NEW LIMITS ON DOMINATING SETS' ENERGY SUM IN PARTICULAR GRAPHS

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ABSTRACT

The energy sum of a dominating subset of connected & undirected graphs was investigated in this article. A graph Ġ = (X, Y) consists of edges & vertices, which are known as nodes. A subset in a graph's nodes is known as the dominant set Đ; every node in the graph is either within Đ or is adjacent in Đ. The dominating number of network Ġ, abbreviated as γ(Ġ), is the least cardinality for the dominant set. The energy of a simple, connected graph Ġ is calculated by adding its absolute eigen values. The eigen value of a graph's dominating matrix Đ(Ġ) is it's eigen value. In this article, we utilized code for MATLAB and a strategy to investigate the bounds of the energy sum using the dominating matrix.

Keywords: Domination number, Adjacency Matrix, Dominant Matrix, Eigen Values, Energy Sum.

Mathematics Subject Classification: 05C50

1. INTRODUCTION

Numerous real-world scenarios can be faithfully represented by a straightforward point-and-line diagram. The graph consists of vertices (also known as nodes) and edges [1]. Dominance is a very interesting topic in graph theory. The number of vertices Đ with every vertex within Đ or next to a one of its members is generally referred to as the dominant set of a graph.

Quantum chemistry is where the idea of a graph's an overview energy [3–4] first appeared. In the 1930s, E. Hückel's theory of molecular orbital's, which incorporated graph eigen values, illustrated the chemical implications of graph theory. The geometry of the appropriate linear transformation is shown by eigen values and eigenvectors [2].

Eigen values have applications in structural analysis that go beyond theoretical chemistry. Eigen values can be used to find cracks or other defects in a solid. One common method used by oil companies to locate oil is eigen value analysis. eigen values are also used to identify new and improved designs in the future. If x_i ∈ Đ [6], then 1 should be used in place of the a_{ii} member in Ġ's matrix of adjacency (referred to as A(Ġ)). The adjacency matrix may be used to generate the matrix for the set Đ, indicated by A(Ġ).

In this paper, we investigate the dominant set and the corresponding matrix, Đ(Ġ). A graph can be connected to several matrices, and the spectrum of these matrices reveal valuable information about the representation [1-2].

This study shows the numerous basic graph energy and their limits. This paper investigates undirected, finite, and simple graphs. We examine how one graph's energy limitations relate to another's energy. Additionally, the provided MATLAB code, Python program, and technique can be used to construct the dominating matrix, eigen value parameters, and energy of particular graph families. We'll go over all the definitions that are required here.

2. PRELIMINARY RESULTS

Definition 2.1 Assume that the graph Ġ = (X, Y) exists. If a vertex x in X dominates any other vertex of its closed neighborhood N[x], it is said to dominate both itself and its neighbours. Consequently, x has control over the points of G given degree 1 + deg x. If at least one vertex in Đ dominates every vertex in Ġ, then set Đ is a dominating set of Ġ.

Definition 2.2 A square matrix is of order n with a (i, j)[th] entry of 1 if i dominates j or if j dominates i, and equal to zero otherwise, is the dominant matrix $Đ_{ij}(Ġ)$ of a graph. That is

$$Đ_{ij}(\dot{G}) = \begin{cases} 1, & \text{if i dominates j (or) i is dominated by j} \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.3 The energy sum of a connected graph is equal to the sum of the absolute eigen values of the graph.

$$E(\dot{G}) = \sum_{i=1}^{n} |\lambda_i|$$

Definition 2.4 The sum of all of the absolute values of the eigen values of $Đ_{ij}(Ġ)$ is the energy sum of the dominating matrix of a graph. To put it another way , n represents the n eigen values of the dominating matrix of a graph Ġ of order n if λ_i , i = 1, 2, 3,...,n. Next, the graph's energy $E_Đ(Ġ)$ is provided by

$$E_{Đ}(\dot{G}) = \sum_{i=1}^{n} |\lambda_i|$$

Definition 2.5 A simple connected graph's adjacency matrix is made up of rows and columns that are named by the graph's vertices. Whether or not x_i and x_j are neighbors determines whether the value at location (x_i, x_j) is 1 or 0.

$$A(\dot{G}) = \begin{cases} 1, & x_i \text{ adjacent to } x_j \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.6 The graph joins K_1+C_n, where K_1 is the a singleton graph & C_n is a cycle graph, can be used to characterize the wheel W_n. This creates a (n,1) conical graph. The graph joining K_1+C_n, in which K_1 is the full graph containing a single vertex and C_n represents the cycle graph, represents the wheel graph W_n, which is a (n,1) conical graph.

