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A STUDY OF PERISTALTIC FLOW OF A COUPLE STRESS FLUID IN AN INCLINED CHANNEL UNDER THE EFFECT OF MAGNETIC FIELD WITH SLIP CONDITION

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ABSTRACT

In this paper we study about peristaltic flow of CSF in a inclined channel and its effect on magnetic field with different parameters and shown it in graphically.

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1. INTRODUCTION

The study of fluid flows through and past porous medium has attracted much attention recently. This is primarily because of numerous applications of flow through porous medium, such as storage of radioactive nuclear waste materials transfer, separation processes in chemical industries, filtration, transpiration cooling, transport processes in aquifers and ground water pollution. Flow through a porous medium has been studied by Scheidegger (1974). Some studies about this point have been made by Varshney (1979) and EL-Dave and EL-Mohendis (1995). Elshehawey *et al.* (2000) studied peristaltic motion of a generalized Newtonian fluid through a porous medium. Ramireddy *et al.* (2010) studied peristaltic transport of a conducting fluid in an inclined asymmetric channel. Peristaltic motion of a generalized Newtonian fluid under the effect of a transverse magnetic field is studied by Elshehawey *et al.* (1999). Rathod and Asha (2014) studied peristaltic transport of couple stress fluids in a uniform and Non-uniform annulus through porous media. Satyanarayana *et al.* (2011) studied hall current effect on magnetohydro dynamics free-convection flow past a semi-infinite vertical porous plate with mass transfer. A study of ureteral peristalsis in cylindrical tube through porous medium is made by Rathod and Channakote (2011). Rathod and Pallavi (2011) studied the effect of slip condition and heat transfer on MHD peristaltic transport through a porous medium with complaint wall.

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Srinivas and Pushparaj (2008) have investigated the peristaltic transport of MHD flow of a viscous incompressible fluid in a two dimensional asymmetric inclined channel. However, the interaction of peristalsis and heat transfer has not received much attention, which may become highly relevant and significant in several industrial processes. Slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel is made by Rathod and Channakote (2012). Lately, the combined effects of magneto hydrodynamics and heat transfer on the peristaltic transport of viscous fluid in a channel with compliant walls have been discussed by Mekheimer and Abd elmaboud *et al*. (2008) and (2009).

The aim of present chapter is to investigate the interaction of peristalsis for the flow of a couple stress fluid in a two dimensional inclined channel with the effect of magnetic field using slip condition. The computational analysis has been carried out for drawing velocity profiles, pressure gradient and frictional force.

2. MATHEMATICAL FORMULATION

We consider a peristaltic flow of a Couple stress fluids through two-dimensional channel of width 2a and inclined at an angle α to the horizontal symmetric with respect to its axis. The walls of the channel are assumed to be flexible.

The wall deformation is

$$H(x,t) = a + bCos(\frac{2\pi}{\lambda}(X - ct)) \tag{1}$$

Where 'b' is the amplitude of the peristaltic wave, 'c' is the wave velocity, 'λ' is the wave length, t is the time and X is the direction of wave propagation.

The governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \mu\nabla^2 u - \eta^*\nabla^4 u + \rho g \sin\alpha - \sigma B_o^2 u \tag{3}$$

$$\rho(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y} + \mu\nabla^2 v - \eta^*\nabla^4 v - \rho g \cos\alpha - \sigma B_o^2 v \tag{4}$$

Where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

u and v are velocity components, 'p' is the fluid pressure, 'ρ' is the density of the fluid, 'μ' is the coefficient of viscosity, 'η^*' is the coefficient of couple stress, 'g' is the gravity due to acceleration, 'α' angle of inclination, 'σ' is electric conductivity and '$B_\circ$' is applied magnetic field.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x = X - ct, y = Y, u = U, v = V, p = P(X,t) \tag{5}$$

We introduce the non-dimensional variables:

$$x^* = \frac{x}{\lambda},\ y^* = \frac{y}{a},u^* = \frac{u}{c},\ v^* = \frac{v}{c\delta},\ \ t^* = \frac{tc}{\lambda}, p^* = \frac{pa^2}{\mu c\lambda}, G = \frac{\rho g a^2}{\mu c}, M = B_\circ\sqrt{\frac{\sigma}{\mu a^2}}, \phi = \frac{b}{a} \tag{6}$$

