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SOME NEW RESULTS ON SEMITOTAL-BLOCK GRAPHS

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ABSTRACT

In this paper, we obtain characterization of graphs is 3- minimally nonouterplanar in terms of forbidden sub-graphs. In addition, we present characterization of graphs whose semitotal-block graph is n-minimally nonouterplanar and also 2n-minimally nonouterplanar.

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1. INTRODUCTION

In [2] kulli introduced the concept of the semitotal-block graphs and total-blackgraps.In [3] and [4], the planarity and outerplanarity of these graph valued functions were discussed. In[5], one finds the minimally nonouterplanarity of these graph valued functions. In [1], D.G.Akka and M.S.Patil finds the 2-minimally nonouterplanarity of these graph valued functions. In [6], M.H. Muddebihal, jayashree.B.Shetty and Shabbir Ahmed finds the 3-minimally nonouterplanarity of these graphs valued functions. In this paper we obtain the characterizations of graph whose semitotal-block graphs are 3–minimally nonouterplanarity in terms of forbidden subgraph and n-minimally, 2n-minimally nonouterplanar semitotal-block graphs.

The folowing defination will be noted for later use. Agraph G is called a block if it has more then one vertex, is connected and has no

cutvertices block of a graph G is a maximal subgraph of G which itself a block.

If B={u1, u2_ _ _ _, ur ; r> 2} is a block of [G] then we say thar vertex u1 and block B are incident with each other as are u2 and B so on.

If two distinct blocks B1 and B2 are incident with a common cutvertex, than they are adjacent blocks. The vertices and blocks of a graph are called the members.

The following will be useful in the proof of our results

Theorem A[6], the semitotal –block graph Tb[G] of a connected graph G is minimally, nonouterplanar if and only if.[1] or[2] or [3] holds.

1) G has exactly three cycle and each cycle is block

Or

2) G is either P4+K1 of K4-X.Cn

Or

3) G is a Cycle Cn [n>6] together with a diagonal edge joining a pair of vertices of length [n-3].

2. FORBIDDEN SUBGRAPHS

By using theorem A, we now characterizations of graphs whose 3-minimally nonouterplanar semitotal-block graphs in terms of forbidden subgraph as follows.

Theorem 1: A connected planar graph G is 3-minimally nonouterplanar semitotal-block if and only if G has no subgraph homeomorphic to

1) A graph G which has exactly four blocks and each block is a Cycle.

Or

2) G has exactly three blocks in which one blocks is isomophic to K4-X and two blocks are Cycles

Or

3) G has exactly two blocks in which each block is isomorphic to K4-X or P5+k1 or H1 or H2 {See fig 1]

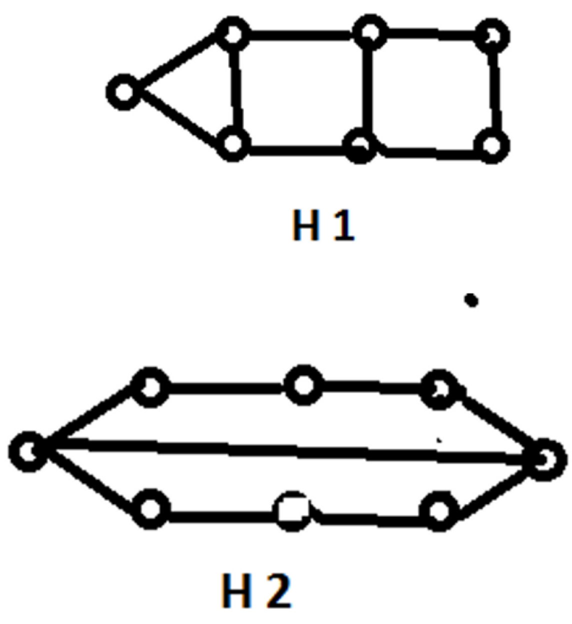

Figure 1

Proof:- If G is a connected planer graph with 3-minimally nonouterplnar semitotal-block graph. Therefore we need to show that all graphs isomorphic to P5+k1 of G has exactly four blocks and each block is homeomorphic to cycle with P>3 vertices or G has exactly three blocks in which one block is isomorphic to K4-X and two blocks which are homoemorphic to cycles with P>3 vertices or G has exactly two blocks in which each block is isomorphic to K4-X and remaining blocks of G are edges of G has homeomorphic to H1 or H2 for all the above conditions, G have no 3-minimally nonouterplanar semitotal-block graph. This follows from theorem A.

