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MASSIVE PARTICLE TUNNELING RATE OF KERR-NEWMAN-ANTI-DE SITTER BLACK HOLE BY HAMILTON-JACOBI METHOD

M. Ilias Hossain [1], M. Jakir Hossain [1]

[1] Professor, Department of Mathematics, Rajshahi University, Rajshahi - 6205, Bangladesh

ABSTRACT

Using Parikh and Wilczek's opinion tunneling rate of Hawking radiations of Kerr-Newman-anti-de Sitter (KNAdS) black hole has been investigated by Hamilton-Jacobi method. Involving the self-gravitation effect of the emitted particles, energy and angular momentum has been taken as conserved and considered the space time background as dynamical. The explored results shown that the massive particle tunneling rate is related to the change of Bekenstein-Hawking entropy and the derived emission spectrum deviates from the pure thermal spectrum.

Keywords: Massive Particle Tunneling, KNAdS Black Hole, Non-Thermal and Purely Thermal Radiations.

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Corresponding Author

M. Ilias Hossain, ilias_math@yahoo.com

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1. INTRODUCTION

Recently, a semiclassical tunnelling process applied to find the Hawking radiation of the static Schwarzschild and Reissner-Nordstr¨om black holes by Parikh and Wilczek Parikh and Wilczek (2000), Parikh (2002), Parikh (2004) and their result shows that the radiation spectrum is not pure thermal but satisfies the unitary principle and support the result of information conservation. In their process, the tunneling potential barrier is produced by the self-gravitation interaction and the position of the horizons before and after the particle's emission. Following this method, several researchers studied the Hawking radiation of various spacetime Hemming and Keski-Vakkuri (2001), Arzano et al. (2005), Medved (2002), Medved (2002), Medved and Vagenas (2005), Medved and Vagenas (2005), Vagenas (2002), Vagenas (2003), Shankaranarayanan et al. (2002), Angheben et al. (2005) by using Painleav´e or dragging or tortoise or Eddington-Finkelstein coordinate transformations and these radiations are limited to uncharged massless particle only.

In this article, we use the Parikh and Wilczek's opinion Parikh and Wilczek (2000), Parikh (2002), Parikh (2004) and employing standard Hamilton-Jacobi method to investigate the Hawking non-thermal and purely thermal tunneling rates of the Kerr-Newman-anti-de Sitter (KNAdS) black hole for massive particle. In order to carry-over this article, KNAdS black hole spacetime is described as follows. The Kerr-Newman anti-de Sitter (KNAdS) black hole which is the KAdS black hole generalized with a charge parameter, described by the metric

$$ds^2 = -\frac{\Delta_r - \Delta_\theta a^2 \sin^2\theta}{\rho^2} dt^2 + \frac{\Delta_\theta (r^2+a^2)^2 - \Delta_r a^2 \sin^2\theta}{\rho^2 \in^2} \sin^2\theta\, d\phi^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 - \frac{2a[(r^2+a^2)\Delta_\theta - \Delta_r]\sin^2\theta}{\rho^2 \in} dt d\phi\,,$$

Equation 1

Where

$$\rho^2 = r^2 + a^2 cos^2\theta\,,\ \Delta_r = \left(1+\frac{a^2}{l^2}\right) - 2Mr + a^2 + \frac{r^4}{l^2} + q^2,\ \Delta_\theta = 1 - \frac{a^2 cos^2\theta}{l^2},\ \in = 1 - \frac{a^2}{l^2}$$

Equation 2

Here the parameters $M, a, l\left(\Lambda = \frac{3}{l^2}\right)$ and q are the associated with the mass, angular momentum, cosmological radius, and charge parameters of the spacetime respectively in the background of the rotating anti de Sitter space. The spacetime causal structure depend strongly on the singularities of the metric given by the zeros of Δ_r as follows

$$\Delta_r = \left(1+\frac{a^2}{l^2}\right) - 2Mr + a^2 + \frac{r^4}{l^2} + q^2 = 0$$

