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SOME PHYSICAL CHARACTERISTICS OF A FIVE-DIMENSIONAL MASS SCALAR ELECTROMAGNETIC COSMOLOGICAL MODEL

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ABSTRACT

In this paper we are interested to study some important physical aspects of a five dimensional space time which is attained by the interaction of magnetic field and zero mass scalar field in Einstein's theory of gravitation, where the cosmic parameters X & A are functions of cosmic time t .The concluding remark is focused on the singularity nullity, uniformity, energy condition and about the possession of gravitational field radiation of the space-time.

Keywords: Magnetic Field, Zero Mass Scalar Field, Cosmic Parameter, Singularity, Gravitational Field

1. INTRODUCTION

Higher-dimensional cosmological models are crucial in describing the universe in its early stages of evolution. Many cosmologists have studied the mechanics of the early cosmos in the context of higher-dimensional spacetime in recent years. (The most well-known five- dimensional theory proposed by Kaluza (1921) and Klein (1926) was the first to unite gravitation into a single geometrical structure). As part of their Electromagnetic cosmology research, several writers Patel & Singh (2002), Pradhan et al. (2006), Mohanty et al. (2007), Singh et al. (2004), Das & Banerjee (1999), Chatarjee (1987), Delice et al. (2013), Al-Haysah & Hasmani (2021), Reddy

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& Ramesh (2019), Ingunn & Ravndal (2004), Pranjalendu & Rajshekhar (2022), Barkha et al. (2017) investigated the physics of the cosmos in higher-dimensional spacetime. The unification of gravitational forces with other natural forces is not conceivable in four-dimensional spacetime. This may be achievable in higher-dimensional quantum field theory Appelquist et al. (1987), Weinberg (1986), Chodos & Detweiler (1980). This concept is significant in cosmology because we know that the universe was much smaller in the early phases of evolution than it is today. As a result, we anticipate that the universe's current four- dimensional spacetime could have been replaced by a higher-dimensional spacetime. With the passage of time, the extra dimensions are reduced to a volume of the order of the pauk length, which is not observable at the current stage of the cosmos.

Freund (1982), Appelquist & Chodos (1983), Randjbar-Daemi et al. (1984), Rahaman et al. (2002), and Singh et al. (2004) asserted, using field equation solutions, that four-dimensional space time expands while the fifth dimension contracts or remains constant. Furthermore, Guth (1981) and Alvarez & Gavela (1983) discovered that extra dimensions generate a huge quantity of entropy during the contraction phase, providing an alternative solution to the flatness and horizon problem when contrasted to the conventional inflationary scenario. The Kasnas were first studied when Albert Einstein applied his general theory of relativity to the structure of the entire universe. Different domains of gravitation in the form of tensor equations are used to investigate various kinematical phenomena properties of the universe. Bloch and colleagues Bloch et al. (2023) investigated the scalar dark matter-induced oscillation of a permanent magnet field; Krongos & Torre (2015) General Relativity Geometrization Conditions for Perfect Fluids, Scalar Fields, and Electromagnetic Fields. Kashyap (1978) investigated the interaction of an electromagnetic field and a scalar field in a cylindrically symmetric space- time. Rao, Tiwari, and Roy solved Einstein's equations for coupled electromagnetic and scalar forces. Rosen metric and other physicists have also made significant contributions in magnetic fields and scalar fields.

Bloch et al. (2023) have studied Scalar dark matter induced oscillation of a permanent-magnet field, Krongos & Torre (2015) Geometrization Conditions for Perfect Fluids, Scalar Fields, and Electromagnetic Fieldsin general relativity. Kashyap (1978) studied about couple electromagnetic field and scalar field in cylindrically symmetric space-time. Ayyangar & Mohanty (1985), Banerjee & Bhuli (1990) found out solutions for coupled electromagnetic and scalar field for Einstein-Rosen metric and other physicists have also done remarkable works in magnetic field and scalar fields.

The magnetic field plays an important role in the energy distribution of the universe as it contains highly ionised matter. The scalar field represents matter field with spin less quanta. The zero-rest mass scalar field describes long range interaction.

In this paper, we are interested to study the various physical property of the five-dimensional space-time with magnetic field and scalar fields.