Equation of motion and boundary conditions in dimensionless form becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$\operatorname{Re}\delta(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x}+(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})-\frac{1}{\gamma^2}(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$-M^2 u + G\sin\alpha \tag{8}$$

$$\operatorname{Re}\delta^3(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y}+\delta^2(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})-\frac{1}{\gamma^2}\delta^2(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$
$$-M^2\delta^2 v - \rho g\delta\cos\alpha \tag{9}$$

Where, $\gamma^2 = \dfrac{\eta^*}{\mu a^2}$ couple-stress parameter and $M^2 = B_o^2\dfrac{\sigma}{\mu a^2}$ Hartmann number.

The dimensionless boundary conditions are:

$$\frac{\partial u}{\partial y}=0;\quad \frac{\partial^2 u}{\partial y^2}=0 \qquad at \quad y=0$$

$$u=-k_n\frac{\partial u}{\partial y};\frac{\partial^2 u}{\partial y^2}\quad finite \quad at \quad y=\pm h = 1+\phi Cos[2\pi x] \tag{10}$$

where, k_n is Knudsen number (slip parameter)

Using long wavelength approximation and neglecting the wave number δ, one can reduce governing equations:

$$\frac{\partial p}{\partial y}=0 \tag{11}$$

$$\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}-\frac{1}{\gamma^2}\frac{\partial^4 u}{\partial y^4}-M^2 u+G\sin\alpha \tag{12}$$

3. METHOD OF SOLUTION

Using Adomian decomposition method, the equation (12) can be written as

$$L_{yy}(L_{yy}u) = \gamma^2 (L_{yy}u - M^2 u - \overline{p}) \tag{13}$$

$$\overline{p} = \frac{dp}{dx} - G\sin\alpha$$

where $L_{yy} = \frac{d^2}{dy^2}$ .Since L_{yy} is a second -order differential operator, L_{yy}^{-1} is a second-fold integration operator. Operating with L_{yy}^{-1} (L_{yy}^{-1}) to (6.13) becomes

$$u = c_1 + c_2 y + c_3 y^2 + c_4 y^3 + L_{yy}^{-1} L_{yy}^{-1} \left(\frac{dp}{dx} \right) + L_{yy}^{-1} L_{yy}^{-1} M^2 u \tag{14}$$

By the standard Adomian decomposition method, one can write

$$u = \sum_{n=0}^{\infty} u_n \tag{15}$$

$$u_0 = c_1 + c_2 y + \left(\frac{dp}{dx} \right) \frac{y^2}{2!},$$

$$u_{n+1} = M^2 L_{yy}^{-1}(u_n), \qquad n \geq 0. \tag{16}$$

Using boundary conditions (10) to the equations from (11) to (16), we obtain

$$u_1 = c_1 \frac{(My)^2}{2!} + \frac{c_2}{M} \frac{(My)^3}{3!} + \left(\frac{dp}{dx} / M^2 \right) \frac{(My)^4}{4!},$$

$$u_2 = c_1 \frac{(My)^4}{4!} + \frac{c_2}{M} \frac{(My)^5}{5!} + \left(\frac{dp}{dx} / M^2 \right) \frac{(My)^6}{6!}$$

$$u_n = c_1 \frac{(My)^{2n}}{(2n)!} + \frac{c_2}{M} \frac{(My)^{2n+1}}{(2n+1)!} + \left(\frac{dp}{dx} / M^2 \right) \frac{(My)^{2n+2}}{(2n+2)!}$$

$$u = c_1 \cosh(My) + \frac{c_2}{M} \sinh(My) + \left(\frac{dp}{dx} / M^2 \right) (\cosh(My) - 1). \tag{17}$$