Equation 3

Depending on the black hole parameters, the function $\Delta_r = 0$ with $l^2 > a^2$ has four distinct roots. For the KNAdS black hole case we are interested to find the real root of $\Delta_r = 0$, namely the real root r_+ corresponds to the radius of the black hole's outer event horizon, while the other real root r_- represents the radius of the inner cauchy horizon and r_c as the cosmological horizon. Equation (3) can be written as

$$r^4 + (l^2 + a^2)r^2 - 2Ml^2 r \quad + l^2(a^2 + q^2) = 0$$

Equation 4

Solving the above equation, the position of the black hole horizons is given by

$$r_\pm = \frac{l\beta}{\sqrt{3}} . \sinh\left[\frac{1}{3}\sinh^{-1}\frac{3M\sqrt{3}}{l\alpha\beta}\right] \times \left(1 \pm \sqrt{1 - \frac{(a^2+q^2)l}{\sqrt{3}M\beta} . \frac{2}{1+\delta} . cosech\left[\frac{1}{3}\sinh^{-1}\frac{3M\sqrt{3}}{l\alpha\beta}\right]}\right)$$

Equation 5

and

$$\mathrm{r_c} = \frac{l\beta}{\sqrt{3}}.\sinh\left[\frac{1}{3}\sinh^{-1}\frac{3M\sqrt{3}}{l\alpha\beta}\right] \times \left(\sqrt{1+\frac{1+\delta}{2}.\frac{3M\sqrt{l}}{\sqrt{3}\beta^2}.cosech^3\left[\frac{1}{3}sinh^{-1}\frac{3M\sqrt{3}}{l\alpha\beta}\right]}\ 1\right), \qquad \textbf{Equation 6}$$

where $\delta = \sqrt{1-\frac{4(a^2+q^2)\beta^2}{3M^2}.sin^2[\frac{1}{3}sinh^{-1}\frac{3M\sqrt{3}}{l\alpha\beta}]}$,

$$\alpha = \sqrt{\left(1-\frac{a^2}{l^2}\right)^2-\frac{4q^2}{l^2}}, \qquad \beta = \sqrt{1+\frac{a^2}{l^2}} \ . \qquad \textbf{Equation 7}$$

and $r_{--} = -(r_+ + r_- + r_c)$ is the another cosmological horizon. With $\delta \approx 1$ the black hole horizon can be approximated as

$$\mathrm{r_\pm} \approx \frac{l\beta}{\sqrt{3}}.sinh\left[\frac{1}{3}sinh^{-1}\frac{3M\sqrt{3}}{l\alpha\beta}\right].\left(1 \pm \sqrt{1-\frac{(a^2+q^2)\alpha}{M^2}}\right). \qquad \textbf{Equation 8}$$

Taking only the positive sign which is the event horizon of KNAds black hole as follows

$$\mathrm{r_+} = \frac{l\beta}{\sqrt{3}}.sinh\left[\frac{1}{3}sinh^{-1}\frac{3M\sqrt{3}}{l\alpha\beta}\right].\left(1 + \sqrt{1-\frac{(a^2+q^2)\alpha}{M^2}}\right). \qquad \textbf{Equation 9}$$

Expanding r_+ in terms of black hole parameters with negative cosmological constant under the condition $(a^2+q^2)\alpha < M^2$, we obtain

$$r_+ = \frac{M}{\alpha}\left(1-\frac{4M^2}{l^2\beta^2\alpha}+\cdots\right).\left(1+\sqrt{1-\frac{(a^2+q^2)\alpha}{M^2}}\right), \qquad \textbf{Equation 10}$$

which can be written as

$$r_+ = \frac{1}{\alpha}\left(1-\frac{4M^2}{l^2\beta^2\alpha}+\cdots\right).\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right). \qquad \textbf{Equation 11}$$

Now if we set $\mu = \frac{1}{\alpha}\left(1-\frac{4M^2}{l^2\beta^2\alpha}+\cdots\right)$, then $r_+ = \left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)\mu$ with $\mu <$ 1 and hence the event horizon of KNAds black hole is less than Kerr-Newmann Chen and Yang (2007) event horizon $r_{KN} = M + \sqrt{M^2-(a^2+q^2)}$. As the event horizon of KNAds black hole coincides with the outer infinite red-shift surface, we apply the geometrical optical limit and the "s-wave" approximation. Using the semiclassical WKB method Massar and Parentani (2000), the tunneling probability is found to be related to the imaginary part of the action of the following form

$$\Gamma \sim (\exp(-2\mathrm{Im}I), \qquad \textbf{Equation 12}$$

where I is the action of the radiating particle and Γ is the emission rate.

