

MELTING HEAT TRANSFER IN MAGNETOHYDRODYNAMIC SLIP NANOFLUID FLOW OVER STRETCHING SHEET WITH OUTER VELOCITY

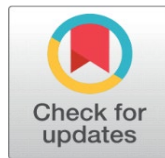
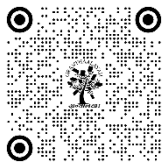
Seema ¹, Kuldeep Tanwar ², Vikas Poply ³✉, Rakesh Kumar ⁴

¹ Research Scholar, MVN University, Palwal, Haryana, India

² Associate Professor MVN University, Palwal, Haryana, India

³ Assistant Professor Department of Applied Mathematics, KLP College, Rewari, India

⁴ Associate Professor, Department of Chemistry, Government P. G. Nehru College Jhajjar, Haryana, India



Corresponding Author

Vikas Poply, vikaspoply@gmail.com

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ABSTRACT

Melting heat transmission characteristics of magnetohydrodynamic (MHD) nanofluid flow over a stretching sheet, considering the effects of velocity slip and an external flow (outer velocity) has been examined. Similarity transitions converted the basic partial differential equations of nanofluid into a set of nonlinear ordinary differential equations. The Runge–Kutta–Fehlberg method is employed in conjunction with a shooting technique to numerically solve these equations. The analysis demonstrates that a narrower thermal boundary layer is because of an increase in the melting parameter. The fluid velocities decrease as the velocity slip and magnetic parameter increases. The velocity distributions are considerably influenced by the presence of an outer velocity, which in turn increases the convective heat transfer rates. Concentration decreases with an increase in Lewis number. These discoveries have practical implications for industrial processes that involve MHD nanofluids, including polymer extrusion, materials processing, and cooling systems, where precision heat transfer control is a critical parameter. This research will be essential for the development of heat sinks and cooling devices of varying morphologies, which will enhance the heat transmission capabilities of nanofluids and thereby expand their engineering potential.

Keywords: Melting Heat Transfer, Nanofluid, Slip Velocity, Outer Velocity, MHD, Stretching Sheet

1. INTRODUCTION

The study of heat transmission over stretching surfaces is a substantial field in fluid dynamics, with applications in industries such as metallurgy, polymer processing, and nanotechnology. One of the primary concerns in polymer extrusion processes is the transfer of heat from a stretching surface. This occurs when a product travels through a die and subsequently enters a cooling fluid to be cooled below a predetermined temperature. The ultimate product's characteristics are significantly influenced by the rate at which these objects cool. Crane [1] was the first to investigate the momentum boundary layer associating with the linear elongation of a sheet in cooling fluids of this nature. A situation of linear horizontal velocity with a fixed temperature impacted by suction or airflow has been reported by Gupta and Gupta [2]. Afzal [3] expanded his research to investigate the heat transmission through a power law stretching sheet. Ali

[4] investigated the implications of power-law contact velocity and temperature fluctuations on heat transmission features, taking into account two basic thermal boundary conditions: uniform surface heat flux and changing surface temperature. Elbashbeshy [5] examines the numerical solutions of the laminar boundary layer models that describe heat and flow in a stationary fluid over an exponentially expanding surface with suction.

In the range of 1–100 nanometers, a nanofluid is a base fluid (such as water, oil, or ethylene glycol) that contains suspended nanoparticles, which are typically metals, oxides, or carbides. In comparison to conventional fluids, these nanoparticles improve the thermal conductivity of the fluid, thereby enhancing its capacity to transfer heat. This phenomenon is particularly significant in industries such as electronics cooling, polymer extrusion, metal forming, and glass fiber production, where the quality of the product and the performance of the system are directly influenced by effective temperature control. Experimental findings indicate that nanofluids have a greater thermal conductivity than basic fluids. A convincing explanation for the extraordinary rise of the thermal conductivity and viscosity has not been found, according to Buongiorno [6], who conducted a thorough investigation of convective transport in nanofluids. Khan and Pop [7] assessed the nanofluid boundary-layer flow through a stretching sheet and their nanofluid model incorporates thermophoresis and Brownian motion. Rana and Bhargava [8] performed a numerical study focusing on nanofluid flow and heat transfer across a nonlinearly stretching surface.