This may be simplified as

$$u = F_1\cosh(My) + F_2\sinh(My) - \left(\frac{dp}{dx}/M^2\right),$$

Where

$$F_1 = \frac{\left(\left(-1+\frac{dp}{dx}/M^2\right)\left(\sinh(Mh_2)-\sinh(Mh_1)-2\beta M\cosh(My)\right)\right)}{\sinh(M(h_2-h_1))-\beta M\left(\cosh\left(M(h_1-y)\right)+\cosh\left(M(h_2-y)\right)\right)}$$

$$F_2 = \frac{\left(\left(-1+\frac{dp}{dx}/M^2\right)\left(\cosh(Mh_1)-\cosh(Mh_2)+2\beta M\sinh(My)\right)\right)}{\sinh(M(h_2-h_1))-\left(\beta M\cosh\left(M(h_2-y)\right)+\beta M\cosh\left(M(h_1-y)\right)\right)}$$

The volume flow rate in the wave frame is given as

$$q = \int_{h_2}^{h_1} u\,dy$$

$$q = -A + B + \frac{dp}{dx}\left(\frac{A}{M^2} - \frac{B}{M^2} - \frac{h_1}{M^2} + \frac{h_2}{M^2}\right) \tag{18}$$

where

$$A = \frac{\left(-4\sqrt{2}\beta\tan^{-1}\left(\frac{\sec h\left(\frac{Mh_1}{2}\right)(C_1)}{\sqrt{2}D}\right)\right)}{D}$$

$$B = \frac{\left(-4\sqrt{2}\beta\tan^{-1}\left(\frac{\sec h\left(\frac{Mh_2}{2}\right)(C_2)}{\sqrt{2}D}\right)\right)}{D}$$

$$C_1 = -\cosh\left(\frac{Mh_1}{2} - M(-h_1+h_2)\right) + \cos\left(\frac{Mh_1}{2} + M(-h_1+h_2)\right) - 2M\beta\sinh\left(\frac{Mh_1}{2}\right)$$
$$+2M\beta\sinh\left(\frac{Mh_1}{2} - Mh_2\right)$$

$$C_2 = -\cosh\left(\frac{Mh_2}{2} - M(-h_1+h_2)\right) + \cosh\left(\frac{Mh_2}{2} + M(-h_1+h_2)\right) - 2M\beta\sinh\left(\frac{Mh_2}{2}\right)$$
$$-2M\beta\sinh\left(Mh_1 - \frac{Mh_2}{2}\right)$$

$$D = \sqrt{1 + 4M^2\beta^2 - \cosh\left(2M(-h_1+h_2)\right) + 4M^2\beta^2\cosh(Mh_1 - Mh_2)}$$

From (18), we have

$$\frac{dp}{dx} = \frac{M^2(q+A-B)}{(A-B-h_1+h_2)}$$

The instantaneous flux at any axial station is given by

$$\overline{Q}(x,t) = \int_{h_2}^{h_1}(u+1)dy = q + h_1 - h_2. \tag{19}$$

The average volume flow rate over one wave period $T = \dfrac{\lambda}{c}$ of the peristaltic wave is defined as

$$Q = \frac{1}{T}\int_0^T \overline{Q}dt = \frac{1}{T}\int_0^T (q+h_1-h_2)dt = q + 1 + d \tag{20}$$

The pressure rise over one wave length of the peristaltic wave is given by

$$\Delta p = \int_0^1 \frac{dp}{dx}dx \tag{21}$$

The dimensional frictional force F at the wall across one wavelength in the channel is given by

$$F = \int_{h_2}^{h_1} h\left(-\frac{dp}{dx}\right)dx \tag{22}$$

4. RESULTS AND DISCUSSIONS

In this section we have presented the graphical results of the solutions axial velocity u, pressure rise ΔP, friction force F for the different values of couple stress (γ), magnetic field (M), angle of inclination (α), gravitational parameter (G) and slip parameter (β). The axial velocity is shown in **Figs. (1 to 5)**.

The Variation of u with γ, we find that u depreciates with increase in γ (**Fig1**). The Variation of u with magnetic field M shows that for u decreases with increasing in M (**fig.2**).The Variation of u with angle of inclination α shows that for u increases with increasing in α (**Fig 3**). The Variation of u with gravitational parameter G shows that for u increases with increasing in G (**Fig 4**). The Variation of u with slip parameter β shows that for u increases with increasing in G (**Fig 5**).