Definition 2.7 A book graph (B_n) consists of n quadrilaterals that share a single edge. In other words, it is a Cartesian product of S_{n+1} & P_2, where P_2 is a two-node route graph and S_n is a star network.

Definition 2.8 A tree with m nodes, a single vertex having size m-1 and the remaining ones with vertex degree 1, is the star graph S_m ($S_m = K_{1,m-1}$) of degree m.

Definition 2.9 When P_m is a path graph of nodes m and $\overline{K_n}$ is the void network on nodes n, the graph joining $\overline{K_n} + P_m$ is called the graph of Fan (F_n). m=1 denotes a normal fan graph, m=2 a double fan network, and so forth.

3. ENERGY SUM OF VARIOUS GRAPHS

We display the energy sum values for a variety of graphs in this section. Additionally, we compare the energy sums of its adjacency matrix and dominant matrix.

3.1.1 ALGORITHM FOR FINDING THE DOMINANT MATRIX FOR WHEEL GRAPH:

Procedure:

$X \leftarrow \{x_1, x_2, x_3, ..., x_m\}$

$Y \leftarrow \{y_1=x_1x_2, y_2=x_1x_3, y_3=x_1x_4, \ldots\}$
if m>2
for k =1 to m
    $x_1x_k \leftarrow 1$
    $x_{k+1}x_1 \leftarrow 1$
endfor
for k =2 to m
for l =2 to m
    $x_kx_l \leftarrow 0$
endfor
endfor
endif
end procedure

3.1.2 PYTHON PROGRAM FOR FINDING THE DOMINANT MATRIX, EIGEN VALUES AND THE ENERGY SUM OF WHEEL GRAPH:

```
import numpy as np
m=int(input())
mat = [[0] * (m) for _ in range(m)]
for k in range(0,m):
   mat[0][k]=1
   mat[k][0]=1
Đ=np.array(mat)
w, v = np.linalg.eig(Đ)
sum_eig=np.absolute(w)
print(Đ)
print(w)
print(np.sum(sum_eig))
```

OUTPUT: for m= 6

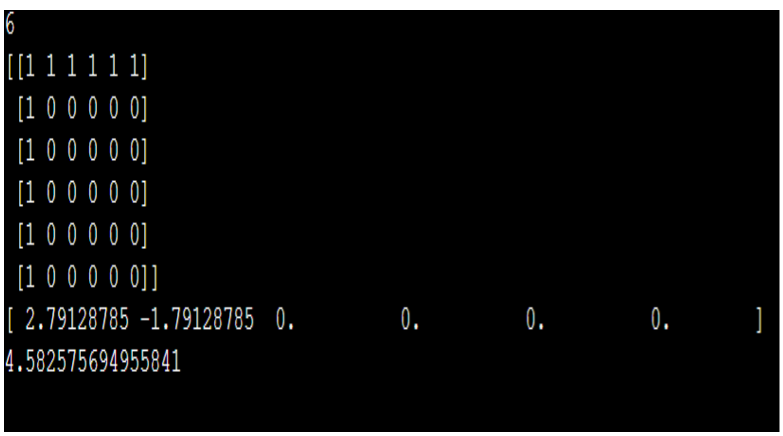


Fig. 1 – python program output- wheel graph

3.1.3 MATHLAB CODE TO GENERATE THE ENERGY OF WHEEL GRAPH:

```
m = ; %enter the number of vertices of m
Đ=zeros(n);
for k = 0:m-1
Đ(1,k+1)=1;
Đ(k+1,1)=1;
end
Đ;
```

eigenvaluesofwheel=eig(Đ);
energy=sum(abs(eigenvaluesofwheel));

RESULT:

Table-1: The table gives the energy sum comparison between Adjacency Matrix and Dominant Matrix of Wheel graph,

W_m	$E_A(W_m)$	$E_Đ(W_m)$
W_3	4	3
W_4	6	3.60555
W_5	6.4721	4.12311
W_6	9.3711	4.5826
W_7	11.2915	5
W_8	12.6448	5.3852
W_9	13.6569	5.74456
W_{10}	15.8421	6.0828
W_{11}	17.5775	6.4031
W_{12}	18.9816	6.7082

3.2 BOOK GRAPH

3.2.1 ALGORITHM FOR FINDING THE DOMINANT MATRIX FOR BOOK GRAPH :

Procedure:

