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INFLUENCE OF GRAVITY MODULATION ON THE INITIATION OF DOUBLE DIFFUSIVE CONVECTION IN BOTH FLUID AND POROUS LAYER

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ABSTRACT

Linear stability analysis of convection in a binary fluid and saturated porous layer subjected to gravity modulation is performed. Regular perturbation method based on small amplitude of modulation is employed to compute onset threshold for synchronous mode, as a function of frequency, . The results of Darcy porous layer and fluid layer form the limiting cases of Brinkman porous layer. The gravity modulation exhibits both stabilizing and destabilizing effect in case of fluid layer while inhibits the porous layer convection. Since the onset of convection is affected by various parameters, by their proper tuning convection may be regulated.

Keywords: Double Diffusive Convection, Gravity Modulation, Fluid Layer, Porous Layer, Perturbation Method

1. INTRODUCTION

Double diffusive convection (DDC) occurs when two diffusing components (e.g., temperature and dissolved concentration) contribute to the buoyancy. In a fluid layer, the temperature and concentration diffuse at different rates, and the process is referred to as DDC. In a porous medium, the system is also double-advective: because heat is shared between fluid and porous medium while the solute is confined only to the void space, temperature is advected more slowly than concentration, and this difference in advection rates is crucial to the stability properties of the system.

The problem of DDC in fluid and saturated porous media has received significant interest during the past few decades because of its wide spread applications, such as convective heat and mass transfer, solidification of binary mixtures to the migration of solutes in water-saturated soils and the migration of moisture through air contained in fibrous insulation and so on. Some of the areas where DDC finds exhaustive applications include oceanography, astrophysics, geophysics, geology, chemistry, and metallurgy. The problem of DDC in fluid and porous media has been extensively investigated both theoretically and experimentally and the exhaustive research of the same is well reported by Turner [1-3], Huppert and Turner [4], Platten and Lagros [5], Ingham and Pop [6, 7], Nield and Bejan [8], Vafai [9, 10] and Vadasz [11].

Nield [12] performed linear stability analysis of thermohaline convection in a porous medium. Finite amplitude convection in a two-component fluid saturated porous layer has been studied by Rudraiah et al. [13]. Further works on double-diffusive convection in porous media include Brand and Steinberg [14], Murray and Chen [15], and Mamou and Vasseur

[16]. The problem of DDC in a fluid saturated porous layer was later on investigated by many authors (see e.g., Nield and Bejan [8] and references therein).

In the recent time, the study of thermal convection induced by oscillating forces resulting from either oscillating wall temperatures or modulated gravitational forces or a combination of these two has received much attention in the fluid dynamics research community. A physically important class of problems involves convection in a fluid layer in the presence of complex body forces. Such forces can arise in number of different ways. For instance, when a system with density gradient is subjected to vibrations, the resulting buoyancy forces which are produced by the interaction of the density gradient with the gravitational field, have a complex spatio-temporal structure. Some of the other situations where the gravity fluctuation becomes predominant include buoyancy-driven convection in microgravity conditions which are of great interest in space laboratory experiments, crystal growth, petroleum production, and large-scale atmospheric convection. Many theoretical and experimental studies dealing with materials processing under the micro-gravity conditions aboard an orbiting spacecraft have been carried out in the past [17]. Owing to several unavoidable sources of residual acceleration experienced by a spacecraft, the gravity field in an orbiting laboratory is not constant in a micro-gravity environment, but is a randomly fluctuating field which is referred to as g-jitter. It is reported in the literature [18, 19] that vibrations can either substantially enhance or retard heat transfer and thus drastically affect the convection. The effect of gravity modulation on a convectively stable configuration can significantly influence the stability of a system by increasing or decreasing its susceptibility to convection. In general, a distribution of stratifying agency that is convectively stable under constant gravity conditions can be destabilized when a time-dependent component of the gravity field is introduced. Certain combinations of thermal gradients, physical properties and modulation parameters may lead to parametric resonance and, hence, to the instability of the system.

The effect of gravity modulation on the stability of a heated fluid layer was first examined by Gresho and Sani [20] and Gershuni et al. [21]. Their results show that the stability of the layer being heated from below is enhanced by gravity modulation; but for the case of heating from above, the layer is destabilized. Wadih and Roux [18] presented a study of convection in an infinitely long cylinder with gravity modulation oscillating along the vertical axis. Their analyses established that the onset of convection is altered under the modulation of constraints. Murray et al. [22] considered the effect of gravity modulation on the onset of convection for the unidirectional solidification problem. Saunders et al. [23] studied the effect of gravity modulation on the stability of a horizontal double-diffusive layer. They considered a fluid layer of stress-free boundaries with linear temperature and solute concentration distributions that would generate instabilities either in the finger or the diffusive mode. Later, Clever et al. [24] studied the problem of two dimensional oscillatory convection in a gravitationally modulated fluid layer. Farooq and Homsy [25] investigated linear and nonlinear convection in a vertical slot in the presence of gravity modulation. Malashetty and Padmavathi [26] studied the effect of small amplitude gravity modulation on the onset of convection in fluid and porous layers. In general, their results indicate that the gravity modulation has destabilizing effect. Li [27] performed a stability analysis of modulated-gravity-induced thermal convection in a heated fluid layer subject to an applied magnetic field. The nearest correction to the critical Rayleigh number for both single and multiple frequency of oscillating-gravity components is obtained by solving the linearized equations using the small parameter perturbation technique. Shu et al. [28] examined the effect of modulation of gravity and thermal gradients on natural convection in a cavity numerically and experimentally. They found that for low Prandtl number fluids, modulations in gravity and temperature produce the same flow field both in structure and in magnitude. Experimental study on the response of Rayleigh-Benard convection to gravity modulation was carried out by Rogers et al. [29]. Yu et al. [30] made an experimental investigation of a horizontal stably stratified fluid layer being heated from below, including its subsequent nonlinear evolution under steady and modulated gravity, using two-dimensional numerical simulations. Recently, Dyko and Vafai [31] investigated effect of gravity modulation on convection in the annulus between two horizontal coaxial cylinders. Their work provides the description of convection in a cylindrical annulus under microgravity, and practical information on the influence of gravity modulation on heat transfer in a space environment.

The study of the effect of gravity modulation on the onset of convection in a porous medium is of comparatively recent origin. Malashetty and Padmavathi [32] studied the effect of small amplitude gravity modulation on the onset of convection in fluid saturated porous layers. Zenkovskaya and Rogovenko [33] investigated filtration convection subject to high frequency oscillations in an arbitrary direction using the averaging method. It is found that horizontal oscillation has a destabilizing effect in the case of zero gravity and microgravity. Bordan and Mojtabi [34] made an analytical and numerical study of convection in a porous cavity in the presence of vertical vibrations. They found that the vibrations stabilize the quiescent state. Govender [35] has made stability analysis to investigate the effect of low amplitude gravity modulation on convection in a porous layer heated from below. It was shown that increasing the frequency of vibration stabilizes the convection. More recently, Razi et al. [36] introduced the time averaged governing equations for Darcy-Brinkman model and they shown that there is a significant deviation from the Darcy model in determining the critical Rayleigh number. The effect of vertical harmonic vibration on the onset of convection in a porous medium is investigated by Strong [37] using continued fraction method. Further, Strong [38] has extended the work to binary fluid mixture saturating a porous medium. Saravanan and Purusothaman [39] carried out an investigation to find the effect of non-Darcian effects in an anisotropic porous medium and found that non-Darcian effects significantly affect the synchronous mode of instability. Recently, Saravanan and Sivakumar [40] studied the effect of vibration on the onset of convection in a horizontal fluid saturated porous layer considering arbitrary amplitude and frequency. It is demonstrated that vibrations can produce a stabilizing or a destabilizing effect depending on their amplitude and frequency. More recently, Malashetty and Swamy [41] asymptotically analyzed the linear stability of a rotating horizontal fluid and fluid-saturated porous layer heated from below for the case of small-amplitude gravity modulation.