2. METRIC & FIELD EQUATIONS

Here we have taken the Kaluza-Klein space time described as

Here we have taken the Kaluza-Klein space time described as

$$ds^2 = -dt^2 + X^2(t)(dx^2 + dy^2 + dz^2) + A^2(t)d\psi^2 \quad \textbf{Equation 1}$$

Where

$g_{11}=X^2(t), g_{22}=X^2(t), g_{33}=X^2(t), g_{44}=A^2(t), g_{55}=-1.$ **Equation 2**

Einstein's gravitational field is given by

$$G_{ij}=R_{ij}-\frac{1}{2}Rg_{ij}=-8\pi G(E_{ij}+T_{ij})$$ **Equation 3**

Where G is the universal gravitational constant. Here E_{ij} is the energy-momentum tensor of source free electromagnetic field and T_{ij} is the energy momentum tensor of zero mass scalar field defined as

$$E_{ij}=\frac{1}{4\pi}\left(-F_{ia}F_j^a+\frac{1}{4}g_{ij}\right)F_{ab}F^{ab}$$ **Equation 4**

Where

$$F_{ij}=\varphi_{j,i}-\varphi_{i,j}$$ **Equation 5**

andφ_i represents scalar potential.

$$T_{ij}=\frac{1}{4\pi}(V_{ji}V_{ij}-\frac{1}{2}g_{ij}V_{ja}V^{ja})$$ **Equation 6**

Here the gravitational potential V_2, V_3 does not exists.

Thus $$T_{11}=\frac{1}{8\pi}\left(V_1^2-\frac{X^2(t)}{A^2(t)}V_4^2+X^2(t)V_5^2\right)$$ **Equation 7**

Similarly

$$T_{22}=\frac{1}{8\pi}(-V_1^2-\frac{X^2(t)}{A^2(t)}V_4^2+X^2(t)V_5^2)$$ **Equation 8**

$$T_{33}=\frac{1}{8\pi}(-V_1^2-\frac{X^2(t)}{A^2(t)}V_4^2+X^2(t)V_5^2)$$ **Equation 9**

$$T_{44}=\frac{1}{8\pi}\left(V_4^2-V_1^2+A^2(t)V_5^2\right)$$ **Equation 10**

$$T_{55}=\frac{1}{8\pi}(V_5^2+\frac{1}{X^2}(V_1^2+A^2(t)V_4^2)$$ **Equation 11**

Since $F_{ij}=\phi_{ji}-\phi_{ij}$

Here only $\phi_2\&\phi_3$ components of the scalar potential ϕ_i, exists. Let us take $\phi_2=m, \phi_3=n$.

Thus $F_{11}=F_{22}=F_{33}=F_{44}=0$ also $F_{23}=F_{32}=0$.

The existing components of F_{ij} are

$F_{12}=m_1, F_{13}=n, F_{21}=-m, F_{24}=-m_4, F_{25}=-m_5, F_{31}=-n_1, F_{34}=-n_4,$
$F_{35}=-n_5, F_{42}=m_4, F_{43}=n_4, F_{52}=m_5, F_{53}=n_5$ **Equation 12**

Using these values of F_{ij} in the existing components are

$$E_{11}=\frac{1}{8\pi}\left(-\frac{m_1^2}{X^2}-\frac{n_1^2}{X^2}+\frac{m_4^2}{A^2}+\frac{n_4^2}{A^2}-m_5^2-n_5^2\right)$$ **Equation 13**

$$E_{22}=\frac{1}{8\pi}\left(\frac{3m_1^2}{X^2}+\frac{n_1^2}{X^2}-\frac{m_4^2}{A^2}+\frac{n_4^2}{A^2}-m_5^2-n_5^2\right)$$ **Equation 14**

$$E_{33}=\frac{1}{8\pi}\left(\frac{3n_1^2}{X^2}+\frac{m_1^2}{X^2}+\frac{m_4^2}{A^2}-\frac{n_4^2}{A^2}-m_5^2+n_5^2\right)$$ **Equation 15**

$$E_{44}=\frac{1}{8\pi}\left(-\frac{m_4^2}{X^2}-\frac{n_4^2}{X^2}+\frac{m_1^2A^2}{X^4}+\frac{n_1^2A^2}{X^2}-\frac{m_5^2A^2}{X^2}-\frac{n_5^2A^2}{X^2}\right)$$ **Equation 16**

$$E_{55}=\frac{1}{8\pi}\left(\frac{3m_5^2}{X^2}-\frac{n_5^2}{X^2}-\frac{m_1^2}{X^4}-\frac{n_1^2}{X^4}-\frac{m_4^2}{A^2X^2}-\frac{n_4^2}{A^2X^2}\right)$$ **Equation 17**

