cdots, y(L)]^T \in R^L$$
$$Y_1(L) := [y_1(1), y_1(2), \cdots, y_1(L)]^T = Y(L) - \psi(L)\vartheta \in R^L$$
$$Y_2(L) := [y_2(1), y_2(2), \cdots, y_2(L)]^T = Y(L) - \phi(L)\theta \in R^L$$
$$\Phi(L) := [\phi(1), \phi(2), \cdots, \phi(L)]^T \in R^{L\times(n_a+n_b)}$$
$$\Psi(L) := [\psi(1), \psi(2), \cdots, \psi(L)]^T \in R^{L\times n_c}$$

Equations (13) and (14) can be equivalently written as:

$$J_1(\theta) = \frac{1}{2}|Y_1(L) - \Phi(L)\theta|^2$$

$$J_2(\vartheta) = \frac{1}{2}|Y_2(L) - \Psi(L)\vartheta|^2$$

By taking advantage of the search of negative gradient to make the criterion functions above minimum, we have:

$$\begin{aligned}\hat{\theta}_k &= \hat{\theta}_{k-1} - \mu_1 grad J_1(\hat{\theta}_{k-1}) \\ &= \hat{\theta}_{k-1} + \mu_1\Phi^T(L)[Y_1(L) - \Phi(L)\hat{\theta}_{k-1}] \\ &= \hat{\theta}_{k-1} + \mu_1\Phi^T(L)[Y(L) - \Psi(L)\vartheta - \Phi(L)\hat{\theta}_{k-1}] \\ &= [I_{n_a+n_b} - \mu_1\Phi^T(L)\Phi(L)]\widehat{\theta_{k-1}} + \mu_1\Phi^T(L)[Y(L) - \Psi(L)\vartheta]\end{aligned}$$

$$\begin{aligned}\hat{\vartheta}_k &= \hat{\vartheta}_{k-1} - \mu_2 grad \\ &= +\mu_2\Psi^T(L)[Y_2(L) - \Psi(L)\hat{\vartheta}_{k-1}] \\ &= \hat{\vartheta}_{k-1} + \mu_2 \\ &= \Psi^T(L)[Y(L) - \Phi(L)\theta - \Psi(L)\hat{\vartheta}_{k-1}] \\ &= [I_{n_c} - \mu_2\Psi^T(L)\Psi(L)]\hat{\vartheta}_{k-1} + \mu_2\Psi^T(L)[Y(L) - \Phi(L)\theta]\end{aligned}$$

To make sure about convergence of $\hat{\theta}_k$ and $\hat{\vartheta}_k$ all the eigenvalues of $[I_{n_a+n_b} - \mu_1\phi^T(L)\phi(L)]$ and $[I_{n_c} - \mu_2\psi^T(L)\psi(L)]$, should be in the unit circle, so we have:

$$-I_n \leq I_{n_a+n_b} - \mu_1\Phi^T(L)\Phi(L) \leq I_n$$

$$-I_n \leq I_{n_c} - \mu_2\Psi^T(L)\Psi(L) \leq I_n$$

Therefore, similar to GI algorithm as a conservative choice, we have the following relation for μ_1 and μ_2:

$$\mu_1 = 2||\Phi(L)||^{-2}$$

$$\mu_2 = 2||\Psi(L)||^{-2}$$

In brief, we have the following set of equations for 2S-GI algorithm:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \mu_1\Phi^T(L)[Y(L) - \Phi(L)\hat{\theta}_{k-1} - \Psi(L)\vartheta_{k-1}], \quad (15)$$

$$\mu_1 = 2||\Phi(L)||^{-2} \quad (16)$$

$$\hat{\vartheta}_k = \hat{\vartheta}_{k-1} + \mu_2 \Psi^T(L)\left[Y(L) - \Phi(L)\hat{\theta}_{k-1} - \Psi(L)\vartheta_{k-1}\right], \quad (17)$$