Although the study of DDC in both fluid and porous media is exhaustively investigated by many researchers, a comparatively little attention has been given to its study under the influence of gravity modulation. The main objective of this article is to analyze the effect of small amplitude gravity modulation on the onset of a binary fluid layer and a saturated porous layer for a wide range of values of frequency of the modulation, solute Rayleigh number, Lewis number, Prandtl number, Darcy number, normalized porosity and viscosity ratio. We intend to provide a fundamental understanding of how the governing parameters would influence natural convection arising from gravity perturbation. As a first attempt, we present a linear stability analysis of a heated fluid layer and a saturated porous layer to explore the effect of various parameters on the onset of DDC in the presence of gravity modulation. In the case of porous layer, both the Darcy and Brinkman models are considered. It may be noted that most of the previous investigators of thermal convection in Brinkman porous medium have assumed that the fluid viscosity is same as the effective viscosity in their study. However, Givler and Altobelli [42] have determined experimentally that $\mu_e = (5 \sim 12)\mu$ where μ_e is the effective viscosity and μ is the fluid viscosity, for water flowing through high porosity porous media. Therefore, consideration of the ratio of effective viscosity to the fluid viscosity different from unity is warranted to know its influence on the critical stability parameters. Another main characteristic of this article is that it deals with fluid layer, Darcy and Brinkman porous layer, which makes this work more general. It is shown that the results corresponding to the two limiting cases, namely viscous and Darcy can be recovered from the Brinkman porous layer. It is believed that the results of this study are useful in the areas of crystal growth in micro-gravity conditions and also in bridging the gap between the results of Darcy and viscous fluid layer limits.

2. MATHEMATICAL FORMULATION

We consider an infinite horizontal binary fluid layer / saturated porous layer confined between the planes $z = 0$ and $z = d$ subjected to time-periodically varying gravity force $\mathbf{g} \equiv (0,\, 0,\, -g(t))$ acting on it, where $g(t) = g_0\left(1 + \varepsilon \cos \varpi t\right)$ with g_0 the constant gravity in an otherwise unmodulated system, ε the small amplitude of modulation, ϖ the frequency and t the time. The temperatures T_l and T_u with $T_l > T_u$ and solute concentrations S_l and S_u with $S_l > S_u$ are imposed at the bottom and top boundaries respectively. A Cartesian frame of reference is chosen with the origin in the lower boundary and z-axis vertically upwards. The interaction between heat and mass transfer, known as Soret and Dufour effects, is supposed to have no influence on the convective flow, so they are ignored. The porous medium is assumed to be isotropic and is in local thermal equilibrium with fluid phase. Within the Oberbeck-Boussinesq approximation, the continuity, momentum, energy and the state equations are:

$$\nabla \cdot \mathbf{q} = 0\,, \tag{1}$$

$$\frac{1}{\phi}\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\phi^2}\left(\mathbf{q}\cdot\nabla\right)\mathbf{q} + \frac{1}{\rho_0}\nabla p - \frac{\rho}{\rho_0}\mathbf{g} = A_1\frac{\mu_e}{\rho_0}\nabla^2\mathbf{q} - A_2\frac{\mu}{\rho_0 K}\mathbf{q}\,, \tag{2}$$

$$\gamma\frac{\partial T}{\partial t} + \left(\mathbf{q}\cdot\nabla\right)T = \frac{k}{\left(\rho_0 c\right)_f}\nabla^2 T\,, \tag{3}$$

$$\frac{\partial S}{\partial t}+\frac{1}{\phi}(\mathbf{q}\cdot\nabla)S=\kappa_S\nabla^2 S, \quad (4)$$

$$\rho=\rho_0\left\{1-\beta_T\left(T-T_0\right)+\beta_S\left(S-S_0\right)\right\}, \quad (5)$$

where $\mathbf{q}\equiv(u,v,w)$ denotes the velocity, p the pressure, ρ_0 the reference density, K and ϕ the permeability and porosity of the porous medium respectively, μ_e the effective viscosity, μ the viscosity of the fluid, T and S the temperature and concentration, respectively, γ the specific heat ratio, k the thermal conductivity, κ_S the solute diffusivity, β_T and β_S the thermal and solutal expansion coefficients. The quantities k_m and $\gamma_m=(\rho_0 c_p)_m/(\rho_0 c_p)_f$ are defined in terms of the fluid and solid components of the porous medium, such that $e_m=\phi e_f+(1-\phi)e_s$, $e_m=k_m$ or γ_m. The suffix m and f refers to the value of the parameter for the porous medium and fluid, respectively. The following table gives the definition of various parameters for clear fluid layer and porous layer.

Parameters	Clear fluid layer	Darcy porous medium	Brinkman porous medium
(A_1, A_2)	$(1, 0)$	$(0, 1)$	$(1, 1)$
ϕ	1	ϕ	ϕ
k	k	k_m	k_m
γ	1	γ_m	γ_m

2.1 Basic state

The basic state of the fluid is assumed to be quiescent and is given by,

$$\mathbf{q}_b\equiv(0,0,0),\ p=p_b(z,t),\ \rho=\rho_b(z),\ T=T_b(z),\ S=S_b(z). \quad (6)$$

Using these into Eqs. (2)-(5) one can obtain

$$\frac{\partial p_b}{\partial z}=-\rho_b g(t),\ \frac{d^2T_b}{dz^2}=0,\ \frac{d^2S_b}{dz^2}=0,\ \rho_b=\rho_0\left[1-\beta_T\left(T_b-T_0\right)+\beta_S\left(S_b-S_0\right)\right]. \quad (7)$$

Thus Eqs. (7) together with boundary conditions possess the following solutions

$$T_b(\mathrm{z})=T_l-\frac{1}{d}(T_l-T_u)z,\ S_b(\mathrm{z})=S_l-\frac{1}{d}(S_l-S_u)z, \quad (8)$$

$$p_b(z,t)=\rho_0 g(t)\left[\left(\beta_T T_l-\beta_S S_l\right)z-\frac{1}{2d}\left(\beta_T(T_l-T_u)-\beta_S\left(S_l-S_u\right)\right)z^2\right]. \quad (9)$$

2.3 Perturbed state

We study the stability of this basic state using the method of small perturbations. On the basic state we superpose infinitesimal perturbations of the form

$$\mathbf{q}=\mathbf{q}_b+\mathbf{q}',\ p=p_b(z)+p',\ \rho=\rho_b(z)+\rho',\ T=T_b(z)+T',\ S=S_b(\mathrm{z})+S', \quad (10)$$

where prime indicates that the quantities are infinitesimal perturbations. Introducing (10) into Eqs. (1)-(5) and using basic state solutions, and neglecting the nonlinear terms in perturbations, we obtain the linearized equations governing the perturbations in the form,

$$\nabla\cdot\mathbf{q}'=0, \quad (11)$$

$$\frac{1}{\phi}\frac{\partial\mathbf{q}'}{\partial t}+\frac{1}{\rho_0}\nabla p'-\left(\beta_T T'+\beta_S S'\right)g_0\left(1+\varepsilon\cos\varpi t\right)\mathbf{k}=A_1\frac{\mu_e}{\rho_0}\nabla^2\mathbf{q}'-A_2\frac{\mu}{\rho_0 K}\mathbf{q}', \quad (12)$$

$$\gamma\frac{\partial T'}{\partial t}-w'\frac{\Delta T}{d}=\kappa_T\nabla^2 T', \quad (13)$$

$$\frac{\partial S'}{\partial t}-\frac{1}{\phi}w'\frac{\Delta S}{d}=\kappa_S\nabla^2 S'. \quad (14)$$