The later section describes near the event horizon the new line element of KAdS black hole. In section 3 and 4, we derived the Hawking non thermal and thermal radiation respectively. In section 5, we present our results and discussion. Finally, in section 6, we present our concluding remarks.

2. THE HJ METHOD FOR KNADS SPACETIME

The Hamilton-Jacobi method was applied extensively to the non-thermal radiation in 1990s and attracted people's attention Srinivasan and Padmanabhan (1999), Angheben et al. (2005), Kerner and Mann (2007). In 2005, applying semiclassical tunneling method, Angheben, Nadalini, Vanzo and Zerbini Angheben et al. (2005) developed Hamilton-Jacobi method Shankaranarayanan et al. (2001), Shankaranarayanan et al. (2002), Shankaranarayanan (2003), Srinivasan and Padmanabhan (1999), Padmanabhan (2004) ignoring the self-gravitational effect of the emitted scalar particles. Here we now consider the method of Chen et al. Chen and Yang (2007), Chen et al. (2008) to calculate the imaginary part of the action from the relativistic Hamilton-Jacobi equation. The action of the radiating particle I satisfies the relativistic Hamilton-Jacobi equation

$$g^{ij}(\partial_i I)(\partial_j I) + m^2 = 0, \quad \textbf{Equation 13}$$

where m and g^{ij} are the mass of the particle and the inverse metric tensors respectively.

In this method, we avoid the exploration of the equation of motion in the Painlev´e coordinates systems for calculate the imaginary part of the action I. For the convenience of our research to study the Hawking radiation, adopting the transformation $\frac{d\phi}{dt} = -\frac{g_{14}}{g_{44}}$ on the line element (1), we obtain the new line element of the Kerr-Newman-anti-de Siter black hole as

$$ds^2 = -\frac{\Delta_r \Delta_\theta \rho^2}{\Delta_\theta (r^2+a^2)^2 - \Delta_r a^2 \sin^2\theta} dt^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 \quad \textbf{Equation 14}$$

The position of black hole horizon of the metric given by Eq. (14) is same as given in Eq. (11). Therefore, the line element near the event horizon rewritten as

$$ds^2 = -\frac{\Delta_{r,r}(r_+)(r-r_+)\rho^2(r_+)}{\left(r_+^2+a^2\right)^2} dt^2 + \frac{\rho^2(r_+)}{\Delta_{r,r}(r_+)(r-r_+)} dr^2 + \frac{\rho^2(r_+)}{\Delta_\theta} d\theta^2, \quad \textbf{Equation 15}$$

where $\rho^2(r_+)$ and $\Delta_{r,r}$ are defined as follows

$$\rho^2(r_+) = r_+^2 + a^2\cos^2\theta \text{ and } \Delta_{r,r} = \frac{d\Delta_r}{dr} = \frac{2}{\epsilon^2}\left(\beta^2 r_+ - M + 2\frac{r_+^3}{l^2}\right). \quad \textbf{Equation 16}$$

Calculating the non-null inverse metric tensors from the metric (15) and employing these in Eq. (13) as follows

$$-\frac{(r_+^2+a^2)^2}{\rho^2(r_+)\Delta_{r,r}(r_+)(r-r_+)}(\partial_t I)^2 + \frac{\Delta_{r,r}(r_+)(r-r_+)}{\rho^2(r_+)}(\partial_r I)^2 + \frac{\Delta_\theta}{\rho^2(r_+)}(\partial_\theta I)^2 + m^2 = 0.$$

Equation 17

To solve action $I(t, r, \theta, \phi)$, we consider the properties of the black hole spacetime and carry out the separation of variables as

$$I = -\omega t + R(r) + H(\theta) + j\phi,$$

Equation 18

where ω is the energy of the emitted particle, $R(r)$ and $H(\theta)$ are the generalized momentums, and j s the angular momentum of the particle with respect to ϕ-axis. Inserting Eq. (18) into Eq. (17) to seek a solution of the following form

$$R(r) = -\frac{r_+^2+a^2}{\Delta_{r,r}(r_+)}\int \frac{dr}{(r-r_+)} \times \sqrt{(\omega - j\Omega_+)^2 \frac{\rho^2(r_+)\Delta_{r,r}(r_+)(r-r_+)}{\left(r_+^2+a^2\right)^2}\left[\frac{\Delta_\theta}{\rho^2(r_+)}(\partial_\theta H)^2 + m^2\right]},$$