$X \leftarrow \{x_1, x_2, x_3, \ldots, x_m\}$

$Y \leftarrow \{x_1x_2, x_2x_3, x_3x_4, \ldots, x_{m-1,m}\}$

if $m \geq 2$

$x_1x_1 \leftarrow 1$
$x_1x_2 \leftarrow 1$
$x_2x_1 \leftarrow 1$
$x_2x_2 \leftarrow 1$

for k =1 to m

$x_1x_{2k+1} \leftarrow 1$
$x_{2k+1}x_1 \leftarrow 1$
$x_2x_{2k+2} \leftarrow 1$
$x_{2k+2}x_2 \leftarrow 1$
$x_1x_{2k+2} \leftarrow 0$
$x_{2k+2}x_1 \leftarrow 0$
$x_2x_{2k+1} \leftarrow 0$
$x_{2k+1}x_2 \leftarrow 0$

end for
for k =1to 2m
for l = 1to 2m

$u_{k+2}u_{l+2} \leftarrow 0$

end for
end if
end procedure

3.2.2 PYTHON PROGRAM FOR FINDING THE DOMINANT MATRIX, EIGEN VALUES AND THE ENERGY SUM OF BOOK GRAPH:

import numpy as np

```
m = int(input())
mat = [[0] * (2 * m + 2) for _ in range(2 * m + 2)]
if m >= 2:
  mat [0][0] = 1
  mat [0][1] = 1
  mat [1][0] = 1
  mat [1][1] = 1
  for k in range (1, m + 1):
    mat [0][2 * k] = 1
    mat [2 * k][0] = 1
    mat [1][2 * k + 1] = 1
    mat [2 * k + 1][1] = 1
    mat [0][2 * k + 1] = 0
    mat [2 * k + 1][0] = 0
    mat [1][2 * k] = 0
    mat [2 * k][1] = 0
  for i in range(2 * m + 2):
    for j in range(2 * m + 2):
      if k >= 2 and l >= 2:
        mat [k][l] = 0
Đ = np.array(mat)
w, v = np.linalg.eig(Đ)
sum_eig = np.absolute(w)
print(Đ)
print(w)
print(np.sum(sum_eig))
```

OUTPUT: for m= 5

```
5
[[1 1 1 0 1 0 1 0 1 0 1 0]
 [1 1 0 1 0 1 0 1 0 1 0 1]
 [1 0 0 0 0 0 0 0 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 0 0]
 [1 0 0 0 0 0 0 0 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 0 0]
 [1 0 0 0 0 0 0 0 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 0 0]
 [1 0 0 0 0 0 0 0 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 0 0]
 [1 0 0 0 0 0 0 0 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 0 0]]
[ 3.44948974e+00  2.23606798e+00 -2.23606798e+00 -1.44948974e+00
 -1.23259516e-32 -7.72870604e-17  1.93918549e-33  3.42113883e-49
 -6.52820283e-66 -8.73138631e-51  0.00000000e+00  0.00000000e+00]
9.371115440565937
```

Fig. 2 – python program output- book graph

3.2.3 MATHLAB CODE TO GENERATE THE ENERGY OF BOOK GRAPH:

```
m =   ;% enter the total number of vertices
n=2*m+2;
Đ=zeros(m);
Đ (1,1)=1;
Đ(1,2)=1;
Đ (2,1)=1;
Đ (2,2)=1;
for k=1:m
Đ (1,2*k+1)=1;
Đ (2*k+1,1)=1;
Đ (2,2*k+2)=1;
```

```
Đ (2*k+2,2)=1;
end
Đ;
eigenvaluesofbook=eig(Đ);
energy=sum(abs(eigenvaluesofbook));
```

RESULT:

Table-2: The table gives the energy sum comparison between Adjacency Matrix and Dominant Matrix of Book graph,

B_m	$E_A(B_m)$	$E_Đ(B_m)$
B_2	7.6569	6.2925
B_3	10.9282	7.4641
B_4	14	8.4721
B_5	16.9443	9.3711
B_6	19.798	10.1905
B_7	22.583	10.9484
B_8	25.3137	11.6569
B_9	28	12.3246
B_{10}	30.6491	12.9578
B_{11}	33.2665	13.5615

3.3 STAR GRAPH

3.3.1 ALGORITHM FOR FINDING THE DOMINANT MATRIX FOR STAR GRAPH:

Procedure:

$X \leftarrow \{x_1, x_2, x_3, ..., x_n\}$

$Y \leftarrow \{y_1=x_1x_2, y_2=x_1x_3, y_3=x_1x_4, ... \}$

if m>2
for k =1 to m
for l =1 to m

$x_1x_l \leftarrow 1$

$x_{k+1}x_1 \leftarrow 1$

end for
for k =2 to m
for l =2 to m

$u_ku_l \leftarrow 0$

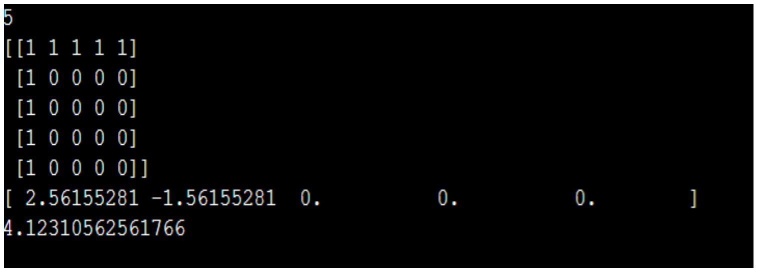
end for
end for
end if
end procedure

3.3.2 PYTHON PROGRAM FOR FINDING THE DOMINANT MATRIX, EIGEN VALUES AND THE ENERGY SUM OF STAR GRAPH:

```
import numpy as np
m=int(input())
mat = [[0] * (n) for _ in range(m)]
for k in range(0,m):
   mat[0][k]=1
   mat[k][0]=1
Đ=np.array(mat)
w, v = np.linalg.eig(D)
```

```
sum_eig=np.absolute(w)
print(D)
print(w)
print(np.sum(sum_eig))
```

OUTPUT: for n= 5



```
5
[[1 1 1 1 1]
 [1 0 0 0 0]
 [1 0 0 0 0]
 [1 0 0 0 0]
 [1 0 0 0 0]]
[ 2.56155281 -1.56155281  0.          0.          0.        ]
4.12310562561766
```

Fig. 3 – python program output- star graph

3.3.3 MATHLAB CODE TO GENERATE THE ENERGY OF STAR GRAPH:

```
m=;   %enter the number of vertices
Đ=zeros(m);
for k=0:m-1
Đ (1,k+1)=1;
Đ (k+1,1)=1;
end
Đ;
eigenvaluesofstar=eig(Đ);
energy=sum(abs(eigenvaluesofstar));
```

RESULT:

Table-3: The table gives the energy sum comparison between Adjacency Matrix and Dominant Matrix of Star graph.

S_m	$E_A(S_m)$	$E_Đ(S_m)$
S_2	2	2.24
S_3	2.8284	3
S_4	3.4641	3.60555
S_5	4	4.12311
S_6	4.4721	4.5826
S_7	4.8990	5
S_8	5.2915	5.3852
S_9	5.6569	5.74456
S_{10}	6	5.913663
S_{11}	6.3246	6.4031
S_{12}	6.6332	6.7082

3.4 FAN GRAPH

3.4.1 ALGORITHM OF FINDING THE DOMINANT MATRIX OF FAN GRAPH

Procedure:

$X \leftarrow \{x_1, x_2, x_3, ..., x_n, x_{n+1}, x_{n+2}, x_{n+3}, ..., x_m\}$

$Y \leftarrow \{ y_1=x_1x_{n+1}, y_2=u_2x_{n+1}, e_3=x_3x_{n+1}, ..., y_n=x_1,x_{n+2}, ..., y_m=x_1x_{n+3,......}y_{m+n}=x_nx_{n+1}, \}$

if n, m ≥ 2

for k =n+1 to n+m

$u_1u_1 \leftarrow 1$

$u_1u_k \leftarrow 1$

$u_ku_1 \leftarrow 1$

end for
for k =1 to n && l=n+1 to n+m

$u_{k+1}u_{n+1} \leftarrow 1$

$u_{n+1}u_{k+1} \leftarrow 1$

end for

$u_{n+1}u_{n+2} \leftarrow 1$

$u_{n+2}u_{n+1} \leftarrow 1$

end if
end procedure

3.4.2 PYTHON PROGRAM FOR FINDING THE DOMINANT MATRX, EIGEN VALUES AND THE ENERGY SUM OF FAN GRAPH:

```
import numpy as np
n=int(input())
m=int(input())
mat = [[0] * (n+m) for _ in range(n+m)]
mat[0][0] = 1
mat[n][n]=1
mat[n][n+1]=1
mat[n+1][n]=1
for k in range(0,n):
   mat[n][k]=1
   mat[k][n]=1
for i in range(n,n+m):
   mat[0][k]=1
   mat[k][0]=1
Đ=np.array(mat)
w, v = np.linalg.eig(Đ)
sum_eig=np.absolute(w)
print(Đ)
print(w)
print(np.sum(sum_eig))
```