$$\mu_2 = 2||\Psi(L)||^{-2} \quad (18)$$

$$Y(L) := [y(1), y(2), \cdots, y(L)]^T \in R^L \quad (19)$$

$$\Phi(L) := [\phi^T(1), \phi^T(2), \cdots, \phi^T(L)]^T, \quad (20)$$

$$\Psi(L) := [\psi^T(1), \psi^T(2), \cdots, \psi^T(L)]^T, \quad (21)$$

$$\phi(t) = [-y(t-1), -y(t-2), \cdots, -y(t-n_a), u(t-1), u(t-2), \cdots, u(t-n_b)]^T, \quad (22)$$

$$\psi(t) = [\nu(t-1), \nu(t-2), \cdots, \nu(t-n_c)]^T, \quad (23)$$

$$\hat{\theta} = \left[\hat{a}_{k-1}, \hat{a}_{k-2}, \ldots, \hat{a}_{k-n_a}, \hat{b}_{k-1},, \hat{b}_{k-2}, \ldots, \hat{b}_{k-n_b}\right]^T, \quad (24)$$

$$\hat{\vartheta} = \left[\hat{c}_{k-1}, \hat{c}_{k-2}, \ldots, \hat{c}_{k-n_c}\right]^T, \quad (25)$$

The steps of attaining $\hat{\theta}_k$ and $\hat{\vartheta}_k$ included in the 2S-GI approach from equation (15)–(25) are brought up as follows:

1) Regarding $t \leq 0$, put every parameter to 0. Imagine k=1 take the length of data as L ($L \gg n\_\{a\} + n\_\{b\}$) and set the initial values as: $\hat{\theta}_0 = \frac{1_{na+n_b}}{p_0}, \hat{\vartheta}_0 = \frac{1_{n_c}}{p_0}$ and the parameter estimation accuracy ε.
2) Gather all the input u(t) and output y(t) for t=1,2,...,L. Attain the information vectors ϕ(22) by equation (22) and ψ(t) by equation (23).
3) Build the vector of stacked output Y(L) by (19) and the matrices of stacked information $\Phi(L)$ and $\Psi(L)$ by (20) and (21), calculate the convergence factor μ_1 and μ_2 regarding (16) and (18).
4) Update the vectors of parameter approximation $\hat{\theta}_k$ *and* $\hat{\vartheta}_k$ by (15) and (17).
5) Compare $\hat{\theta}_k$ with $\hat{\theta}_{k-1}$and $\hat{\vartheta}_k$ with $\hat{\vartheta}_{k-1}$:If $||\hat{\theta}_k - \hat{\theta}_{k-1}||^2$+$||\hat{\vartheta}_k - \hat{\vartheta}_{k-1}||^2 >$ ε, extend k by 1 and start from step 4. In all other respects attain iteration k and the vectors of estimation of parameters $\hat{\theta}_k$ and $\hat{\vartheta}_k$.

4. CONTROL THEORY

In this part of the paper, theory of a ziegler nichols PID controller for third order processes introduced in (Bobal, 2006) is brought up. The control law which we took advantage of is:

$$u_k = K_P\left[e_k - e_{k-1} + \frac{T_0(e_k - e_{k-1})}{2T_I} + \frac{T_D}{T_0}(e_k - 2e_{k-1} + e_{k-2}) + u_{k-1}\right]. \quad (26)$$

$$e_k = w_k - y_k \quad (27)$$

Here e_k is the controller error. The feedback form of control law is:

$$u\_\{k\} = q\_\{0\}e\_\{k\} + q\_\{1\}e\_\{k-1\} + q\_\{2\}e\_\{k-2\} + u\_\{k-1\} \quad (28)$$

Where q_0, q_1 *and* q_2 respectively are:

$$q_0 = K_P\left(1 + \frac{T_0}{2T_I} + \frac{T_D}{T_0}\right)$$
$$q_1 = -K_P\left(1 - \frac{T_0}{2T_i} + \frac{2T_D}{T_0}\right)$$
$$q_2 = K_P\frac{T_D}{T_0}$$