The predetermined values of $G_{ij}=R_{ij}-\frac{1}{2}Rg_{ij}$

as described in equation (3) for our metric are

$$G_{11}=G_{22}=G_{33}=\frac{X''}{X}+2\left(\frac{X'}{X}\right)+\frac{X'A'}{XA}-\frac{3}{2}X^3X''-3X^2\left(X'\right)^2-\frac{3}{2}X^3X'\frac{A'}{A}-\frac{1}{2}A^3A'-\frac{3}{2}A^3A'\frac{X'}{X}-\frac{3}{2}\frac{X''}{X}-\frac{1}{2}\frac{A''}{A}$$ **Equation 18**

$$G_{44}=\frac{1}{2}\frac{A''}{A}+\frac{3X'A'}{XA}-\frac{3}{2}X^3X''-3X^2\left(X'\right)^2-\frac{3}{2}X^3X'\frac{A'}{A}-\frac{1}{2}A^3A''-\frac{3}{2}A^3A'\frac{X'}{X}-\frac{3}{2}\frac{X''}{X}$$ **Equation 19**

$$G_{55}=\frac{3}{2}\frac{X''}{X}+\frac{1}{2}\frac{A''}{A}+\frac{3X'A'}{XA}-\frac{3}{2}X^3X''-3X^2\left(X'\right)^2-\frac{3}{2}X^3X'\frac{A'}{A}-\frac{1}{2}A^3A''-\frac{3}{2}A^3A'\frac{X'}{X}$$ **Equation 20**

Computing the right-hand side of equation (3), using the equation's (7) to (17) and putting the left hand side values from (18) to (20) we get

$$\frac{X''}{X}+2\left(\frac{X'}{X}\right)+\frac{X'A'}{XA}-\frac{3}{2}X^3X''-3X^2\left(X'\right)^2-\frac{3}{2}X^3X'\frac{A'}{A}-\frac{1}{2}A^3A'-\frac{3}{2}A^3A'\frac{X'}{X}-\frac{3}{2}\frac{X''}{X}-\frac{1}{2}\frac{A''}{A}=\frac{-k}{8\pi}\left(-\frac{m_1^2}{X^2}-\frac{n_1^2}{X^2}+\frac{m_4^2}{A^2}+\frac{n_4^2}{A^2}-m_5^2-n_5^2-V_1^2-\frac{X^2(t)}{A^2(t)}V_4^2+X^2(t)V_5^2\right)$$ **Equation 21**

$$\frac{X''}{X}+2\left(\frac{X'}{X}\right)+\frac{X'A'}{XA}-\frac{3}{2}X^3X''-3X^2\left(X'\right)^2-\frac{3}{2}X^3X'\frac{A'}{A}-\frac{1}{2}A^3A'-\frac{3}{2}A^3A'\frac{X'}{X}-\frac{3}{2}\frac{X''}{X}-\frac{1}{2}\frac{A''}{A}=\frac{-k}{8\pi}\left(\frac{3m_1^2}{X^2}+\frac{n_1^2}{X^2}-\frac{m_4^2}{A^2}+\frac{n_4^2}{A^2}-m_5^2-n_5^2-V_1^2-\frac{X^2}{A^2}V_4^2+X^2V_5^2\right)$$ **Equation 22**

$$\frac{X''}{X}+2\left(\frac{X'}{X}\right)+\frac{X'A'}{XA}-\frac{3}{2}X^3X''-3X^2\left(X'\right)^2-\frac{3}{2}X^3X'\frac{A'}{A}-\frac{1}{2}A^3A'-\frac{3}{2}A^3A'\frac{X'}{X}-\frac{3}{2}\frac{X''}{X}-\frac{1}{2}\frac{A''}{A}=\frac{-k}{8\pi}\left(\frac{3n_1^2}{X^2}+\frac{m_1^2}{X^2}+\frac{m_4^2}{A^2}-\frac{n_4^2}{A^2}-m_5^2+n_5^2-V_1^2-\frac{X^2}{A^2}V_4^2+X^2V_5^2\right)$$ **Equation 23**