The study of heat transfer in magnetohydrodynamic (MHD) nanofluid flows over stretching sheets has garnered significant attention due to its applications in industrial processes like polymer extrusion, cooling of electronic devices, and materials processing. Ghasemi et al. [9] shown a numerical investigation on the implications of nanoparticles MHD fluid across a stretching sheet with solar radiation. A theoretical investigation by Sadighi et al. [10] is carried out to examine the thermodynamic first law in the context of MHD nanofluid flow over a porous stretching surface embedded in a permeable medium, taking into account the effects of thermal radiation, heat generation or absorption. Pourmehran et al. [11] conducted a study on the heat and flow transmission of nanofluid flow that is induced by an external magnetic field.

All the studies mentioned above carried out their discussions by assuming the no-slip boundary condition. This means they believed the fluid sticks completely to the solid surface it touches, which is a basic rule in the Navier–Stokes theory. However, there are some situations where this rule doesn't apply. Sometimes, a fluid can slip a little along a stretching surface, especially when it contains particles, like in emulsions, suspensions, foams, or polymer solutions. Bhattacharyya et al. [12] have investigated the impact of slip on the stagnation-point flow and heat transmission of a boundary layer toward a shrinking sheet. This slipping of the fluid, called velocity slip, has been noticed in certain cases by Mukhopadhyay [13]. Mishra and Kumar [14] investigated the effects of thermal slip and velocity on the flow of MHD nanofluid past a stretching cylinder, which includes viscous dissipation and Joule heating. Zubaidi et al. [15] conducted an investigation of the slip circumstance in MHD nanofluid flow over an extending sheet in the presence of viscous dissipation.

Monitoring the degree of heat transfer is essential for the quality, safety, and efficacy of products in numerous engineering processes. An effective method of managing this is to introduce an outer flow over the surface of the material. Poply et al. [16] done an analysis of the effects of fluid properties on MHD flow and heat transmission past a nonlinearly stretching sheet with free stream. The influence of exterior velocity MHD slip flow on the transfer of heat of nanofluid past a stretching cylinder has been investigated by Vinita and Poply [17]. The fluid properties of an erratic MHD free stream flow over a stretching sheet in the presence of radiation were investigated by Taneja et al. [18].

Efficiency and quality control are essential in high-temperature processes such as additive manufacturing, soldering, casting, and melting of ice in contact with heated plates. It is essential to comprehend the manner in which heat is conveyed during melting on a moving (stretching) surface. Ishak et al. [19] studied the transmission of heat from a moving surface to a constant laminar flow. Mabood and Das [20] have investigated the melting heat transmission in the hydromagnetic flow of a nanofluid over a stretching sheet with exposure to radiation and slip. Pardeep et al. [21] examined the influence of melting heat transmission and outer-velocity of nanofluid on a stretching sheet.

Incorporating effects such as melting, velocity slip, and external flows adds complexity to the problem but provides a more realistic representation of practical scenarios. Previous studies have explored various aspects of MHD nanofluid flows, including the effects of thermal radiation, viscous dissipation etc. However, comprehensive analyses that simultaneously consider melting heat transmission, velocity slip, and outer velocity effects remain limited.

2. MATHEMATICAL FORMULATION

Consider a steady, two-dimensional, laminar flow of an incompressible, electrically conducting nanofluid over a stretching sheet. An external magnetic field is applied perpendicular to the sheet. The nanofluid experiences melting at the interface, and a velocity slip with outer flow condition is imposed at the boundary. Figure 1 illustrates a schematic representation of the geometry.

Figure 1

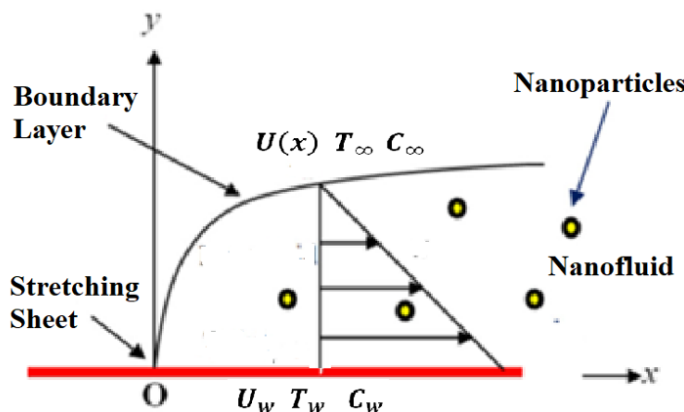


Figure 1 Schematic diagram of problem

The governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\partial U}{\partial x} + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} (u - U) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left\{ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right\} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