OUTPUT: for n= 5, m= 6

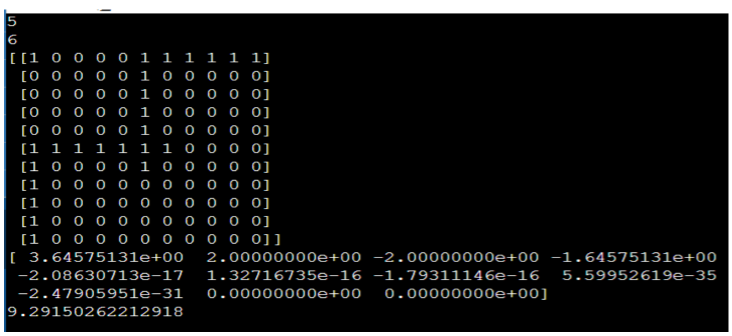


Fig. 4 – python program output- fan graph

3.4.3 MATHLAB CODE TO GENERATE THE ENERGY OF FAN GRAPH:

```
n=;          %enter the number of vertices
m=;          %enter the number of vertices
Đ=zeros(n);
Đ (1,1)=1;
Đ (n+1,n+1)=1;
Đ (n+1,n+2)=1;
```

```
Đ (n+2,n+1)=1;
for k=1:n
Đ (n+1,k)=1;
Đ(k,n+1)=1;
end
for k=n+1:n+m
Đ(1,k)=1;
Đ(k,1)=1;
end
Đ;
eigenvaluesoffangraph=eig(Đ);
energy=sum(abs(eigenvaluesoffangraph));
```

RESULT:

Table-4: The table gives the energy sum comparison between Adjacency Matrix and Dominant Matrix of Fan graph

S. No.	$E_A(F_{n,m})$	$E_Đ(F_{n,m})$
$F_{2,4}$	8.6632	6.6601
$F_{2,6}$	12.4342	7.6769
$F_{3,4}$	9.8877	7.3006
$F_{3,7}$	15.6125	8.7249
$F_{4,4}$	10.931	8
$F_{4,9}$	20.9688	9.9944
$F_{5,7}$	18.237	9.7106
$F_{5,12}$	28.3071	11.4165
$F_{6,8}$	21.6513	10.5141
$F_{6,13}$	31.6443	12.1240

4. OBSERVATION

The energy sums for the matrix of adjacency with the dominant matrix are compared. For networks with a minimum of two vertices (n greater than 2), the adjacency matrix consistently outperforms the dominant matrix. This minimal value indicates that the main matrix's format contributes most to the overall energy sum compared to the adjacency matrix's. i.e.

$$E_A(G)>E_Đ(G).$$

A graph has been utilized to illustrate the variation. This visual technique provides an obvious and intuitive knowledge of the link among the energy sums produced from an adjacency matrix as well as the dominant matrix. Analyzing the graph reveals that for networks with at least two nodes, the energy sum for each adjacency matrix is always bigger than the energy sum of the dominant matrix.

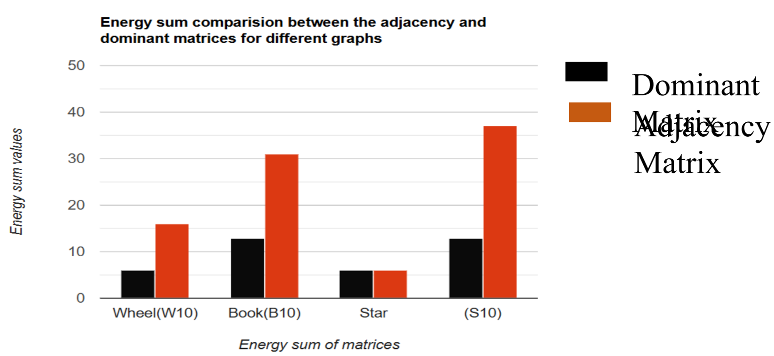


Figure 5. Energy sum comparison between matrices

This comparison shows a significant difference in energy output between the two graph forms.

5. CONCLUSION

The majority of the results presented in this article are based on an analysis of the real energy levels seen in the diagrams under discussion. However, broad conclusions can be conveyed and validated by the data generated. In this case, we compare the graph energy sum for the dominant matrix to the adjacent matrix of networks. Calculating graph energies for a collection of graphs under inquiry is a fascinating challenge.

CONFLICT OF INTERESTS

None.

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None.

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