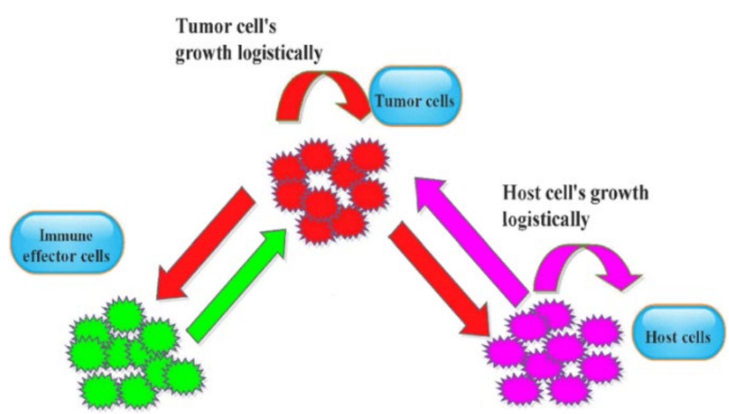
And we have:

$K_P = 0.6K_{Pu}$, $T_I = 0.5T_u$, $T_D = 0.125T_u$.

And T_u *and* K_{pu} are ultimate period and ultimate gain respectively.

5. TUMOR MODEL

I indicate the immune cells number at time t, T denotes the tumor cells number at time t, N describes the normal (host) cells number at time t, and u is the plan of control.\begin{figure}[h] \centering **\includegraphics**[width=.5\linewidth]{T-I-N.eps} \caption{Random tumor and immune cells interactions.}



$$\dot{N}(t) = r_2N(t)\big(1 - b_2N(t)\big) - c_4T(t)N(t) - a_3u(t) \qquad N(0) = N_o,$$

$$\dot{T}(t) = r_1T(t)\big(1 - b_1T(t)\big) - c_2I(t)T(t) - c_3T(t)N(t) - a_2u(t) \qquad T(0) = T_o,$$

$$\dot{I}(t) = s + \frac{\rho I(t)T(t)}{\alpha + T(t)} - c_1I(t)T(t) - d_1I(t) - a_1u(t)I(0)$$

$$I(0) = I_o.$$

Values of known parameters in above equations are listed below Lobato (2016)

Parameter	Values	Parameter	Values
a_1	0.2	a_2	0.3
a_3	0.1	b_1	1
b_2	1	α	0.5
c_1	1	c_2	0.5
c_3	1	c_4	1
d_1	0.2	ρ	0.01
r_1	1.5	r_2	1
S	0.33		

Therefore, we yield: \begin{equation*} \begin{split}

$\dot{N}(t) = N(t) - N^2(t) - T(t)N(t) - 0.1u(t),$

$d(t) = 1.5T(t) - 1.5T^2(t) - 0.5I(t)T(t) - T(t)N(t) - 0.3u(t),$

$\dot{I}(t) = 0.33 + \frac{0.01I(t)T(t)}{0.3+T(t)} - I(t)T(t) - 0.2I(t) - 0.2u(t).$

6. SIMULATIONS

6.1. ESTIMATION OF T(T)

In this paper, we aim to identify T(t) as the quantity of tumor cells at time t and I(t) as the quantity of immune cells at time t, by presenting novel parameter estimation method. In simulations assume $n_a = 2$, $n_b = 2$and $n_c = 2$. In simulations,$N_o = 2$, T_0=1 and I_0=1. **\subsection{Estimation of T(t)}**

The CARMA model of T(t) as the output and u(t) as the input is:

$$A(q) = 1 - 0.0862q^{-1} - 0.8937q^{-2}$$
$$B(q) = -1.7580q^{-1} + 1.7580q^{-2}$$
$$C(q) = 1 + 0.6264q^{-1} - 0.3459q^{-2}$$