Here **k** denotes the unit vector in the *z*-direction and $\kappa_T = k/(\rho_0 c)_f$ the thermal diffusivity with appropriate definition for fluid layer and porous layer. For the clear fluid layer and Brinkman porous medium, the boundaries are assumed to be stress-free, isothermal and isohaline. Accordingly, the boundary conditions at $z = 0$ and $z = d$ are

$$w'=\frac{\partial^2 w'}{\partial z^2}=T'=S'=0. \quad (15a)$$

For Darcy porous medium case,
the boundaries are impermeable, isothermal and isohaline. Therefore, the boundary conditions at $z = 0$ and $z = d$ are

$$w'=T'=S'=0. \quad (15b)$$

By operating curl twice on Eq.
(12) we eliminate p' from it, and then render the resulting equation and the Eqs. (11)-(14)
dimensionless using the following transformations

$$\begin{aligned}&(x',y',z')=d(x^*,y^*,z^*),\ t'=(d^2/\kappa_T)t^*,\ (u',v',w')=(\phi\kappa_T/d)(u^*,v^*,w^*),\\ &p'=(\mu\kappa_T\phi/d^2)p^*,\ T'=(\Delta T)T^*,\ S'=(\Delta S)S^*,\end{aligned} \quad (16)$$

to obtain non-dimensional
equations as (on dropping the asterisks for simplicity),

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t}-MA_1\nabla^2+A_2Da^{-1}\right)\nabla^2 w=(1+\varepsilon\cos\omega t)\nabla_1^2(Ra_T T-Ra_S S), \quad (17)$$

$$\left(\frac{1}{\chi}\frac{\partial}{\partial t}-\nabla^2\right)T=w, \quad (18)$$

$$\left(\frac{\partial}{\partial t}-Le^{-1}\nabla^2\right)S=w, \quad (19)$$

where $Pr=\phi\mu/\rho_0\kappa$, the Prandtl number, $Ra_T=\rho_0\beta_T g\Delta T d^3/\mu\kappa_T$, the thermal Rayleigh number, $Ra_S=\rho_0\beta_S g\Delta S d^3/\mu\kappa_T$, the solute Rayleigh number, $Da=K/d^2$, the Darcy number, $\omega=\varpi d^2/\kappa_T$, the nondimensional frequency of modulation, $Le=\kappa_T/\kappa_S$, the Lewis number, $M=\mu_e/\mu$, the ratio of effective viscosity and fluid viscosity, and $\chi=\phi/\gamma$, the normalized porosity.

The boundary conditions (15a,b) in the non-dimensional form are given by

$w=\partial^2 w/\partial z^2=T=S=0$ at $z=0,\ 1$ and (20a)

$w=T=S=0$ at $z=0,\ 1$ (20b)

After eliminating the coupling between the equations (17)-(19) we obtain the single equation for vertical component of velocity in the form

$$\left[\left(\frac{1}{Pr}\frac{\partial}{\partial t}-MA_1\nabla^2+A_2\,Da^{-1}\right)\left(\frac{1}{\chi}\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-Le^{-1}\nabla^2\right)\nabla^2\right.$$
$$\left.-Ra_T\left(\frac{\partial}{\partial t}-Le^{-1}\nabla^2\right)(1+\varepsilon\cos\omega t)\nabla_1^2+Ra_S\left(\frac{1}{\chi}\frac{\partial}{\partial t}-\nabla^2\right)(1+\varepsilon\cos\omega t)\nabla_1^2\right]w=0. \quad (21)$$

The boundary conditions (20a,b) in terms of the vertical component of velocity become

$$w=\frac{\partial^2 w}{\partial z^2}=\frac{\partial^4 w}{\partial z^4}=\frac{\partial^6 w}{\partial z^6}=\frac{\partial^8 w}{\partial z^8}=0,\ \text{at}\quad z=0,\ 1. \quad (22)$$

Now the disturbances in the normal modes can be expressed as

$$w=W(z,t)e^{i(lx+my)+\sigma t} \quad (23)$$

where $W(z,t)$ is a periodic function of time with the same period as the gravity modulation, $l,\ m$ are the wavenumbers of the disturbances in the x, y directions, respectively, $\sigma=\sigma_r+i\sigma_i$ is the growth rate of the disturbances. Let σ^* be the eigenvalue with greatest real part. The basic state, with respect to the infinitesimal disturbances, is unstable if the real part σ_r^* is greater than zero or stable if σ_r^* is less than zero. Here, unstable means that a disturbance experiences net growth over each modulation cycle, or grows during part of the cycle, but ultimately decays, while stable means that every disturbance decay at every instant. At the neutral stable state σ_r^* is zero. If the imaginary part σ_i^* is also zero simultaneously, the disturbance is synchronous with the periodic basic state. We consider in the present paper only synchronous mode.

Substituting the normal modes (23) into the disturbance equation (21), we obtain

$$\left[\left(\frac{1}{Pr}\frac{\partial}{\partial t}-MA_1(D^2-a^2)+A_2\,Da^{-1}\right)\left(\frac{1}{\chi}\frac{\partial}{\partial t}-(D^2-a^2)\right)\left(\frac{\partial}{\partial t}-Le^{-1}(D^2-a^2)\right)(D^2-a^2)\right.$$
$$\left.+\left\{Ra_T\left(\frac{\partial}{\partial t}-Le^{-1}(D^2-a^2)\right)-Ra_S\left(\frac{1}{\chi}\frac{\partial}{\partial t}-(D^2-a^2)\right)\right\}(1+\varepsilon\cos\omega t)a^2\right]W=0. \quad (24)$$

with $D\equiv\frac{\partial}{\partial z}$ and $a^2=l^2+m^2$. The associated boundary conditions are

$$W=D^2W=D^4W=D^6W=D^8W=0\ \text{at}\quad z=0,\ 1. \quad (25)$$

Equations (24) with boundary conditions (25), is homogeneous system and thus constitute an eigenvalue problem.

3. METHOD OF SOLUTION

We seek the eigenfunctions W and the eigenvalues Ra_T associated with the system of Eqs. (24)-(25) for a modulated gravity field that is different from the constant gravity field by a small quantity of order ε. The eigenfunction W and eigenvalue Ra_T should be a function of ε and they should be obtained for a given Darcy number Da, Prandtl number Pr, solute Rayleigh number Ra_S, Lewis number Le and frequency ω. Since ε is very small for the problem under consideration, we expand these eigenfunctions and eigenvalues in a power series of ε in accordance with the theory of small parameter perturbation, in the form

$$(W,\ Ra_T)=(W_0,\ R_0)+\varepsilon(W_1,\ R_1)+\varepsilon^2(W_2,\ R_2)+\cdots \qquad (26)$$

Here W_0 and R_0 are the eigenfunctions and eigenvalues respectively of the unmodulated system and W_i and R_i, $(i\geq 1)$ are the corrections to W_0 and R_0 in the presence of gravity modulation.

Substituting Eq. (26) into Eq. (24) and equating the corresponding terms, we obtain the following system of equations

$$LW_0=0, \qquad (27)$$

$$LW_1=-a^2R_0L_3\cos\omega tW_0-a^2R_1L_3\sin\pi z+a^2Ra_SL_2\cos\omega tW_0, \qquad (28)$$

$$LW_2=-a^2R_1L_3\cos\omega tW_0-a^2R_2L_3W_0-a^2R_0L_3\cos\omega tW_1-a^2R_1L_3W_1 +a^2Ra_SL_2\cos\omega tW_1, \qquad (29)$$

Where the operators L and L_i's are as given in Appendix through Eqs. (A.1)-(A.4). Each of W_n are required to satisfy the boundary conditions (25). Equation (27) which is obtained at $O(\varepsilon^0)$ is the one used in the study of thermal convection in a horizontal fluid layer/ fluid-saturated porous layer subject to the constant gravitational field. The marginally stable solutions for that problem are

$$W_0^{(n)}=\sin n\pi z, \qquad (30)$$

with the corresponding eigenvalues

$$R_0^{(n)}=MA_1\frac{\left(n^2\pi^2+a^2\right)^3}{a^2}+A_2\,Da^{-1}\frac{\left(n^2\pi^2+a^2\right)^2}{a^2}+Ra_SLe. \qquad (31)$$

For a fixed wavenumber the least eigenvalue occurs for n = 1, and is given by

$$R_0=MA_1\frac{\left(\pi^2+a^2\right)^3}{a^2}+A_2\frac{\left(\pi^2+a^2\right)^2}{a^2}Da^{-1}+Ra_SLe, \qquad (32)$$

corresponding to $W_0=\sin\pi z$.