The boundary conditions for the nanofluid flow are

$$u = bx + l \frac{\partial u}{\partial y}, \quad k \frac{\partial T}{\partial y} = \rho [LH + C_s (T_m - T_0)] v(x, 0), \quad T = T_m, C = C_w \text{ at } y = 0 \quad (5)$$

$$u \rightarrow U(x), \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \quad (6)$$

Here K , LH , T_0 , ρ and C_s are thermal conductivity of the nanofluid, latent heat of the nanofluid, solid surface temperature, density, and heat capacity of the solid surface respectively.

The similarity and dimensionless variables

$$\psi = \sqrt{bv} xf(n), \quad \eta = \sqrt{\frac{b}{v}}y$$

$$\theta(n) = \frac{T - T_\infty}{T_m - T_\infty}, \quad \phi(n) = \frac{C - C_\infty}{C_w - C_\infty},$$

Let define the stream function $\psi(x, y)$

$$v = -\frac{\partial \psi}{\partial x}, \quad u = \frac{\partial \psi}{\partial y}$$

The governing equations reduce to

$$f''' + ff'' - (f'^2 - \lambda^2) - M(f' - \lambda) = 0 \quad (7)$$

$$\theta'' + Pr\{N_b\phi'\theta' + f\theta' + N_t\theta'^2\} = 0 \quad (8)$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + Le\phi'f = 0 \quad (9)$$

The corresponding boundary conditions are given:

$$Me\theta'(\eta) + Prf(n) = 0, \quad f'(\eta) = 1 + Sf''(\eta) \quad \theta(\eta) = 1, \quad \phi(\eta) = 1 \text{ at } \eta = 0 \quad (10)$$

$$f'(\eta) \rightarrow \lambda \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (11)$$

The physical parameters that govern the flow are defined as follows:

$$\sqrt{Re_x}Cf_x = f''(0), \quad \frac{Nu}{\sqrt{Re_x}} = -\theta'(0), \quad \frac{Sh}{\sqrt{Re_x}} = -\phi'(0)$$

3. RESULTS AND DISCUSSION

The converted nondimensional boundary layer equations (7) to (9) with wall and free stream boundary conditions (10) and (11) are solved by the Newton-Fehlberg iteration method. The effects of the following parameters on the pertinent flow variables are detailed in detail: the melting heat parameter Me , slip velocity parameter S , outer velocity parameter λ , magnetic parameter M , and Lewis number Le

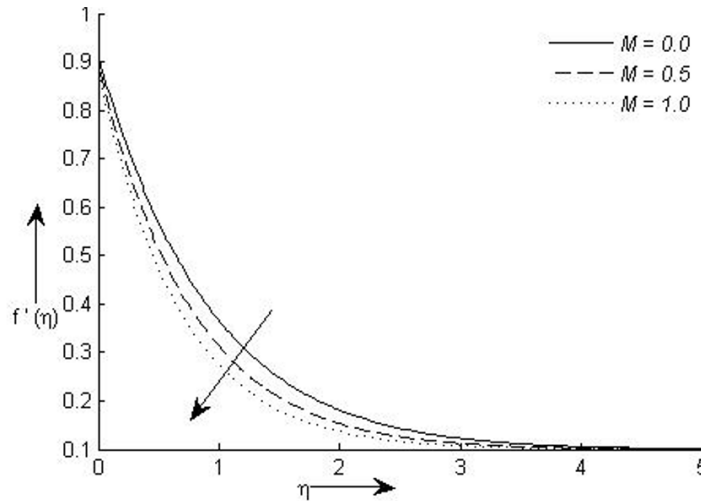
Figure 2**Figure 2** Velocity profile for various M

Figure 2 illustrates the influence of the magnetic parameter M on the velocity profiles of the fluid. It is observed that as the magnetic parameter M increases, the velocity of the fluid decreases progressively throughout the boundary layer region. In magneto-hydrodynamic (MHD) flow, the application of a transverse magnetic field to an electrically conducting fluid generates a resistive force known as the Lorentz force. This force acts in the direction opposite to the fluid motion. As the magnetic parameter M increases — which represents the ratio of electromagnetic force to viscous force — the strength of the Lorentz force also increases. The enhanced Lorentz force opposes the motion of the fluid particles, creating an additional drag force within the boundary layer. As a result, it reduces the momentum of the fluid, leading to a decrease in the velocity profiles near the surface and throughout the boundary layer.