Equation 19

where the angular velocity of the particle at the event horizon is

$$\Omega_+ = \frac{d\phi}{dt} = \frac{a \in}{r_+^2 + a^2}$$

Equation 20

We treat the emitted particle as an ellipsoid shell of energy ω to tunnel across the event horizon. Finishing the above integral by using the Cauchy's integral formula, we obtain

$$R(r) = \pm \frac{2\pi i(r_+^2 + a^2)}{\Delta_{r,r}(r_+)}(\omega - j\Omega_+),$$

Equation 21

where $\pm$ sign comes from the square root. Therefore, the imaginary part of the action I corresponding to the outgoing particle is obtained by π times the residue of the integrand

$$\mathrm{Im}I = \frac{2\pi(r_+^2 + a^2)}{\Delta_{r,r}(r_+)}(\omega - j\Omega_+) = \frac{\in^2 \pi(r_+^2 + a^2)}{\beta^2 r_h - M + 2\frac{r_+^3}{l^2}}(\omega - j\Omega_+).$$

Equation 22

Using Eqs. (11) and (20) into Eq. (22), we get the imaginary part of the true action of the radiation particle as

$$\mathrm{Im}I = \frac{\frac{\pi \in^2}{\alpha^2}\left(1 - \frac{4M^2}{l^2\alpha\beta^2} + \cdots\right)^2\left(M + \sqrt{M^2 - (a^2+q^2)\alpha}\right)^2}{\frac{\beta^2}{\alpha}\left(1 - \frac{4M^2}{l^2\alpha\beta^2} + \cdots\right)\left(M + \sqrt{M^2 - (a^2+q^2)\alpha}\right) - M + A}\omega + \frac{\in^2 \pi a^2}{\frac{\beta^2}{\alpha}\left(1 - \frac{4M^2}{l^2\alpha\beta^2} + \cdots\right)\left(M + \sqrt{M^2 - (a^2+q^2)\alpha}\right) - M + A}\omega - \frac{\in^3 \pi a}{\frac{\beta^2}{\alpha}\left(1 - \frac{4M^2}{l^2\alpha\beta^2} + \cdots\right)\left(M + \sqrt{M^2 - (a^2+q^2)\alpha}\right) - M + A}j,$$

where $A = \frac{2}{l^2\alpha^3}\left(1 - \frac{4M^2}{l^2\alpha\beta^2} + \cdots\right)^3\left(M + \sqrt{M^2 - (a^2+q^2)\alpha}\right)^3$.

$$\mathrm{Im}I = \frac{\pi \in^2 \left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)^2}{\beta^2\alpha\left[\left(1+\frac{4M^2}{l^2\alpha\beta^2}+\cdots\right)\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)+B\right]}\omega + \frac{\in^2 \pi a^2}{\frac{\beta^2}{\alpha}\left[\left(1-\frac{4M^2}{l^2\alpha\beta^2}+\cdots\right)\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)-\frac{M\alpha}{\beta^2}+\frac{\alpha A}{\beta^2}\right]}\omega - \frac{\in^3 \pi a}{\frac{\beta^2}{\alpha}\left[\left(1-\frac{4M^2}{l^2\alpha\beta^2}+\cdots\right)\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)-\frac{M\alpha}{\beta^2}+\frac{\alpha A}{\beta^2}\right]}j,$$

where $B = -\frac{M\alpha}{\beta^2}\left(1+\frac{8M^2}{l^2\alpha\beta^2}+\cdots\right)+\frac{2}{l^2\beta^2\alpha^2}\left(1-\frac{4M^2}{l^2\alpha\beta^2}+\cdots\right)C$ and $C = \left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)^3$.