3.1 Fluid layer

For a fluid layer, $A_1=1,\ A_2=0$ and then Eq. (32) yields

$$R_0=\frac{\left(\pi^2+a^2\right)^3}{a^2}+Ra_SLe, \qquad (33)$$

which assumes the minimum value R_{0c} for $a=a_c$, where a_c satisfies the equation

$$3a_c^2\left(\pi^2+a_c^2\right)^2-\left(\pi^2+a_c^2\right)^3=0. \qquad (34)$$

We observe that Eqs. (33) and (34) are the classical results obtained for DDC in binary fluid layer without gravity modulation (see e.g., Turner [1]). Further, these equations yield the values $R_{0c}=27\pi^4/4=657.5$ and $a_c=\pi/\sqrt{2}$, which are associated with the classical Rayleigh-Benard problem.

3.2 Darcy model

For Darcy porous medium, i.e. for a densely packed porous layer, $A_1=0,\ A_2=1$, then Eq. (32) reads

$$R_0 = \frac{\left(\pi^2 + a^2\right)^2}{a^2} Da^{-1} + Ra_S Le . \tag{35}$$

When both sides of Eq. (35) are multiplied by *Da* one can obtain the expression for Darcy-Rayleigh number

$$R_{D0} = \frac{\left(\pi^2 + a^2\right)^2}{a^2} + Ra_{SD} Le , \tag{36}$$

with modified solute Rayleigh number, $Ra_{SD} = \rho_0 \beta_S g \Delta S d K / \mu \kappa_T$ for the Darcy porous layer. The corresponding critical values of Darcy-Rayleigh number and wavenumber are, respectively, given by

$$R_{Doc} = 4\pi^2 \text{ and } a_c = \pi . \tag{37}$$

which are the classical results of Horton and Rogers [43] and Lapwood [44] for convection in a porous layer.

3.3 Brinkman model

For a Brinkman porous medium, i.e. for a sparsely packed porous layer, $A_1 = 1,\ A_2 = 1$, so that Eq. (32) reduces to

$$R_{B0} = M \frac{\left(\pi^2 + a^2\right)^3}{a^2} + \frac{\left(\pi^2 + a^2\right)^2}{a^2} Da^{-1} + Ra_S Le . \tag{38}$$

The minimum value of the Rayleigh number R_{B0c} occurs at the critical wavenumber $a = a_c$, where a_c satisfies the equation

$$\left(\pi^2 + a_c^2\right)\left(2a_c^2 - \pi^2\right)\left\{MDa\left(\pi^2 + a_c^2\right) + 1\right\} = 0 , \tag{39}$$

The above results (38) and (39) are obtained by Poulikakos for DDC in a horizontal sparsely packed porous layer in the absence of modulation [45].

The equation for W_1, then reads

$$LW_1 = -a^2 \left[\left(R_0 \tilde{L}_3 - Ra_S \tilde{L}_2\right) \mathrm{Re}\left\{e^{-i\omega t}\right\} + R_1 \delta^2 Le^{-1}\right] \sin \pi z , \tag{40}$$

where $\tilde{L}_2$, $\tilde{L}_3$ and δ^2 are as given in the Appendix through Eqs. (A.5)-(A.7).

The above equation is inhomogeneous and its solution poses a problem, because of the presence of resonance term. The mathematical properties and solvability conditions of the differential equations with time periodic coefficients have been extensively discussed by Yakubovich and Starzhinskii [46]. If this equation is to have a solution, the right-hand side must be orthogonal to the null space of the operator *L*. This requires that the time-independent part of the right-hand side should be orthogonal to its steady state solution W_0 i.e. $\sin \pi z$. Since $\cos \omega t$ varies sinusoidally with time, the only steady term is $-a^2 R_1 \delta^2 Le^{-1} \sin \pi z$, so that R_1 is zero. It follows that all the odd coefficients $R_1, R_3, \ldots$, in Eq. (26) are zero because a change of the sign of ε shifts the time origin by half period but does not change the physical problem.

Now we solve Eq. (40) by inverting the operator *L* term-by-term and obtain the expression for W_1, in the form

$$W_1 = -a^2 \left(R_0 \tilde{L}_3 - Ra_S \tilde{L}_2\right) \mathrm{Re}\left\{\sum_{n=1}^{\infty} \frac{e^{-i\omega t}}{L(\omega, n)}\right\} \sin n\pi z , \tag{41}$$

where $L(\omega, n) = B_1 + i B_2$, with B_1 and B_2 as given in the Appendix through the Eqs. (A.8)-(A.9).

Equation (29) now takes the form

$$LW_2 = -a^2\delta^2 Le^{-1}R_2 \sin\pi z - a^2\left(R_0\tilde{\tilde{L}}_3 - Ra_S\tilde{\tilde{L}}_2\right)\mathrm{Re}\left\{e^{-i\omega t}\right\}W_1, \tag{42}$$

where $\tilde{\tilde{L}}_2$ and $\tilde{\tilde{L}}_3$ are as given in the Appendix through the equations (A.10)-(A.11).

We shall not require the solution of this equation, but we use it to determine R_2, the first non-zero correction to Ra. The solvability condition requires that the steady part of right-hand side must be orthogonal $\sin\pi z$. Thus,

$$R_2 = -\frac{2\,Le}{\delta^2}\left(R_0\tilde{\tilde{L}}_3 - Ra_S\tilde{\tilde{L}}_2\right)\mathrm{Re}\left\{\int_0^1 \overline{e^{-i\omega t}W_1}\sin\pi z\,dz\right\}, \tag{43}$$

where the over bar indicates the time average. Now, from Eq. (41), one can obtain

$$\mathrm{Re}\left\{\overline{e^{-i\omega t}W_1}\sin\pi z\right\} = -\frac{1}{a^2\left(R_0L_3 - Ra_SL_2\right)}\overline{W_1LW_1}. \tag{44}$$

Using Eq. (41) and (40) in Eq. (44) and finding the time average, substituting the resulting expression in Eq. (43) we obtain

$$R_2 = \frac{a^2Le}{2\delta^2}\sum_{n=1}^{\infty}\frac{B_1\left(B_3^2 - \omega^2B_4^2\right) + 2\omega B_2B_3B_4}{B_1^2 + B_2^2}, \tag{45}$$

with the values of B_3 and B_4 as given in Appendix through the Eqs. (A.12)-(A.14).

Eq. (42) could now be solved for W_2 if desired, and the procedure may be continued to obtain further corrections to W and Ra_T. However we shall stop at this step. The value of thermal Rayleigh number Ra_T obtained by this procedure is the eigenvalue corresponding to the eigenfunctions W, which, though oscillating, remains bounded in time. Ra_T is a function of horizontal wavenumber a and the amplitude of modulation ε, accordingly we expand

$$Ra_T\left(a,\,\varepsilon\right) = R_0\left(a\right) + \varepsilon^2R_2\left(a\right) + \varepsilon^4R_4(a) + \cdots, \tag{46}$$

$$a = a_0 + \varepsilon^2a_2 + \cdots, \tag{47}$$

where R_0 and a_0 are the Rayleigh number and wavenumber, respectively for the unmodulated system. The thermal Rayleigh number Ra_T as a function of wavenumber a has a least value Ra_{Tc} which occurs at $a = a_c$ and the critical wavenumber occurs when $\frac{\partial Ra_T}{\partial a} = 0$. In view of Eq. (46) we have

$$\frac{\partial R_0}{\partial a} + \varepsilon^2\frac{\partial R_0}{\partial a} + \cdots = 0. \tag{48}$$