$$\frac{1}{2}\frac{A''}{A}+\frac{3X'A'}{XA}-\frac{3}{2}X^3X''-3X^2(X')^2-\frac{3}{2}X^3X'\frac{A'}{A}-\frac{1}{2}A^3A''-\frac{3}{2}A^3A'\frac{X'}{X}-\frac{3}{2}\frac{X''}{X}$$
$$=\frac{-k}{8\pi}\left(-\frac{m_4^2}{X^2}-\frac{n_4^2}{X^2}+\frac{m_1^2A^2}{X^4}+\frac{n_1^2A^2}{X^2}-\frac{m_5^2A^2}{X^2}-\frac{n_5^2A^2}{X^2}-V_1^2+V_4^2+A^2V_5^2\right)$$ **Equation 24**

$$\frac{3}{2}\frac{X''}{X}+\frac{1}{2}\frac{A''}{A}-\frac{3}{2}X^3X''-3X^2(X')^2-\frac{3}{2}X^3X'\frac{A'}{A}-\frac{1}{2}A^3A''-\frac{3}{2}A^3A'\frac{X'}{X}$$
$$=\frac{-k}{8\pi}\left(\frac{3m_5^2}{X^2}-\frac{n_5^2}{X^2}-\frac{m_1^2}{X^4}+\frac{n_1^2}{X^4}-\frac{m_4^2}{A^2X^2}-\frac{n_5^2}{A^2X^2}+V_5^2+\frac{V_1^2}{X^2}+A^2V_4^2\right)$$ **Equation 25**

Let us impose the following restrictions on the electromagnetic field to get some exact value of the cosmological parameters.

- **Case-I:**

$$m\neq 0, n\neq 0, v\neq 0$$

- **Case-II:**

$$m=0, n\neq 0, v\neq 0$$

- **Case-III:**

$$m\neq 0, n=0, v\neq 0$$

- **Case-I: As**

$$m\neq 0, n\neq 0, v\neq 0$$

Solving (21) to (25) we get

$$X=\frac{n_4^2(2m_1^2+n_1^2)-m_4^2(2n_1^2+m_1^2)}{m_4^2(-v_1^2+v_5^2)-n_4^2-v_1^2}$$ **Equation 26**

&

$$A=m_4\left[\frac{(2m_1^2+n_1^2)[(m_4^2-v_1^2+v_5^2)-n_4^2(-v_1^2)]}{n_4^2(2m_1^2+n_1^2)-m_4^2(2n_1^2+m_1^2)+v_1^2}\right]^{\frac{1}{2}}$$ **Equation 27**

- **Case-II:**

Putting $m=0, n\neq 0 \& v\neq 0$ in equation (21) to (25) and solving we get

$$X=\left[\frac{m_1^2}{v_1^2}\right]^{\frac{1}{2}}=\frac{m_1}{v_1}$$ **Equation 28**

&

$$A=\left[\frac{m_4^2}{v_1^2}\right]^{\frac{1}{2}}=\frac{m_4}{v_1}$$ **Equation 29**

- **Case-III:**

Putting $m\neq 0, n=0 \& v\neq 0$ in equations (21) to (25) and solving we get

$$X=\left[\frac{m_1^2}{v_1^2}\right]^{\frac{1}{2}}=\frac{m_1}{v_1}$$ **Equation 30**

&

$$A=\left[\frac{m_4^2}{v_1^2}\right]=\frac{m_4}{v_1}$$ **Equation 31**

3. PHYSICAL PROPERTIES

1) **Nulity:** The null electromagnetic field indicates the propagation of e-m radiation with fundamental velocity. As per Synge (1958)

$$\omega = \left[\left(F_{ij}F^{ij}\right)^2 + \left(F_{ij}F^{-ij}\right)\right]^{\frac{1}{2}} \quad \textbf{Equation 32}$$

$\omega = 0$ (Null electromagnetic field)

$\omega \neq 0$ (Non-null electromagnetic field)

After calculation of equation (32) for all the three cases using equation (26) to (31) and using the values of $F_{ij}'s$, it is seen that the electromagnetic field isof non-null nature for our space-time.

2) **Singularity:** We study the regularity of a solution using, Bonnor (1958) who stated that a point's (may be a point of spatial or temporal infinity) is a regular point in a natural co-ordinate system, where the following sufficient conditions are satisfied.