Table 1

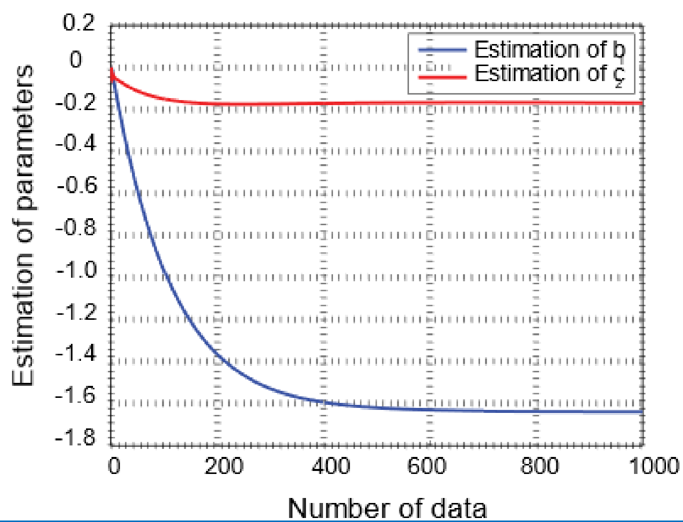
Table 1 Estimation Result for $\sigma^2 = (1.00)^2$

Algorithms	t=L	a_1	a_1	b_1
	1000	-0.1005	-0.8864	-1.7053
GI	2000	-0.0970	-0.8816	-1.7751
	3000	-0.0974	-0.8822	-1.7473
	1000	-0.0976	-0.8835	-1.7570
2S-GI	2000	-0.0946	-0.8836	-1.7145
	3000	-0.0921	-0.8797	-1.7161
True value		-0.0862	-0.8937	-1.7580

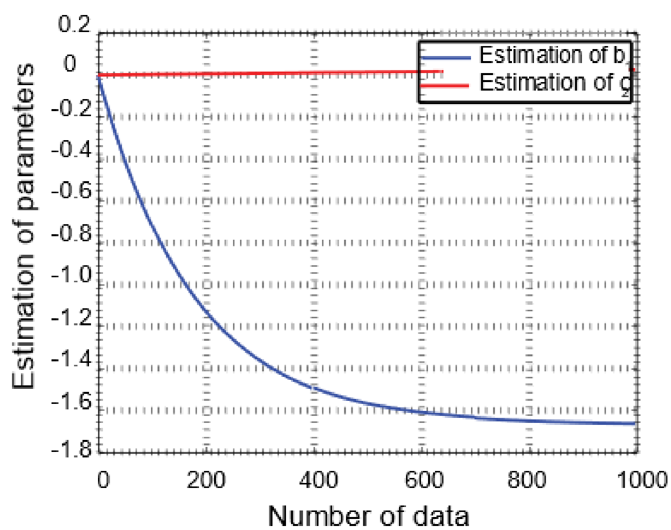
Algorithms	t=L	b_2	c_1	c_2
	1000	1.7859	-0.0460	-0.0020
GI	2000	1.7389	0.0229	0.0426
	3000	1.7378	0.0231	0.0009