Eq. (48) in view of Eq. (47) takes the form

$$\frac{\partial R_0}{\partial a_0} + \varepsilon\left(\frac{\partial^2R_0}{\partial a_0^2}\right)a_1 + \varepsilon^2\left[\frac{1}{2}\left(\frac{\partial^3R_0}{\partial a_0^3}\right)a_1^2 + \left(\frac{\partial^2R_0}{\partial a_0^2}\right)a_2 + \frac{\partial R_2}{\partial a_0}\right] + \cdots = 0. \tag{49}$$

Equating the coefficients of like powers of ε on both sides of Eq. (49) we get

$$\frac{\partial R_0}{\partial a_0} = 0,\ a_1 = 0,\ a_2 = -\left(\partial R_2/\partial a_0\right)/\left(\partial^2R_0/\partial a_0^2\right). \tag{50}$$

The critical thermal Rayleigh number is then given by

$$Ra_{Tc}(a,\varepsilon) = R_{0c} + \varepsilon^2R_{2c} + \varepsilon^4R_{4c} + \cdots$$

$$= R_0(a_0) + \varepsilon\left(\partial R_0/\partial a_0\right)a_1 + \varepsilon^2\left[\frac{1}{2}\left(\partial^2R_0/\partial a_0^2\right)a_1^2 + \left(\partial R_0/\partial a_0\right)a_2 + R_2(a_0)\right] + \cdots \tag{51}$$

In view of Eq. (50), the above equation reduces to

$$Ra_{Tc}(a,\varepsilon) = R_0(a_0) + \varepsilon^2R_2(a_0) + \cdots \tag{52}$$

The critical value of the thermal Rayleigh number Ra_{Tc} is thus computed up to $O\left(\varepsilon^2\right)$ by evaluating R_0 and R_2 at $a = a_0$. It is only when one wishes to evaluate R_4, a_2 must be taken into account (see also Venezian [47]). In that case the gravity modulation affects the critical wavenumber. If R_{2c} is positive, the effect of modulation is to stabilize the system as compared to the unmodulated system. When R_{2c} is negative, the effect of modulation is to destabilize the system as compared to the unmodulated system.

To the order of $O\left(\varepsilon^2\right)$, R_{2c} is obtained for the three cases, *viz*., (i) fluid layer, (ii) Darcy porous layer and (iii) Brinkman porous layer. The variation of R_{2c} with ω for different values of Ra_S, *Le, Pr*, *Da*, χ and *M* is depicted in Figs. 1-14 and the results are discussed in the next section.

4. RESULTS AND DISCUSSION

The onset of binary convection in a horizontal fluid layer and fluid saturated porous layer, under the influence of time-periodically varying gravitational force, is investigated analytically using the linear stability theory. Due to gravity modulation there is a shift in the onset criteria. The critical Rayleigh number and the wavenumber are computed using regular perturbation technique based on the assumption that amplitude of imposed modulation is very small. Because of this we have restricted our analysis only for the first order correction to the critical Rayleigh number, *viz*., R_{2c}. The critical correction Rayleigh number is evaluated as a function of the frequency of modulation ω, the solute Rayleigh number Ra_S, the Lewis number *Le*, the Prandtl number *Pr*, the Darcy number *Da*, the viscosity ratio *M* and the normalized porosity χ. The influence of these governing parameters on the onset of DDC is displayed through the Figs. 1-14.

The value of frequency of modulation plays an important role in validating the results obtained in this analysis. When ω is very small the period of modulation becomes sufficiently large and the disturbances grow to a large extent and therefore, the entire system under consideration becomes unstable. This is justified by the magnitude of R_{2c}, which is found to be sufficiently small or even negative in some cases. On the other-hand when ω is very large the effect of gravity modulation is confined only to a narrow boundary layer near the boundary. This is due to the fact that the high frequencies correspond to renormalization of the static gravity field. Thus, outside this thickness the buoyancy force takes a mean value tending towards the equilibrium sate value of the unmodulated case. The effect of gravity modulation is therefore significant only for the moderate values of ω. Further, due to the assumption that the amplitude of modulation is small and Darcy resistance dampens the convection currents, the nonlinear effects may be neglected.

In Figs. 1-3 the variation of critical correction Rayleigh number R_{2c} with the frequency of modulation ω is revealed, for the case of fluid layer. It is observed that R_{2c} is negative for small ω while for moderate values of ω, there is a considerable increase in the value of R_{2c}. Thus, the low frequency gravity modulation destabilizes the system where as the convection is delayed when ω is quite large. The system becomes most stable when R_{2c} attains a maximum value corresponding to a specific frequency $\omega = \omega^*$. If ω is increased beyond ω^* we notice that R_{2c} goes on decreasing and becomes independent of ω for large values of frequency. Thus, critical Rayleigh number tends to its equilibrium value of unmodulated state.

The variation of R_{2c} with ω for different values of Ra_S is displayed in Fig. 1. When $Ra_S = 0$ a curve similar to that of a single component case is obtained. In this case R_{2c} is positive over the entire range of values of ω. This indicates the stabilizing effect of gravity modulation on the onset of thermal convection in a viscous fluid layer. However, when $Ra_S \neq 0$, R_{2c} is negative for small values of ω. Thus the presence of second diffusing agent namely the solute concentration leads the gravity modulation to advance the convection as compared to the unmodulated case. For moderate frequency the stabilizing effect is noticed and at $\omega = \omega^*$, the system becomes most stable due to both gravity modulation and solute gradient. Further it is found that ω^* increase with Ra_S.

In Fig. 2 the effect of *Le* on the stability of the binary fluid layer is exhibited. The role of *Le* is to stabilize the system. The frequency ω^* at which the system is most stable is independent of *Le*. When *Le* = 0, we observed that R_{2c} is negatively very small over the entire domain of ω. Thus in this case the gravity modulation shows a very weak destabilizing effect. The Fig. 3 depicts the variation of R_{2c} with ω for different values of *Pr*. It is reported that the influence *Pr* is to enhance the stabilizing effect of Ra_S and *Le*. This figure also indicates that ω^* increases with *Pr*.

For the case of a densely packed porous layer saturated with a binary fluid the variation of R_{2c} with ω for various governing parameters is exposed through the Figs. 4-8. We detect that R_{2c} is positive over the entire realm of ω. Therefore, the onset of DDC is delayed as compared to the unmodulated system. It is important to note that the range of values of ω over which the effect of gravity modulation is significant is comparatively larger than the cases of viscous fluid layer and the Brinkman porous layer.

In Fig. 4 the influence of *Da* is revealed. It is found that with the increasing values of *Da*, there is a decrease in R_{2c}. Further the range of values of frequency for which R_{2c} becomes independent of ω is reduced considerably for the larger *Da*. Thus when *Da* is very small the effect of gravity modulation is more pronounced and is sustained for larger range of frequency. It is also noticed from this figure that ω^* decreases with *Da*. Thus the *Da* retards the stabilizing effect of gravity modulation.

It is important to note from Fig. 5 that R_{2c} decreases significantly with Ra_S. Thus the increasing solute gradient shifts R_{2c} towards the lower value. Therefore, there is a net decrease in the value of critical Rayleigh number. This shows a destabilizing effect of Ra_S, which is in contrast to the case of onset of DDC in Darcy porous layer in the absence of gravity modulation. This figure also shows that ω^* is almost independent of Ra_S.

Fig. 6 indicates the stabilizing effect of *Le*. When *Le* = 0, R_{2c} decreases monotonically with ω, where as when $Le \neq 0$, the stabilizing effect of gravity modulation attains maximum at $\omega = \omega^*$ and beyond ω^* the reverse trend is reported. The effect of *Pr* on the stability of the system is revealed in Fig. 7. When *Pr* > 50, the effect of gravity modulation is prevailed over a larger range of frequency, where as for small ω the modulation effect is constrained to a comparatively lower frequency range. This figure also shows that *Pr* leads R_{2c} to increase and thus inhibits the DDC.