To get the maximum value of the integration, neglecting higher order terms above and equal M^3 in the denominator, we then get

$$\mathrm{Im}I = \frac{\pi \in^2}{\beta^2\alpha}\frac{\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)^2}{\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)-\frac{M\alpha}{\beta}}\omega + \frac{\in^2 \pi a^2\alpha}{\beta^2\left[\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)-\frac{M\alpha}{\beta^2}\right]}\omega - \frac{\in^3 \pi a\alpha}{\beta^2\left[\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)-\frac{M\alpha}{\beta^2}\right]}j,$$

Equation 23

3. NON-THERMAL TUNNELING RATE

Since the emitted particle can be treated as a shell of energy ω, Eqs. (22) and (23) should be modified when the particle's self-gravitational interaction is incorporated. Taking into account the energy conservation as well as angular momentum, the mass parameter and the angular momentum in these equations will be replaced with $M \rightarrow M - \omega$ and $j \rightarrow J - j$ when the particle with energy ω and angular momentum j tunnels out of the event horizon. We fix the ADM mass, charge and angular momentum of the total spacetime and in presence of comological constant KNAdS spacetime is dynamic and allow mass and angular momentum of the black hole to fluctuate. Then the imaginary part of the true action can be calculated from Eq. (23) in the following integral

$$\mathrm{Im}I = \frac{\pi \in^2}{\beta^2\alpha}\cdot\int_0^\omega \frac{\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)^2}{\sqrt{M^2-(a^2+q^2)\alpha}+\left(M-\frac{M\alpha}{\beta}\right)}d\omega' + \frac{\in^2 \pi a^2\alpha}{\beta^2}\cdot\int_0^\omega \frac{1}{\sqrt{M^2-(a^2+q^2)\alpha}+\left(M-\frac{M\alpha}{\beta^2}\right)}d\omega' - \frac{\in^3 \pi a\alpha}{\beta^2}\cdot\int_0^j \frac{1}{\sqrt{M^2-(a^2+q^2)\alpha}+\left(M-\frac{M\alpha}{\beta^2}\right)}dj',$$

Equation 24

For the maximum value of integration, neglecting $\left(1-\frac{\alpha}{\beta^2}\right)M$. Equation (24) becomes

$$\mathrm{Im}I=\frac{\pi\in^2}{\beta^2\alpha}\cdot\int_0^{\omega}\frac{\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)^2}{\sqrt{M^2-(a^2+q^2)\alpha}}d\omega'+\frac{\in^2\pi a^2\alpha}{\beta^2}\cdot\int_0^{\omega}\frac{1}{\sqrt{M^2-(a^2+q^2)\alpha}}d\omega' -\frac{\in^3\pi a\alpha}{\beta^2}\cdot\int_0^{j}\frac{1}{\sqrt{M^2-(a^2+q^2)\alpha}}dj',$$

Equation 25

Replacing M and j by $M-\omega$ and $J-j$ respectively, we obtain

$$\mathrm{Im}I=\frac{\pi\in^2}{\beta^2\alpha}\cdot\int_M^{M-\omega}\frac{\left(M-\omega+\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}\right)^2}{\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}}d\left(M-\omega'\right)+\frac{\in^2\pi a^2\alpha}{\beta^2}\cdot\int_M^{M-\omega}\frac{1}{\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}}d\left(M-\omega'\right)-\frac{\in^3\pi a\alpha}{\beta^2}\cdot\int_J^{J-j}\frac{1}{\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}}d\left(J-j'\right),$$

Equation 26

where $J-j'=\frac{(M-\omega')\alpha}{\in^2}$ and so there is

$$\mathrm{Im}I=-\frac{\pi\in^2}{\beta^2\alpha}\cdot\int_M^{M-\omega}\frac{2(M-\omega)^2+2(M-\omega)\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}}{\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}}d\left(M-\omega'\right)+\frac{\in^2\pi}{\alpha\beta^2}\cdot\int_M^{M-\omega}\frac{(a^2+q^2)\alpha}{\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}}d\left(M-\omega'\right)$$

Equation 27

Finishing the ω' integral, we obtain

$$\mathrm{Im}I=-\frac{\pi\in^2}{\beta^2\alpha}\left\{(M-\omega)\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}+(M-\omega)^2-M\sqrt{M^2-(a^2+q^2)\alpha}-M^2\right\}$$

$$=-\frac{\pi\in^2}{2\beta^2\alpha}\left\{2(M-\omega)\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}+2(M-\omega)^2-2M\sqrt{M^2-(a^2+q^2)\alpha}-2M^2\right\}$$

$$=-\frac{\pi\in^2}{2\beta^2\alpha}\left\{(M-\omega)+\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}\right\}^2-\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)^2$$