Since graph isomorphic to P5+K1 is a block which contains exactly four interior regions and is also a maximal outerplanar in condition 1 G has four cycles of length Cn [n>3] a blocks, in condition 2 G has four cycle as three blocks, in condition 3 G has four Cycle as exactly two blocks each isomorphic to K4-X and remaining blocks are edges, graph homeomorphic to h1 has a cycle Cn [n>7] together with two diagonal edges each joining a pair vertices of length two and three and graph homeomorphic to H2 has a cycle Cn [>8] together with a diagonal edge joining a pair of vertices of length at least [n-4].

To prove sufficiency, assume G is 3-minimally nonouterplanar graph which does not contain subgraph homeomorphic to P5+k1 or G has exactly four blocks and eace block is a cycle or G has exactly three blocks in which one block is isomorphic to K4-X and two block are cycles or G has exactly two blocks in which each block is isomorphic to K4-X remaining blocks of G are edges or H1 or H2.

3. WE CONSIDER THE FOLLOWING CASES

Case1 Suppose G has four cycles. Then we have two following subcase of case 1.

Subcase 1.1: Assume G is a block which contains four cycles. Then G has subgraph homeomorphic to P5+K1, a contradiction. Thus g has a P4+K1 as a block which contains three cycles.

Subcase 1.2: Assume G has exactly two blocks in which each block is isomorphic to K4-X and remaining blocks of G are edges. Let V1 and V2 are two cutvertices in G such that vertices v1 and v2 lies on exactly two blocks each. In which one block is K4-Xand other block is an edge then G has subgraph isomorphic to condition 3.

Subcase1.3: Assume G has exactly three blocks in which one block is isomorphic to K4-X and remaining two blocks are cycles Cn [n>3], Let G has exactly one cut vertices of degree >6, such thar vertex V lies on three blocks in which one block is isomorphic K4-X and remaining two blocks are cycle Cn [n>3]. Than G has a subgraph isomorphic to condition 2.

From the above subcase 1.2 and 1.3 we conclude that g has exactly two blocks such that one block is isomorphic to K4-X and other one is a cycle Cn [n>3].

Subcase 1.4: Assume G has exactly four cycles as blocks. Then consider the following subcases of subcases 1.4

Subcase1.4.1: Assume G has four blocks. Let v be a cut vertices in G such that v lies on four blocks in which each block contains a cycle. Than G has a subgraph homeomorphic to condition 1, a contradiction.

Subcase 1.4.2: supposes G has exactly four blocks as cycles and remaining blocks are edges. Let V1, V2 and v3 are the vertices in G such that vertex is v1 lies on exactly three blocks in which each block contains a cycle, the vertex in v2 lies on exactly two blocks in which one blocks is a cycle and other is an edges. Similarly vertex V3 lies in exactly two blocks in which one block is a cycle and other is an edge. Than G has a subgraph homeomorphic to condition 1, a contradiction.

Subcase 1.4.3: Suppose G has exactly four blocks as cycles and remaining blocks are edges. Let v1,v2,v3, and v4, are the cutverties in G, such that each cutvertex vi [i=1,2,3,4] lies on exactly two blocks in which one block is a cycle and other block is an edge. That G has a subgraph homeomorphic to condition a contradiction.

From the above subcase 1.4.1,1.4.2 and 1.4.3. We conclude that G has exactly three cycles as blocks.

Case 2: Suppose G has atleast two diagonal edges. Then there are two subcases to consider depending on whether the two diagonal edges exits in one cycle or in two different edge disjoint cycles.

Subcsase 2.1: Assume two diagonal edges exist in one cycle than G has a subgraph isomorphic to h1 a Contradiction.

Subcase 2.2 Assume two diagonal edges exist in different edge disjoint cycles. Than G has subgraph homeomorphic to condition3, a contradiction in each subcase we have a contradiction Hence G has exactly one diagonal edge. Than we discuss in the following case.