Fig. 8 reports the influence of normalized porosity χ on the onset criterion. The increasing χ enhances the value of R_{2c}. Thus the normalized porosity reinforces the stabilizing effect of gravity modulation, the Darcy number *Da* and Prandtl number *Pr* towards the onset of convection in a Darcy porous layer saturated with a binary fluid.

The effect of gravity modulation on the onset of DDC in a sparsely packed porous layer is exhibited through the Figs. 9-14. From these figures it is clear that R_{2c} is positive over the entire domain of ω. Thus the onset of binary convection is suppressed due to the gravity modulation. Similar to the case of clear binary fluid layer the range of ω over which the effect of gravity modulation is significant is comparatively smaller than that for the Darcy porous layer.

The Darcy number which characterizes the porous matrix is considered with a wide range of values, to encompass the limits of Darcy porous layer and the binary fluid layer. From this figure we observe that R_{2c} decreases with *Da*. Therefore, the Darcy number retards the stabilizing effect of gravity modulation. It is important to note that when *Da* is very small (i.e., $Da << 10^{-3}$) the curve corresponding to the Darcy case is recovered while for large *Da* (i.e., $Da \to \infty$), we recover the result of viscous fluid layer case. The curves characterizing the Brinkman porous layer are confined between those representing the cases of viscous fluid layer and the Darcy porous layer.

Figs. 10-14 respectively exhibit the stabilizing effect of Ra_S, *Le*, *Pr*, χ and *M*. The frequency ω^* at which the system becomes most stable is independent of Ra_S, *Le* and χ, where as it increases with *Pr* and *M*. It is interesting to note from the Fig. 11 that when *Le* = 0, the gravity modulation shows a very weak destabilizing effect. Further Fig. 12 indicates that when *Pr* > 50 there is a considerable increase in the range of ω over which the influence of gravity modulation is significant. From the Fig. 14 it is worth reporting that for the small values of ω (< ω^*), R_{2c} decreases with *M* and the maximum value of R_{2c} which occurs at ω^* decreases with *M*. Thus *M* shows a dual effect on the onset of thermohaline convection in a sparsely packed porous layer under the influence of gravity modulation.

5. CONCLUSIONS

The analytical study of the effect of gravity modulation on the onset of DDC in a binary fluid layer and a saturated porous layer is carried out. The critical correction Rayleigh number is computed using regular perturbation method and the variation of the same with frequency of the imposed modulation is shown graphically and the following conclusions are drawn:

- Due to imposed gravity modulation there is a shift in the onset criteria.
- For small ω, R_{2c} is sufficiently small or even negative therefore the low frequency gravity modulation advances the convection in a binary fluid / saturated porous layer.
- When ω is very large the effect of gravity modulation is confined only to a narrow boundary layer. Outside this thickness the buoyancy force takes a mean value tending towards the equilibrium sate value of the unmodulated case.
- The effect of gravity modulation is significant only for the moderate values of ω.
- The nonlinear effects are neglected due to the assumption that the amplitude of modulation is small and Darcy resistance dampens the convection currents.
- The system becomes most stable when R_{2c} attains a maximum value corresponding to a specific frequency $\omega = \omega^*$.

For the case of binary fluid layer

- When $Ra_S = 0$, R_{2c} is positive over the entire range of values of ω this is in agreement to the result of single component case.
- When $Ra_S \neq 0$, R_{2c} is negative for small values of ω and for moderate frequency the stabilizing effect is noticed.
- *Le* and *Pr* enhance the stabilizing effect of gravity modulation and Ra_S.
- When *Le* = 0, gravity modulation shows a very weak destabilizing effect.
- ω^* increase with Ra_S and *Pr* while it is independent of *Le*.

For the case of binary fluid-saturated densely packed porous layer

- R_{2c} is positive over the entire realm of ω, indicating the inhibition of onset of DDC as compared to the unmodulated system.
- The range of values of ω over which the effect of gravity modulation is significant is comparatively larger than the cases of viscous fluid layer and the Brinkman porous layer.
- *Da* retards while *Le*, χ and *Pr* reinforce the stabilizing effect of gravity modulation.
- The range of values of frequency for which R_{2c} becomes independent of ω is reduced considerably for the larger *Da*.
- When *Da* is very small the effect of gravity modulation is more pronounced and is sustained for larger range of frequency.
- ω^* decreases with *Da*, while increases with *Pr* and it is almost independent of Ra_S, *Le* and χ.
- R_{2c} decreases significantly with Ra_S. This shows a destabilizing effect of Ra_S, which is in disparity with the case of onset of DDC in Darcy porous layer in the absence of gravity modulation.
- When *Pr* > 50, the effect of gravity modulation is prevailed over a larger range of frequency.

For the case of binary fluid-saturated sparsely packed porous layer

- R_{2c} is positive over the entire domain of ω.

- *Da* destabilizes the two component system while Ra_S, *Le*, *Pr*, χ and *M* supplement to the stabilizing effect of gravity modulation.
- The cases of Darcy porous layer and viscous fluid layer are recovered respectively for very small *Da* (i.e., $Da << 10^{-3}$) and very large *Da* (i.e., $Da \to \infty$).
- ω^* is independent of Ra_S, *Le* and χ, where as it increases with *Pr* and *M*.
- When *Le* = 0, the gravity modulation shows a very weak destabilizing effect.
- *M* shows a dual effect i.e., for small values of ω ($<\omega^*$), R_{2c} decreases while for moderate frequencies R_{2c} increases with *M*.

CONFLICTS OF INTEREST
None.

ACKNOWLEDGMENTS
None.

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APPENDIX

$$L \equiv L_1 L_2 L_3 \left(D^2 - a^2\right) + a^2 R_0 L_3 - a^2 Ra_S L_2 , \qquad \text{(A.1)}$$

$$L_1 \equiv \frac{1}{Pr}\frac{\partial}{\partial t} - MA_1 (D^2 - a^2) + A_2\, Da^{-1} , \qquad \text{(A.2)}$$

$$L_2 \equiv \chi \frac{\partial}{\partial t} - (D^2 - a^2) , \qquad \text{(A.3)}$$

$$L_3 \equiv \frac{\partial}{\partial t} - Le^{-1}(D^2 - a^2) , \qquad \text{(A.4)}$$

$$\tilde{L}_2 = -\frac{i\omega}{\chi} + \delta^2 \qquad \text{(A.5)}$$

$$\tilde{L}_3 = -i\omega + \frac{\delta^2}{Le} \qquad \text{(A.6)}$$

$$\delta^2 = \pi^2 + a^2 \qquad \text{(A.7)}$$

$$\tilde{\tilde{L}}_2 = -2i\omega\chi + \delta_n^2 \qquad \text{(A.8)}$$

$$\tilde{\tilde{L}}_3 = -2i\omega\phi + \delta_n^2 \qquad \text{(A.9)}$$

$$B_1 = \frac{\delta_n^2}{Pr}\Big[a^2\, Pr\left(Le^{-1}R_0 - Ra_S\right) - Le^{-1}\, Pr\, \delta_n^4 \left(MA_1\delta_n^2 + Da^{-1}A_2\right) + \omega^2\, Pr\, A_2 \left(\chi Da\right)^{-1} + \delta_n^2 \omega^2 \left(1 + \chi^{-1}\left(M\, Pr\, A_1 + Le^{-1}\right)\right)\Big] , \qquad \text{(A.10)}$$

$$B_2 = \frac{\omega}{Pr}\Big[\delta_n^2 \left(\delta_n^4 Le^{-1} + \delta_n^2\, Pr\left(MA_1\delta_n^2 + Da^{-1}A_2\right)\left(1 + \left(Le\chi\right)^{-1}\right) - \omega^2\chi^{-1}\right) + a^2\, Pr\left(Ra_S\chi^{-1} - R_0\right)\Big] , \qquad \text{(A.11)}$$