- $det\ g = |g_{ij}|$ is nonzero.
- $g_{ij}$and their first derivatives are finite and continuous at "s",
- The second derivatives of $g_{ij}$are finite and continuous at "s"

For various values of $g_{ij}$and their derivatives for all the three cases and putting the values of $X\&A$ from equations (21) to (31), it is seen that the solutions are regular at the point "s".

3) **Gravitational radiation:** Gravitational field radiation of a space-time exist, if $T = \sum T_{ij} \neq 0$.

Using the values of $X\&A$ for all the three cases it is seen that $T \neq 0$.

Hence the space-time possesses gravitational field radiation.

4) **Curvature Scalar(R):** As per Einstein the curvature scalar is defined as

$$R = -\frac{k}{4\pi} v_i v^i \quad \textbf{Equation 33}$$

and for our metric it takes the form.

$$R = -\frac{k}{4\pi}[X^2 v_1^2 + A^2 v_4^2 - v_5^2] \quad \textbf{Equation 34}$$

Using the various values of $X\&A$for all the three cases and calculating we get

$R = at^2 - bt^4 - ct^6$ for **case- I**

$R = \alpha t^2$ for **case- II**

$\&R = \beta t^2$ for **case- III**

Where $a, b, c, \alpha\&\beta$are constants. As$t \rightarrow \infty$, the variation of curvature scalar with cosmic time for all the three cases the scale factor diverges to infinity.

5) Energy conditions:

- **Scalar Field:** The energy condition for scalar mesan field is studied from the component $T_{44}$of the energy momentum tensor as given in equation (10)

$$T_{44} = \frac{1}{8\pi}\left(V_4^2 - V_1^2 + A^2(t)V_5^2\right)$$

The different energy values of Awe get,

$T_{44} = \delta t^4$ for case-I

$T_{44} = \lambda t^2$ for case-II

$T_{44} = \sigma t^2$ for case-III

Where $\delta, \lambda \& \sigma$are constants. As $T_{44} \to t$, the variation of energy of the scalar mesan field with cosmic time.

- **Electromagnetic field:** The energy condition of electromagnetic field is studied from the component E_{44} of the energy momentum tensor as given in equation (17)

$$E_{44} = \frac{1}{8\pi}\left(-\frac{m_4^2}{X^2} - \frac{n_4^2}{X^2} + \frac{m_1^2 A^2}{X^4} + \frac{n_1^2 A^2}{X^2} - \frac{m_5^2 A^2}{X^2} - \frac{n_5^2 A^2}{X^2}\right)$$

The energy values of the electromagnetic field for all the three cases after using the different values of $X \& A$ we get

$E_{44} = \rho t^2 - \gamma t^4 + \xi$ for case-I

$E_{44} = \eta t^2$ for case-II

$E_{44} = k t^2$ for case-III

Where $\rho, \gamma, \xi, \eta \& k$ are constants, the $E_{44} \to t$shows the variation of electromagnetic energy with cosmic time of our space-time for all the three cases.

- **Uniformity:** A space-time is uniform, if it satisfies the condition $F_{ij,k} = 0$.The existing values of $F_{ij}$are given in equation (13) using the above condition, it is found that our space-time is found uniform along $X, Y, Z \& \psi$ direction only

4. CONCLUSION

In this paper, we have constructed a five-dimensional space time with the interaction of magnetic field and zero-mass scalar field. Considering various cases when

Case-I:

$m \neq 0, n \neq 0, v \neq 0$

Case-II:

$m = 0, n \neq 0, v \neq 0$

Case-III:

$m \neq 0, n = 0, v \neq 0$

Where *m* & *n* are components of scalar potential assumed to be function of "t" which is clear from equation (27) to (32). The new form of the space time is found to posses gravitational field radiation, uniformity, non-null electromagnetic field. Moreover, the singularity, curvature scalar and the energy conditions of both the fields are discussed clearly.

5. SUMMARY

This article deals with Kaluza-Klein electromagnetic model in the presence of a mass scalar meson field. This model is obtained by solving Einstein field equations using hybrid expansion law and a relation between metric potentials. It is observed that the variation of electromagnetic energy with cosmic time of our space-time for all the three cases diverges for late time. Moreover, I found that the space-time possesses gravitational field radiation, and the space time is found to possess gravitational field radiation, uniformity, non-null electromagnetic field.

CONFLICT OF INTERESTS

None.

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