Case 3: Suppose has exactly one diagonal edge exist in a cycle Cn [n>8] together with a diagonal eged hoining a pair of vertices of length [n-4]. Than has a subgraph homeomorphic to H2, a contradiction.

From above cases we conclude that G is a Cycle Cn [n>6] together with a diagonal edge joining a pair of vertices of length [n-3].

Thus by theorem A,G has 3-minimally nonouterplanar semitotal-block graph.

Hence the Proof.

THEOREM 2: For any connected graph G with n number of blocks, i [Tb[a]=n if and only if each block is a cycle Ck, [K>3].

Proof: Suppose Tb [G] is n-minimally nonouterplanar.

Than Tb [G] is planar.

We consider the following cases

Case 1: Assume G is three with n blocks. Than every block of Tb [G] is triangle.

Hence Tb [G] is outerplanar, a contradiction.

Case 2: Assume G is not a tree. Than we consider the following of subcase 2.

Subcase 2.1: Suppose G has n blocks in which atleast one block is isomorphic to K4-X and remaining blocks are cycles of length Than in planar embedding of Tb [G] in any plane i[K4-X]=2 and each cycle in Tb [G] gives a wheel since every wheel is a minimally nonouterplaner. Thus i [K4-X] .n a contradiction.

Subcase 2.2: Suppose G has blocks in which atleast one block is isomorphic to K4 or K2,3 and remaining blocks are cycles of length Ck [K>3] Than in Planer embedding of Tb [G] In any plane Tb [G] is nonplanar, a condiction.

Subcase 2.3: Suppose G has n number of blocks in which atleast one block is isomorphic to P4+k1 and remaining blocks are cycles of length Ck [K>3]. It is easy to see that in planar embedding of Tb [G], i [P4+k1] =3 and each cycle in Tb [G] gives a wheel. Thus Tb [G]>3, a Contradiction.

Subcase 2.4: Suppose G has n number of block in which atleast one block is isomorphic to cycle Ck [K>3]. And remaining blocks are edges which are adjaction of a cycle Ck [K>3]. In planar embedding of Tb [G] in any plane i [Tb [G]]=1, a contradiction.

Conversely, suppose G has n blocks and each block is a cycle Ck (K≥3). Then we prove that I [Tb (G)]=n, this proved by mathematical induction on the number of blocks n of G it is easy to observe that semitotal block graph of a cycle Ck (k≥3) is a wheel with I [Tb(G)] =1.

As the inductive hypotheses,. it the semitotal block graph with m number of blocks and each block is a cycle Ck(k≥m), we consider the smallest cycle C3. In Tb(G), I [Tb(G)]=m.

We now show that the semitotal block graph of G with n=m+1 blocks with (m+1)-minimally nonouterplanar, let G has {B1, B2……, Bm,bm+1} number of blocks each block is isomorphic to a cycle C3. Let {b1,b2…..,bm,bm+1} are the block vertices corresponding to the cycles C3. Without loss of generality if we delete an end block BM+1 as a cycle C3 and also it is corresoponding vertices edges, the resulting graph G has m number of blocks and each block is a cycle C3. Thus by inductive hypothesis i[Tb(G)]=m. if we rejoin an end block BM+1 as a cycle C3 and also its corresponding vertices and edges. The resulting graph G has (m+1)-number of blocks and each block is isomorphic to a cycle C3.

In any planner embedding of Tb (G), each block {B1, B2……..Bm, Bm+1} is adjacent to its corresponding block vertices {b1,b2…….bm, bm+1] thus each cycle C3 forms a wheel. We know that every wheel is a minimally nonouterplanner.

Hence i [Tb(G)]=m+1. Thus Tb(G) is (m+1)- minimally non outerplanner, since, Tb(G) is m-minimally nonouterplanner, with m=n number of blocks.

In G then above argument can be extended to each block which is a cycle of length greater than 3. Hence it satisfies the above hypothesis.

Hence the proof.

THEOREM 3: for any connected graph G with n number of blocks, i[Tb (G)]=2n if and only if each block is a K4-X

Proof: suppose Tb (G) is 2n-minimallynonouterplanner than Tb (G) planner.