	1000	1.7690	0.0019	-0.0030
2S-GI	2000	1.7380	0.0281	-0.0189
	3000	1.7726	0.0455	-0.0485
		1.7570	0.6264	-0.3459

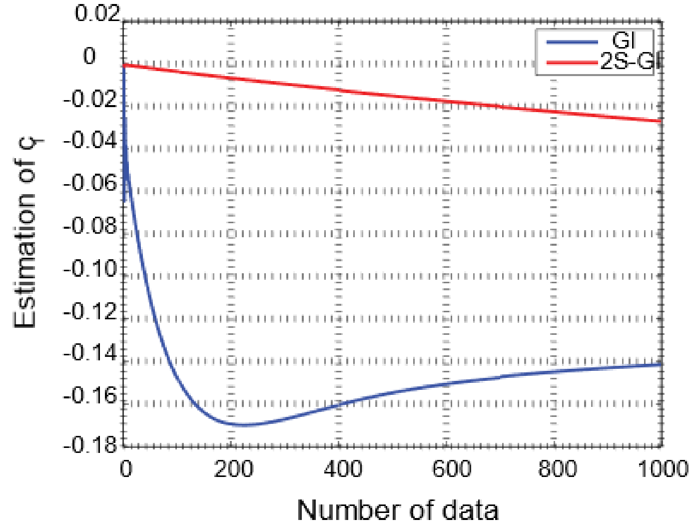
Algorithms	**t=L**	$\delta(\%)$
	1000	7.6604
GI	2000	6.8844
	3000	6.4698
	1000	6.8316
2S-GI	2000	6.2351
	3000	5.7110



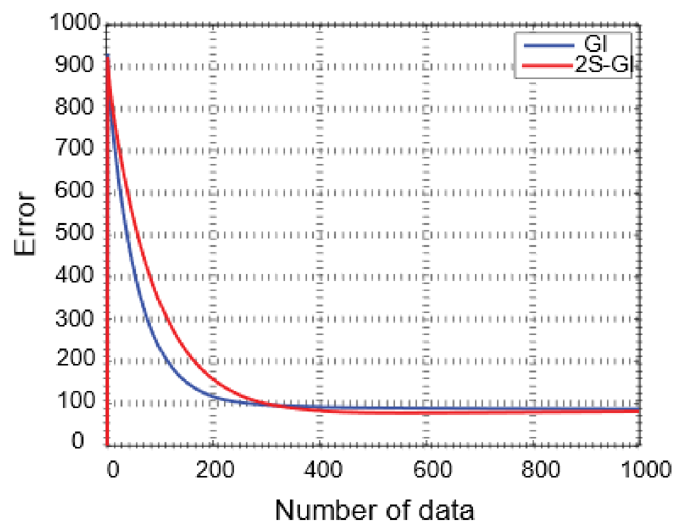
Estimation of $b_1$and c_2 for CARMA System with Variance $\sigma^2 = 2.00^2$ and Number of Data L=1000 with GI Algorithm



Estimation of b_1 *and* $c_2$for CARMA System with Variance $\sigma^2 = 2.00^2$ and Number of Data L=1000 with 2S-GI Algorithm



Estimation of c_1 for CARMA System with Variance $\sigma^2 = 2.00^2$ and Number of Data L=1000



Estimation Error for CARMA System with Variance $\sigma^2 = 2.00^2$and Number of Data L=1000

Table 2

Table 2 Estimation Results for $\sigma^2 = (2.00)^2$

Algorithms	t=L	a_1	a_1	b_1
	1000	-0.1182	0.8405	-1.6400
GI	2000	-0.1412	-0.8478	-1.7594
	3000	-0.136	-0.8474	-1.7737
	1000	-0.1379	-0.8400	-1.6632
2S-GI	2000	-0.1332	-0.8451	-1.6933
	3000	-0.1270	-0.8465	-1.7573
True value		-0.0862	-0.8937	-1.7580

Algorithms	t=L	b_2	c_1	c_2
	1000	1.8659	-0.1413	-0.1693
GI	2000	1.9069	0.0147	0.0319
	3000	1.8679	0.0142	0.0082
	1000	1.9208	-0.0274	0.0265
2S-GI	2000	1.8392	0.0123	-0.0094
	3000	1.8060	0.0385	-0.0566
		1.7570	0.6264	-0.3459

Algorithms	t=L	$\delta(\%)$
	1000	8.6731
GI	2000	6.7497
	3000	6.9010
	1000	8.1050
2S-GI	2000	6.2351
	3000	5.8114