$$B_3 = \delta^2\left(Le^{-1}R_0 - Ra_S\right), \qquad \text{(A.12)}$$

$$B_4 = Ra_S\chi^{-1} - R_0 , \qquad \text{(A.13)}$$

$$\delta_n^2 = n^2\pi^2 + a^2 . \qquad \text{(A.14)}$$

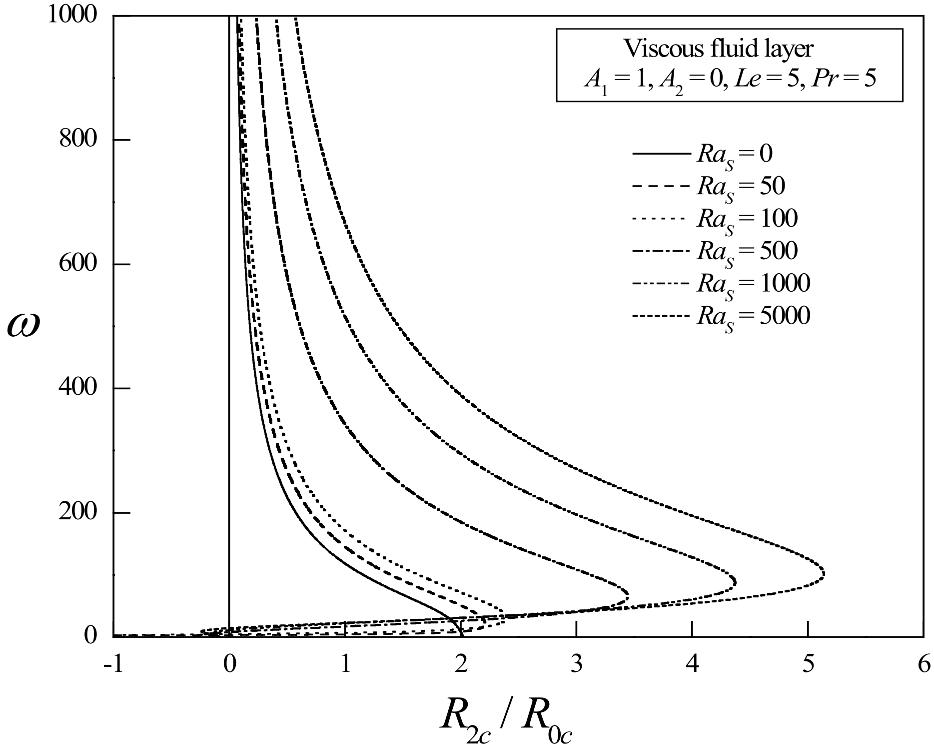

Fig. 1. Variation of R_{2c} with ω for different values of Ra_S for the viscous fluid layer.

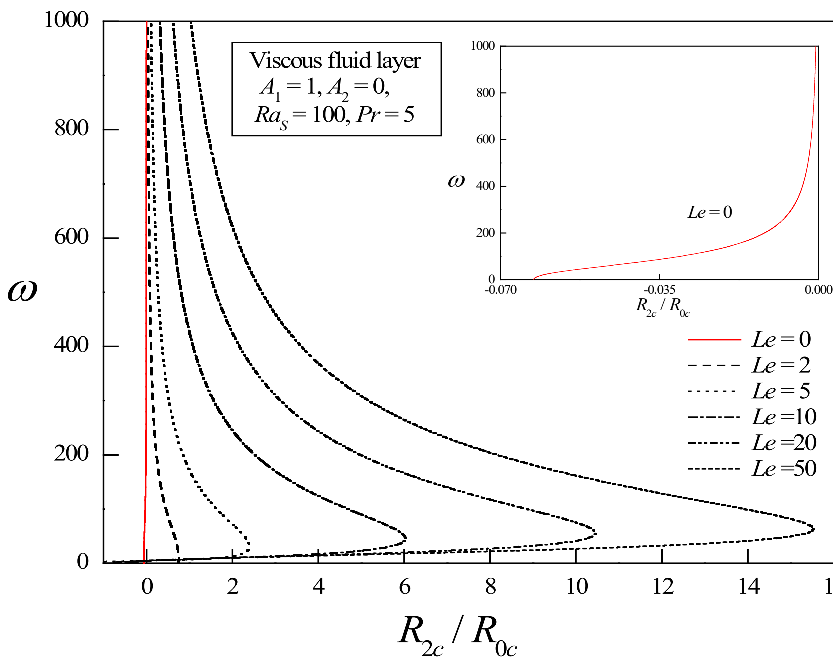

Fig. 2. Variation of R_{2c} with ω for different values of Le for the viscous fluid layer.

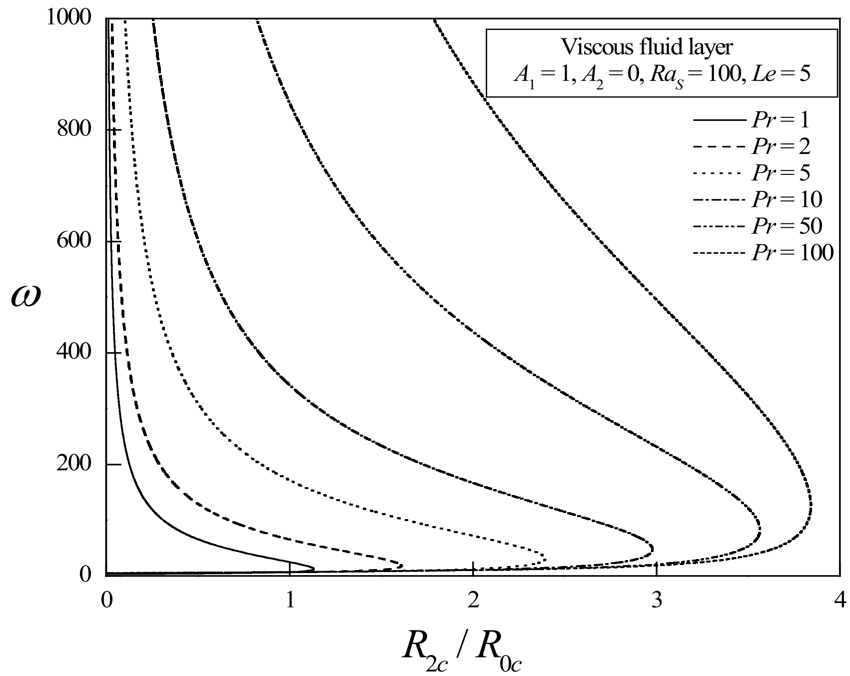

Fig. 3. Variation of R_{2c} with ω for different values of Pr for the viscous fluid layer.

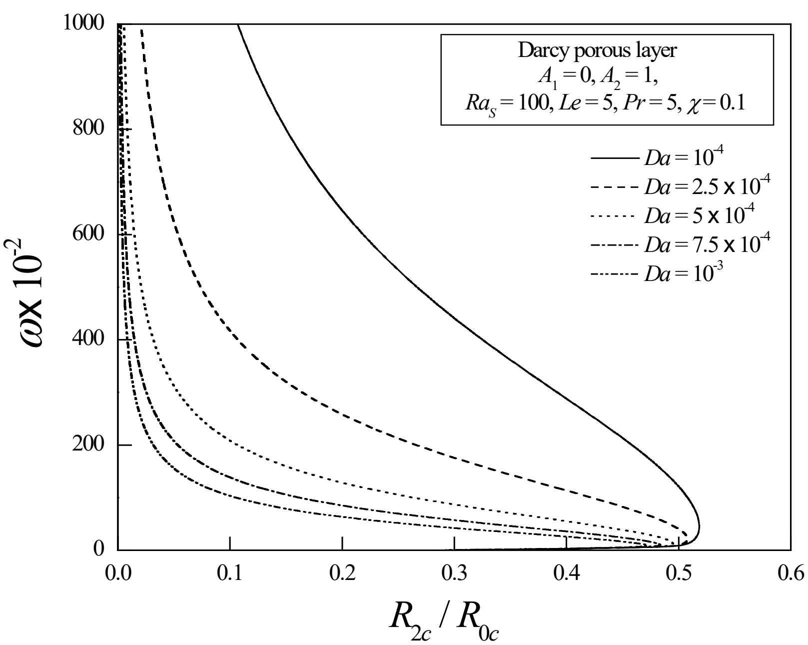

Fig. 4. Variation of R_{2c} with ω for different values of Da for the Darcy porous layer.