We consider the following cases.

Case 1: assume G is a tree. Than every block of Tb(G) is a triangle. Hence Tb (G) is outerplanner, a contradiction.

Case 2: Assume G is not a tree. we consider the following subcase of case 2.

Subcase 2.1: suppose G has n block in which at least 1 block is isomorphic 2 K4 of K2,3 and remaining blocks are K4-X .then in planner embedding of Tb (G) in any plan Tb (G)] is non planner, a contradiction..

Suppose G has n blocks in which at least 1 block is isomorphic to CK (K≥3) and remaining blocks are K4-X then Tb (G) each (k4-X) gives i[K4-X] equal to 2 and each cycle gives a w2heel. Since every wheel ia a minimally nonouterplaner thus i[Tb(G)] < 2n, a contradiction.

Subcase2.3: suppose G has n blocks in which at least one block is isomorphic 2 P4+k1 and remaining blocks are K4-X. In planner embedding of Tb (G) in any plan each i[K4-X]=2 and i[P4+k1)=3. The inner vertex number increase in planner embedding of Tb (G). Thus i [Tb G)]>2n, a contradiction.

Subcase 2.4: suppose for a connected graph G has n blocks and thbei9r exist a block which is isomorphic to K4-X and remaining blocks are edges. In formation of Tb (G)], each block of G which is an edge generate triangles which are outer planner in Tb (G)] and in planner embedding of Tb (G)] in any plane, i[K4-X]=2, thus i [Tb (G)=2, a contradiction.

Conversely, suppose G has n number of blocks is isomorphic to k4-x . Than we use Mathematical induction on n number of blocks. Suppose n=1 then G=K4-x Let has V1.V2.V3. and V4 Vertices, without lose generality the diagonal edge 'e' is having V1 and V3 as adjacent vertices. In planar embedding of Tb [G] in any plane either V1,V2, of V1, V4 or V3, V4 are only two inner vertices in Tb (a). Thus i [Tb (a)]=2, Hence Tb (a) is 2-minimally nonouterplaner.

Thus the result is true for n=1

Assume that the result is true n=m then G has m number of blocks each block is isomorphic to K4-X Thus each in Tb (G) gives inner vertex number 2.

Since there are m blocks clearly Tb (G) is 2m-minimally nonouterplaner, i [Tb (G)] =2m

It is also for n=m.

Suppose n=m+1. Then (a) has m+1 Blocks, each block is a K4-X we have to prove that I [Tb (a)] =2m+2,

let G1 has m blocks each block is a K4-X such that each K4-X have Vi, Vi,+1 Vi+2 Vi+3 (i=1,2,………….n) vertices with diagonal edge (i=1,2,……………..n) Joining Vi and Vi+3.

{B1, B2,……………B) are the number of blocks and each block is isomorphic to a K4-x {b1,b2,…………bm} are the block vertice corresponding to each Ku-x if we add one more K4-x with vertices V1,V2,V3, and V4 such that Vi=Vi+3 then the resulting graph G having (m+1) number of blocks and block is isomorphic to a K4-x without loss of generality suppose a diagonal e, edge is incident to the vertices Vi+3 and V3 .if we delete an vertex V3 by deleting all the edges incident to V31. We obtain a graph G, having m number of blocks and each block is isomorphic to K4-X remaining blocks are edges. By inductive K4-x and remaining blocks are edges. By inductive hypothesis Tb (a1) is 2m-minimally nonouterplanar now rejoin a vertex V3 by joining all edges incident to V3, resulting a graph G having (m+1) number of blocks and each blocks is a K4-x

In planer embedding of Tb (G) in any plane that is any one vertex from each K4-x and other is a block vertex corresponding to each K4-x we only two inner vertices in K4-x Also the end block B m+1 as K4-x have 2 inner vertices that is one vertex V2 from the end block K4-x and other is the block bm+1 corresponding to a K4-x

Hence i [Tb (a)] =2(m+1) =2m+2 inner vertices.

Thus Tb (G) has 2m+2 minimally nonouterplanar.

Hence the Proofs.

CONFLICT OF INTERESTS

None.

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None.

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