6.2. ESTIMATION OF I(T)

The CARMA model of $I(t)$ as the output and u(t) as the input is:

$$A(q) = 1 - 1.0420q^{-1} + 0.0541q^{-2}$$

$$B(q) = -0.8791q^{-1} + 0.8782q^{-2}$$

$$C(q) = 1 - 0.1586q^{-1} + 0.0987q^{-2}$$

Table 3

Table 3 Estimation Results for $\sigma^2 = (1.00)^2$				
Algorithms	t=L	a_1	a_1	b_1
	1000	-1.042	0.0541	0.8791
GI	2000	-0.9243	-0.0612	-0.8699
	3000	-0.9436	-0.0405	-0.9078
	1000	-0.9297	-0.05	-0.8541
2S-GI	2000	-0.941	-0.0471	-0.8699
	3000	-0.127	-0.8465	-1.7573
True value		0.9499	-0.0345	-0.8921

Algorithms	t=L	b_2	c_1	c_2
	1000	0.8782	-0.1586	0.0987
GI	2000	0.7725	0.0158	-0.0080
	3000	0.7917	0.0165	0.0248
	1000	0.7871	-0.0219	-0.0229
2S-GI	2000	0.7537	0.0324	0.0342
	3000	0.7697	-0.0238	0.0200
		1.7570	0.6264	-0.3459

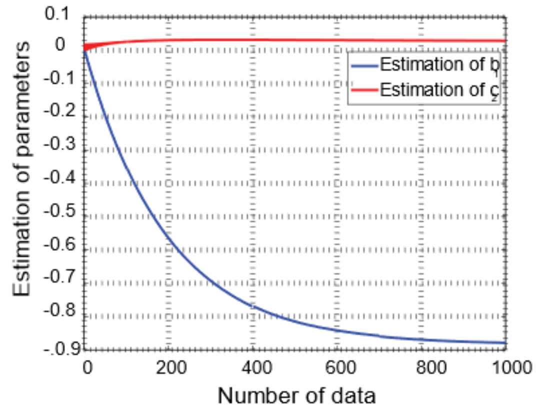
Algorithms	t=L	$\delta(\%)$
	1000	3.7344
GI	2000	3.0066
	3000	2.3635
	1000	2.9884
2S-GI	2000	2.1040
	3000	1.9732

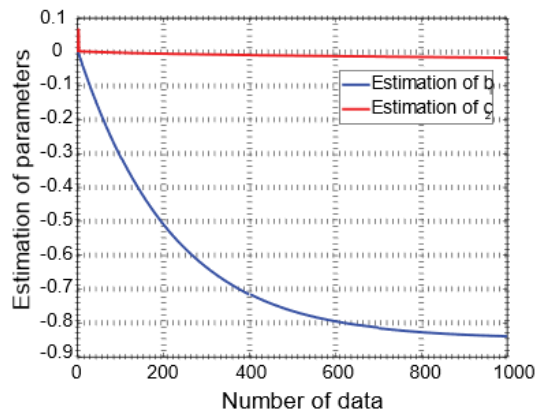
Table 4

Table 4 Estimation Results for $\sigma^2 = (2.00)^2$				
Algorithms	**t=L**	a_1	a_1	b_1
	1000	-0.9189	-0.0712	-0.8799
GI	2000	-0.8958	-0.0928	-0.9277
	3000	-0.9008	-0.0857	-0.8848
	1000	-0.9282	-0.0555	-0.8405
2S-GI	2000	-0.9542	-0.0366	-0.9301
	3000	-0.1270	-0.8465	-1.7573
True value		-0.9175	-0.0647	-0.8817

Algorithms	t=L	b_2	c_1	c_2
	1000	0.6899	-0.0324	0.0286
GI	2000	0.8068	0.0209	0.0159
	3000	0.8065	-0.0254	-0.0163
	1000	0.6995	0.0206	-0.0171
2S-GI	2000	0.8130	0.0207	-0.0189
	3000	0.7981	0.0217	0.0273
		1.7570	0.6264	-0.3459

Algorithms	t=L	$\delta(\%)$
	1000	4.039
GI	2000	3.3538
	3000	2.8344
	1000	3.8944
2S-GI	2000	2.5776
	3000	2.2446