Figure 3 illustrates the effect of the outer velocity parameter λ on the velocity profiles of the fluid. It is observed that as λ increases, the velocity of the fluid correspondingly increases throughout the boundary layer. The outer velocity parameter λ represents the strength of the external or free-stream velocity relative to the stretching or moving surface velocity. As λ increases, it indicates that the external driving force or far-field flow speed is increasing. This enhanced outer velocity induces a stronger momentum transfer from the free stream into the boundary layer region. Consequently, the fluid particles near the boundary layer experience a greater driving force in the direction of flow. This reduces the retarding influence of viscous forces within the boundary layer. As a result, the momentum boundary layer thickness decreases and the velocity within the boundary layer increases.

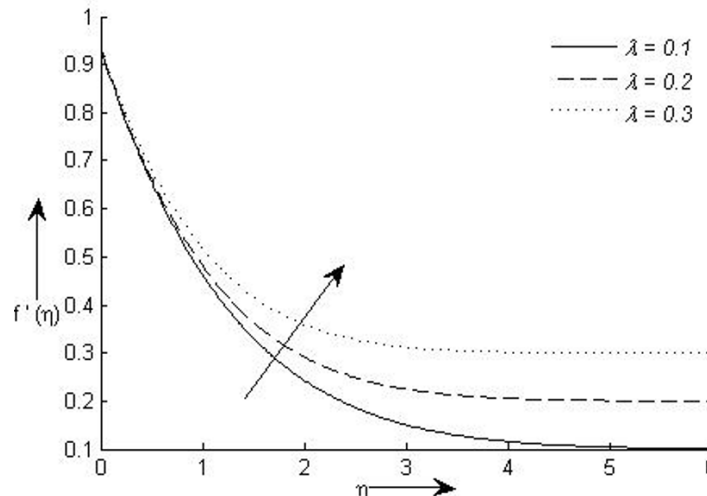
Figure 3

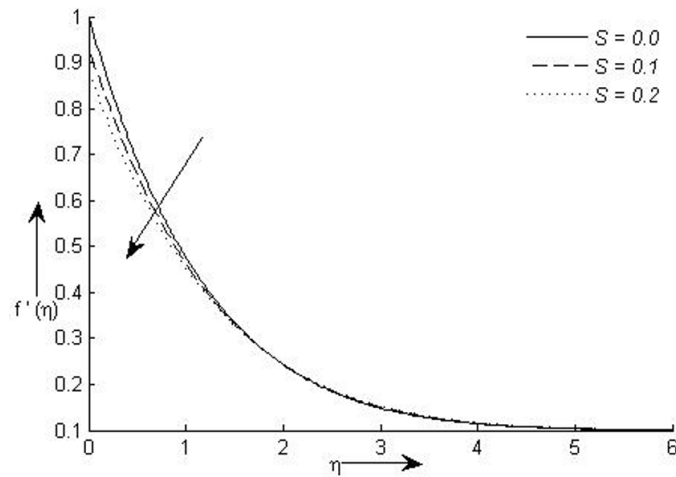
Figure 3 Velocity profile for various λ **Figure 4****Figure 4** Velocity profile for various S

Figure 4 illustrates the effect of the slip velocity parameter S on the velocity profiles of the fluid. From the figure, it is evident that as the slip parameter S increases, the fluid velocity within the boundary layer decreases. In classical fluid mechanics, the no-slip boundary condition assumes that the fluid velocity at a solid surface is equal to the surface velocity. However, in certain engineering applications such as microfluidics, nano-scale flows, or flows over hydrophobic surfaces, a velocity slip condition is applied — meaning the fluid at the surface has a finite velocity relative to the wall. The slip velocity parameter S quantifies the amount of slip allowed at the surface: When $S = 0$ means no-slip condition (fluid velocity matches surface velocity). As S increases, a larger slip occurs, meaning the fluid near the surface moves more slowly relative to a moving/stretching surface. As a result, the momentum transfer from the surface to the fluid reduces. The driving force for fluid motion within the boundary layer diminishes. Consequently, the velocity throughout the boundary layer decreases. This is because a larger slip weakens the interaction between the wall and the adjacent fluid particles, reducing the shear-induced acceleration of the fluid.