Figure 1

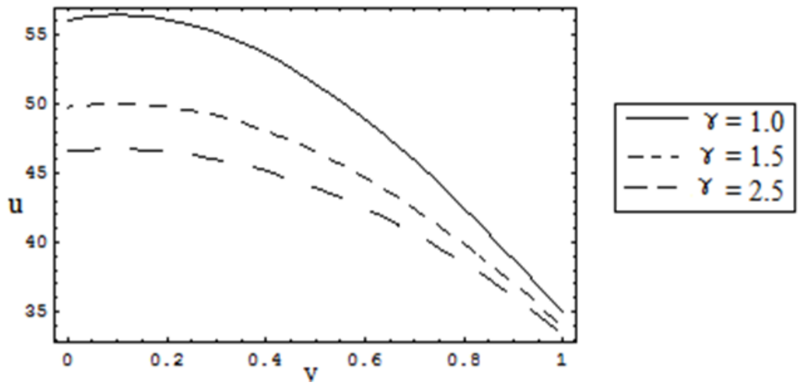

Figure 1 Effect of γ on u, when ϕ =0.2,x=0.1 & p=-25, G= 6, M =1, β = .1 & α = π/4.

Figure 2

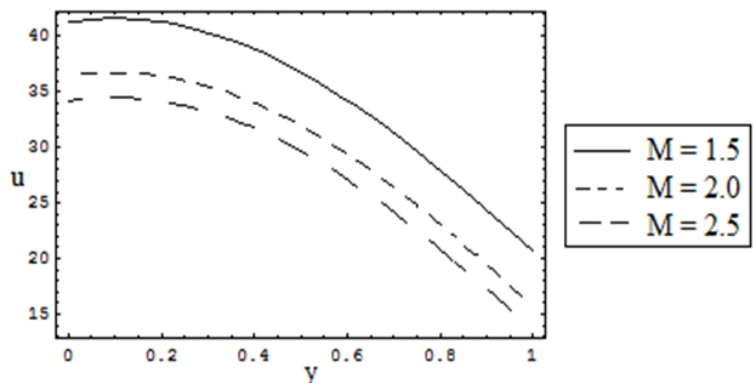

Figure 2 Effect of **M** on u,when γ =1, ϕ =0.2,x=0.1,p=-25, G=4, α = π/4, ϕ=0.2 & β=.1

Figure 3

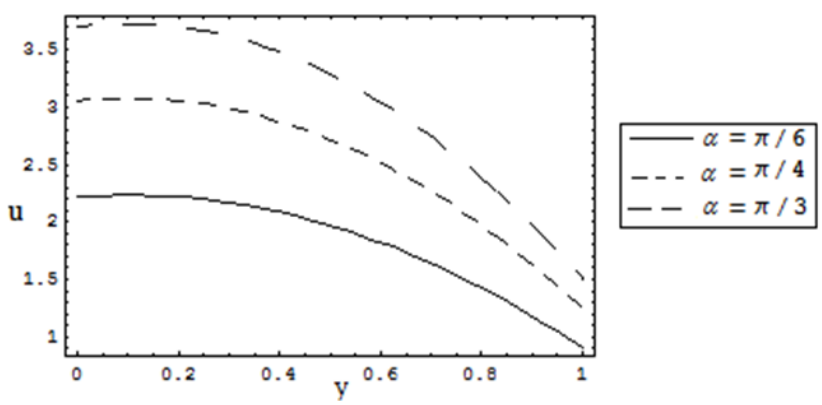

Figure 3 Effect of α on u, when γ =3, ϕ =0.2,x=0.1,p=-.25, M=5, β = .1&G= 6.

Figure 4

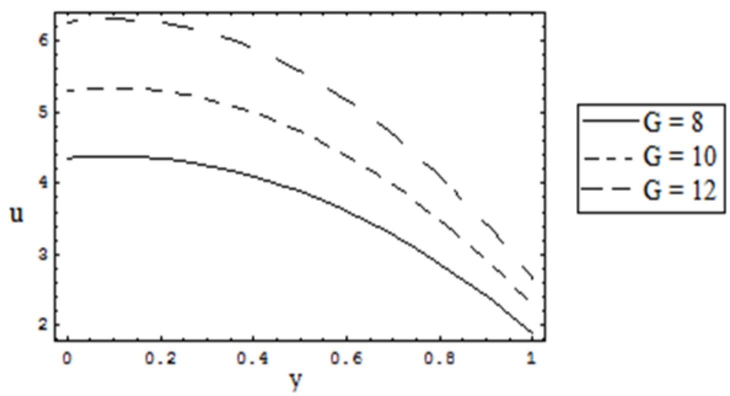

Figure 4 Effect of G on u, when γ =1,ϕ=.2,x=0.1, p=-.25, M =1, β = .1 & α = π/4.

