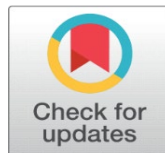
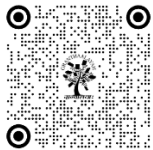


COMPARATIVE STUDY OF HMM TOPOLOGIES FOR SIGNATURE VERIFICATION (ERGODIC VS. LEFT-TO-RIGHT)

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ABSTRACT

This research explores the impact of two distinct Hidden Markov Model (HMM) topologies—Ergodic and Left-to-Right—on the performance of online signature verification systems. Using the SVC 2004 dataset and pressure-based hybrid wavelet transform (HWT) features, we systematically evaluate each topology's classification accuracy, convergence speed, and computational cost. Our experimental framework includes varying the number of HMM states, training samples, and observation symbols to examine how these topologies influence Equal Error Rate (EER), False Acceptance Rate (FAR), and False Rejection Rate (FRR). Results indicate that while Ergodic HMMs provide superior accuracy due to their flexibility, Left-to-Right models converge faster and demand fewer computational resources. This study provides practical recommendations for selecting an HMM topology based on the intended application's performance and efficiency requirements.

1. INTRODUCTION

In recent years, biometric authentication has evolved beyond traditional fingerprint and iris scans to include behavioral characteristics such as online signatures. These dynamic traits capture temporal variations like pen pressure, velocity, and stroke order, making them inherently time-dependent [1]. As a result, sequential modeling techniques have gained traction in this domain, with Hidden Markov Models (HMMs) emerging as a foundational framework for signature classification tasks [2].

An HMM represents a statistical model where the system is assumed to follow a Markov process with hidden states emitting observable symbols. This generative model suits dynamic biometric data well, particularly when capturing time-dependent patterns [3]. However, the structure or topology of an HMM significantly influences both learning and classification outcomes.

Among the most studied HMM topologies are: - Ergodic (Fully Connected) HMM: Every state can transition to every other state, offering high flexibility [4]. - Left-to-Right (Bakis) HMM: Transitions are unidirectional, mimicking progression through time [5].

Although both topologies have found applications in biometric systems, the choice between them is often based on heuristics rather than systematic evaluation. This paper aims to provide a comparative analysis of these topologies under consistent preprocessing and feature extraction conditions.

2. BACKGROUND AND RELATED WORK

HMMs were first popularized in speech recognition applications and later extended to various sequence modeling tasks [6]. Rabiner's tutorial on HMMs [7] remains a cornerstone reference for many biometric researchers.

In signature verification, both Ergodic and Left-to-Right HMMs have been deployed, though under different assumptions. Starner et al. used Left-to-Right models for gesture recognition, relying on the inherent sequence structure of human gestures [8], while Justino et al. used Ergodic models to accommodate the non-linear structure of handwritten signatures [9].

Recent work has focused on leveraging deep learning enhancements and hybrid wavelet features in biometric verification, such as Galbally et al. [10], Rattani and Derakhshani [11], and Kisku and Rattani [12]. However, explicit comparison of HMM topologies across consistent datasets remains limited.

3. DATASET AND FEATURE EXTRACTION

The SVC 2004 online signature dataset was selected for this study: - 40 users - 20 genuine and 20 forged samples per user - Each sample contains 512 events, including pressure, azimuth, and timing [13]

Preprocessing: - Extracted pressure signals only - Interpolated to 128 values per sample - Normalized using z-score normalization

Feature Vector Generation: - HWT constructed using orthogonal transform pairs (DCT, DHT, HAAR, HADAMARD, KEKRE) [14][15] - 48 coefficients extracted (first 16 + middle 32 samples) - Quantized into 300–500 symbols using K-means clustering [16]

4. HIDDEN MARKOV MODEL TOPOLOGIES

- 1) **Ergodic HMM:** Fully connected transitions - Capable of modeling complex temporal dependencies - Higher training complexity and time
- 2) **Left-to-Right HMM:** Unidirectional transitions only - Models progression naturally over time - Simpler structure, faster convergence
- 3) **Configuration:** Hidden states: 3, 4, 5 - Observation symbols: 300, 400, 500 - Uniform initial probabilities - Baum-Welch (EM) training algorithm [7]

5. EXPERIMENTAL METHODOLOGY

Training: Separate HMMs trained per user per topology (10–15 genuine samples)

Testing: Evaluated using 5 genuine and 20 forged signatures

Metrics: FAR, FRR, EER, convergence time, memory usage

Tools: MATLAB HMM toolbox, custom feature extractor

6. RESULTS AND COMPARATIVE ANALYSIS

- 1) **Accuracy (EER):** Ergodic HMM: 5.1% ($\pm 0.6\%$) - Left-to-Right HMM: 6.9% ($\pm 0.8\%$)
- 2) **Convergence Speed:** Left-to-Right: 20–25 iterations - Ergodic: 35–50 iterations

- 3) **Computational Cost:** Left-to-Right: ~30% fewer parameters - Ergodic: More memory usage due to dense transition matrices
- 4) **State Impact:** Accuracy increases up to 4 states; marginal after 5 - Left-to-Right saturates earlier
- 5) **Symbol Sensitivity:** Optimal around 300–350 symbols - No major improvement beyond 500 [17]
- 6) **Visualization:** ROC curves: tighter for Ergodic - Loss plots: slower convergence in Ergodic

7. DISCUSSION

These results affirm that topology selection significantly affects HMM performance in biometric systems. Ergodic HMMs provide superior accuracy due to their flexible transition dynamics but at higher computational cost [18].

Left-to-Right HMMs are more suitable for real-time systems or mobile platforms where resources are constrained [19]. For high-stakes authentication (e.g., banking or forensics), Ergodic models offer superior reliability, whereas Left-to-Right models suit constrained environments.

8. CONCLUSION

This paper systematically compares Ergodic and Left-to-Right HMM topologies for online signature verification using consistent feature inputs. The findings suggest: - Ergodic HMMs outperform in accuracy (lower EER) - Left-to-Right models train faster and use fewer resources - Optimal configuration: 4–5 states, 300 observation symbols

Future work should explore hybrid or adaptive HMMs and compare their performance with deep neural models integrated with temporal modeling.

CONFLICT OF INTERESTS

None.

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