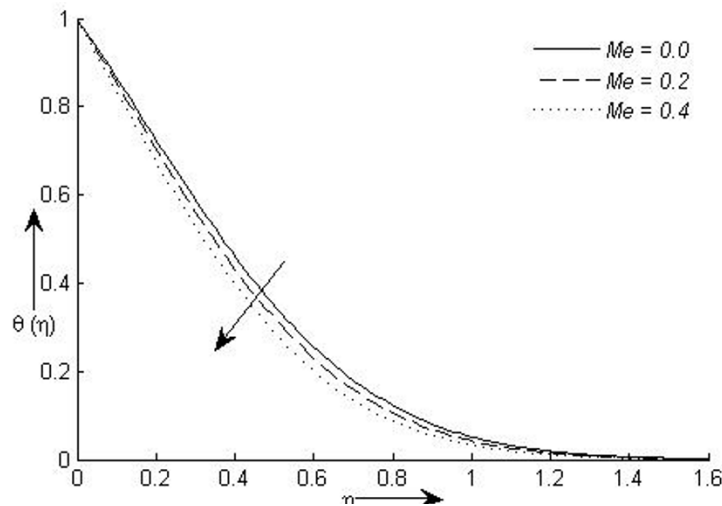
Figure 5**Figure 5** Temperature Profile for various Me

Figure 5 illustrates the effect of the melting heat parameter Me on the temperature profiles within the boundary layer. From the figure, it is evident that as the melting heat parameter Me increases, the temperature of the fluid decreases throughout the thermal boundary layer region. The melting heat parameter Me represents the influence of phase change (melting) at the surface on the heat transfer characteristics of the fluid flow. It is typically defined as a ratio involving the latent heat of melting and the thermal properties of the fluid. As melting occurs at the surface, it absorbs thermal energy from the fluid adjacent to the surface to convert the solid into liquid, a process requiring latent heat. As Me increases: A larger quantity of thermal energy is consumed at the surface for the melting process. This reduces the amount of heat available to increase the temperature of the fluid within the boundary layer. Consequently, the thermal energy absorbed during melting acts as a heat sink, lowering the temperature distribution within the boundary layer region.

Figure 6

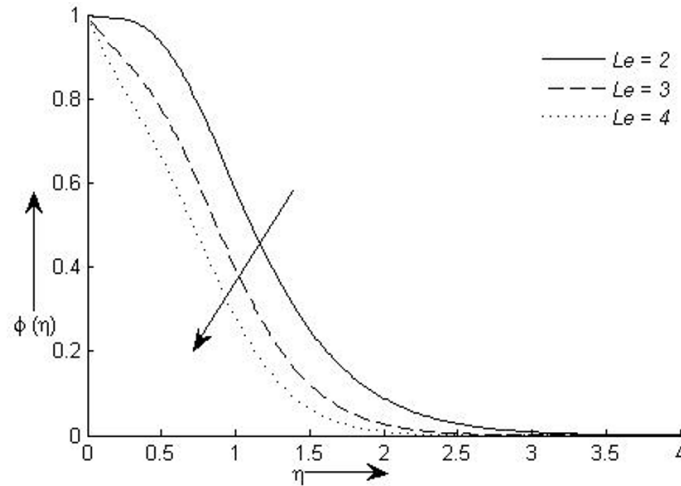


Figure 6 Concentration Profile for various Le

Figure 6 illustrates the effect of the Lewis number Le on the concentration profiles within the boundary layer. From the figure, it is evident that as the Lewis number increases, the concentration of the fluid decreases throughout the concentration boundary layer. With increasing Le , the ability of species to diffuse away from the surface diminishes. This leads to a thinner concentration boundary layer and a more rapid decay of concentration away from the surface. Consequently, the concentration profiles within the boundary layer decrease more sharply with increasing Le .

4. CONCLUSION

- 1) Velocity decreases with increase in M and S ,
- 2) Velocity increases with increase in λ .
- 3) Temperature decreases with increase in Me .
- 4) Concentration decreases with increase in Le .

CONFLICT OF INTERESTS

None.

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