Figure 5

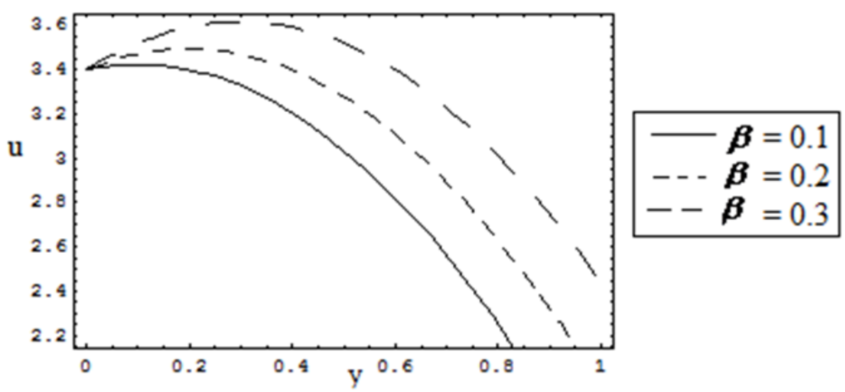

Figure 5 Effect of β on u, when γ =1,x=0.1, p=-.25, G = 6, ϕ=0.2, M=1 & α = π/4.

5. CONCLUSION

1) In this chapter we have presented a theoretical approach to study the peristaltic flow of a couple stress fluid in an inclined channel with the effect of magnetic field using slip condition.
2) The governing Equations of motion are solved analytically. Furthermore, the effect of various values of parameters on velocity, pressure rise and friction force have been computed numerically and explained graphically.
3) We conclude the following observations:
4) The velocity u increases with increasing in gravitational parameter G, angle of inclination α , slip parameter β but, decreases with increasing in couple stress parameter γ & magnetic field M.
5) The pressure ΔP increases with increasing in gravitational parameter G, angle of inclination α , couple stress parameter γ & magnetic field M but, decreases with increasing in slip parameter β .
6) The friction force F decreases with increasing in gravitational parameter G, angle of inclination α , couple stress parameter γ & magnetic field M but, increases with increasing in slip parameter β .

CONFLICT OF INTERESTS

None.

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REFERENCES

J. C. Misra and S. K. Pandey, Peristaltic transport in a tapered tube, Mathl. Comput. Modelling, 22 (1995) 137-151.

J.G. Brasseur, S. Corrsin and Q. LU Nan, The influence of a peripheral layer of different viscosity on peristaltic pumping with Newtonian fluids, J. Fluid.Mech,174 (1987) 495-519.

K. Vajravelu, G. Radhakrishnamacharya and V. Radhakrishnamurty, Peristaltic flow and heat transfer in a vertical porous annulus with long wavelength approximation, Int. J. Nonlinear Mech, 42 (2007) 754-759.

K. Vajravelu, S. Sreenadh and V. Ramesh Babu, Peristaltic transport of a Hershel Bulkley fluid in an inclined tube,Int. J.Non linear Mech, 40 (2005) 83-90.

K. Vajravelu, S. Sreenadh and V.Ramesh Babu, Peristaltic pumping of a Hershel- Bulkley fluid in a channel, Appl. Math. and Computation, 169 (2005) 726-735.

K.Vajravelu, S.Sreenadh and P.Lakshminarayana, Influence of heat transfer on peristaltic transport of Jeffrey fluid in a vertical porous stratum, Communications in Nonlinear Science and Numerical Simulation, 16 (2011) 3107-3125.

V.P. Rathod and S.K. Asha, Peristaltic transport of a couple stress fluid in a uniform and non uniform annulus, International journal of mathematical modeling, simulation and application, 2 (2009) 414–426.