Equation 28

Therefore, the non-thermal tunnelling rate for the KNAdS black hole is given by

$$\begin{aligned}
\Gamma &\sim \exp(-2\mathrm{Im}I) \\
&= \exp\left[\frac{\pi \in^2}{\beta^2\alpha}\left\{(M-\omega)+\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}\right\}^2-\left(M+\sqrt{M^2-(a^2+q^2)\alpha}\right)^2\right] \\
&= \exp\left[r_f^2-r_i^2\right] \\
&= \exp(\Delta S_{BH}).
\end{aligned}$$

Equation 29

Here we find that $\Delta S_{BH}=S_{BH}(M-\omega)-S_{BH}(M)$ is the change of Bekenstein-Hawking entropy of the KNAdS black hole before and after the massive particles emission by taking into account $r_i=\frac{\in}{\beta\sqrt{\alpha}}\left[M+\sqrt{M^2-(a^2+q^2)\alpha}\right]$ and $r_f=\frac{\in}{\beta\sqrt{\alpha}}\left[(M-\omega)+\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}\right]$.

4. PURELY THERMAL RADIATION

The radiation spectrum given by Eq. (29) is not pure thermal, which gives a correction to the Hawking radiation of the KNAdS black hole and is consistent with an underlying unitary theory. We now expand (29) in power of ω upto second order as discussed by Hossain et al. Hossain and Rahman (2013) of the form

$$\Gamma \sim \exp(\Delta S_{BH}) = \exp\left\{-\omega\frac{\partial S_{BH}(M)}{\partial M}-\frac{\omega^2}{2}\frac{\partial^2 S_{BH}(M)}{\partial M^2}\right\}.$$

Equation 30

From Eq. (10.29), we can write

$$S_{BH}(M-\omega)=\frac{\pi \in^2}{\beta^2\alpha}\left\{(M-\omega)+\sqrt{(M-\omega)^2-(a^2+q^2)\alpha}\right\}^2.$$

Equation 31

Using Eq. (31) in Eq. (30), we obtain

$$\Gamma \sim \exp(\Delta S_{BH}) = \exp\left[-\frac{2\pi \in^2 \omega}{\beta^2\alpha}\left\{\left(2M+\sqrt{M^2-(a^2+q^2)\alpha}+\frac{M^2}{\sqrt{M^2-(a^2+q^2)\alpha}}\right)-\frac{\omega}{2}\left(2+\frac{3M}{\sqrt{M^2-(a^2+q^2)\alpha}}-\frac{M^3}{(M^2-(a^2+q^2)\alpha)^{\frac{3}{2}}}\right)\right\}\right].$$

Equation 32

If we put $-\ell^2$ in the place of ℓ^2, the Hawking non-thermal spectrum and pure thermal spectrum agree with that of KNdS black hole.

5. RESULTS AND DISCUSSION

The results we have obtained in this articler provides further evidence to support the Parikh and Wilczek's opinion Parikh and Wilczek (2000), Parikh (2002), Parikh (2004) from spherically symmetric black holes. The study of this article gives the result for the Kerr-Newman black hole Chen and Yang (2007) when $\ell \to \infty$. For $q = 0$, the study provides the result for the Kerr-anti-de Sitter black hole Hossain (2017), while for $\ell \to \infty, q = 0$, the result reduces for the Kerr black hole Zhang and Zhao (2005). The result for the Schwarzschild-anti-de Sitter black hole Rahman and Hossain, (2013) is obtained if one sets $a = 0$ and $q = 0$. Moreover, the choice $\ell \to \infty, a = 0$ and $q = 0$ gives the result for the Schwarzschild black hole Parikh and Wilczek (2000).

6. CONCLUSION

In a nutshell, we have investigated the Hawking non-thermal and purely thermal radiations of massive particles as a semiclassical tunneling process from the KNAdS black hole event horizon by taking into account the self-gravitation effect of the emitted particles, the unfixed background spacetime. The results of our work show that the radiant spectrum is not a pure thermal one and the tunneling rate is related to the change of Bekenstein-Hawking entropy and is consistent with an underlying unitary theory.

CONFLICT OF INTERESTS

None.

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