Estimation of parameters
Number of data
Estimation of b
Estimation of ç
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-.0.9
0
200
400
600
800
1000


Estimation of parameters
Number of data
Estimation of b
Estimation of ç
0.1
0
-0.1
-0.2
-0.3
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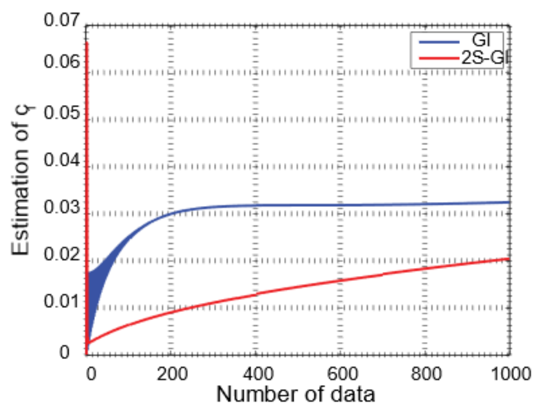
-0.5
-0.6
-0.7
-0.8
-0.9
0
200
400
600
800
1000


Estimation of ç
Number of data
GI
2S-GI
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0.06
0.05
0.04
0.03
0.02
0.01
0
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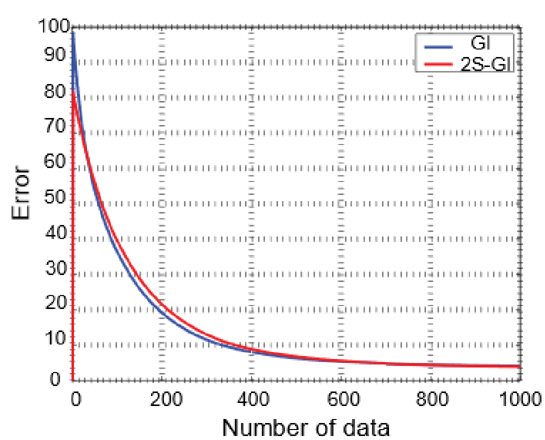
200
400
600
800
1000


Error
Number of data
GI
2S-GI
100
90
80
70
60
50
40
30
20
10
0
0
200
400
600
800
1000

7. CONTROL OF TUMOR MODELS

The final goal of this research is to make the amount of tumor cells minimum, therefore we take T(t)=0 as the desired output of the system. Based on control theory introduced in the third section and the identified polynomial model of T(t), the ultimate period and ultimate gain is $K_{pu} = 1.0722$ and $T_u = 0.4428$ Therefore $q_0 = 4.2383$ and $q_1 = -7.7866$ and $q_2 = 3.5774$.

The output and input of the feedback form is depicted in the next two figures.

From tables and figures above, the below results are derived:

- The system identification errors of the GI and 2S-GI approaches decrease as the data length increases.
- 2S-GI method, compared to GI method, produces less error and therefore is more effective at estimating parameters.
- As the noise to ratio signal rises, both introduced algorithms produce a larger amount of error.
- From figures, it is perceived that both introduced algorithms converge at a final point and have a competent convergence rate.
- The introduced controller proved that, it is able to make the amount of tumor cells in a specific period of time minimum.

8. CONCLUSION

In this contribution, mathematical theories and algorithms of two identification methods of GI and 2S-GI for CARMA systems were developed. GI is an old method but 2S-GI is a novel method which introduced in this paper. Furthermore, a tumor model with one input and three outputs were presented by works of other scholars. By means of introduced parameter estimation approaches, the model were identified. Above that, by taking advantage of a ziegler nichols PID controller the amount of tumor cells were controlled and it was illustrated that the controller could minimize amount of tumor cells in a specific span of time. Also, the GI and 2S-GI algorithm showed that they both are able to estimate parameter of a polynomial CARMA configuration in fast convergence rate and by producing an insignificant amount of error.

CONFLICT OF INTERESTS

None.

ACKNOWLEDGMENTS

None.

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