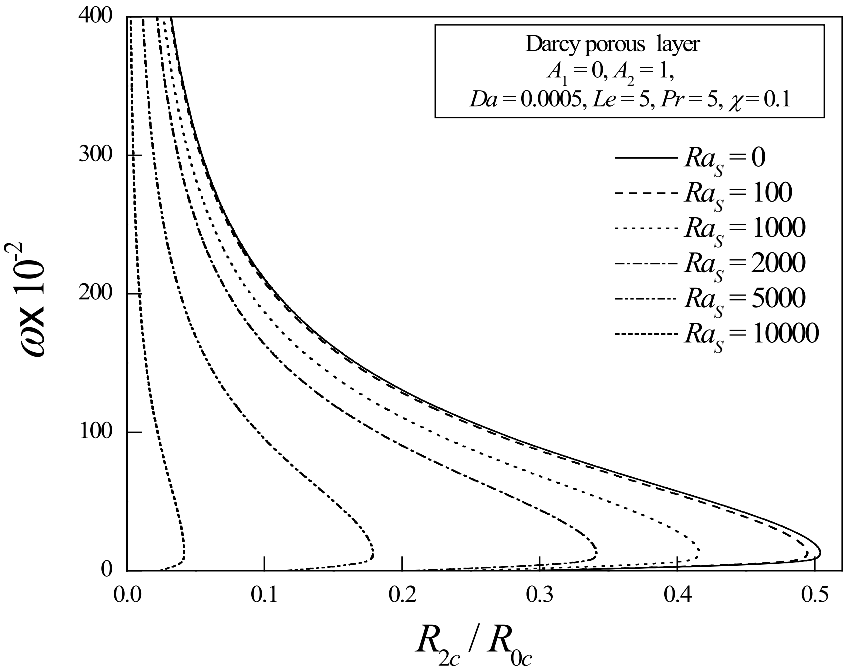

Fig. 5. Variation of R_{2c} with ω for different values of Ra_S for the Darcy porous layer.

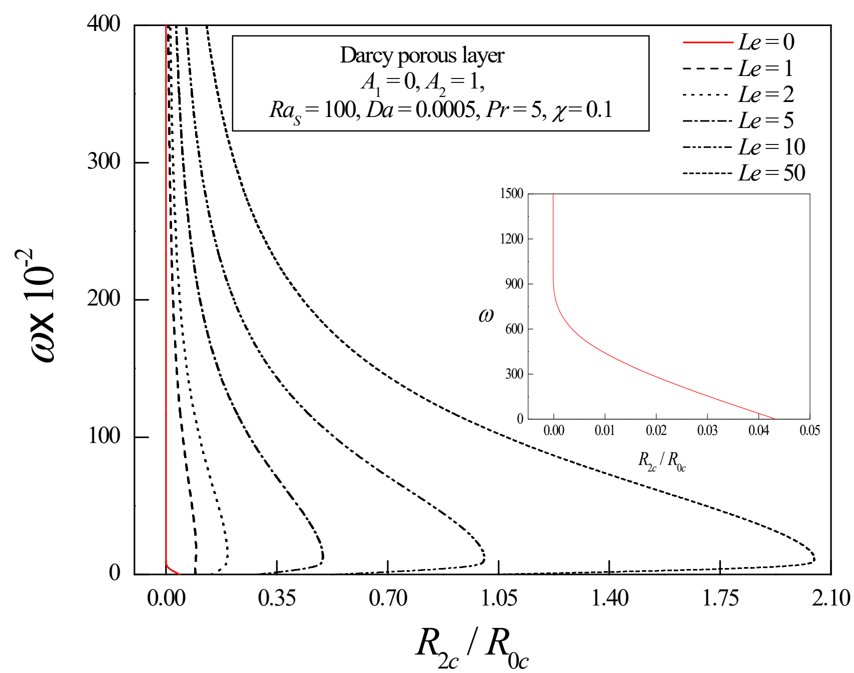

Fig. 6. Variation of R_{2c} with ω for different values of Le for the Darcy porous layer.

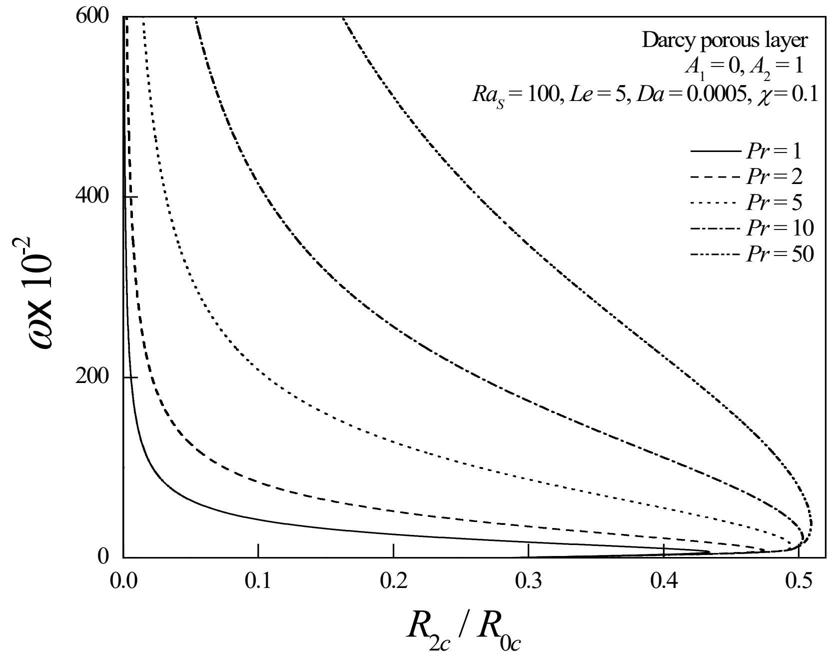

Fig. 7. Variation of R_{2c} with ω for different values of Pr for the Darcy porous layer.

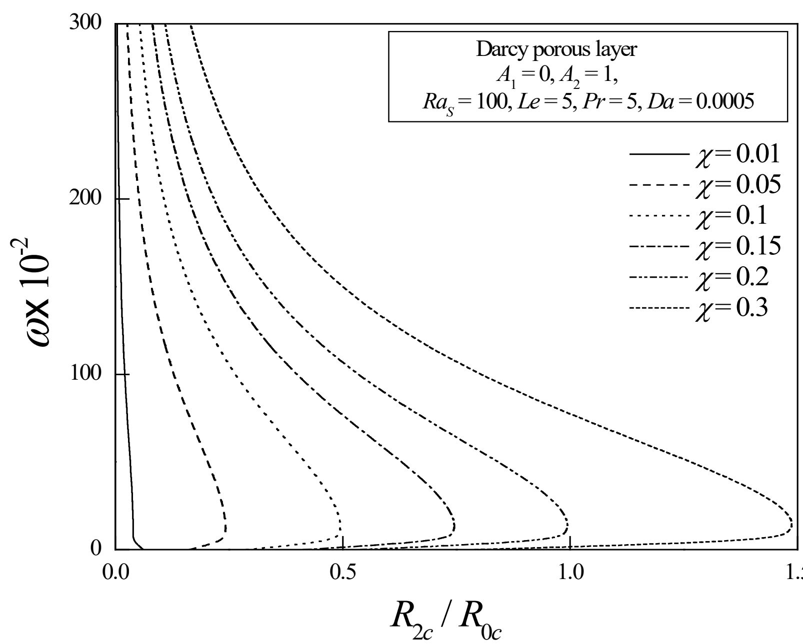

Fig. 8. Variation of R_{2c} with ω for different values of χ for the Darcy porous layer.

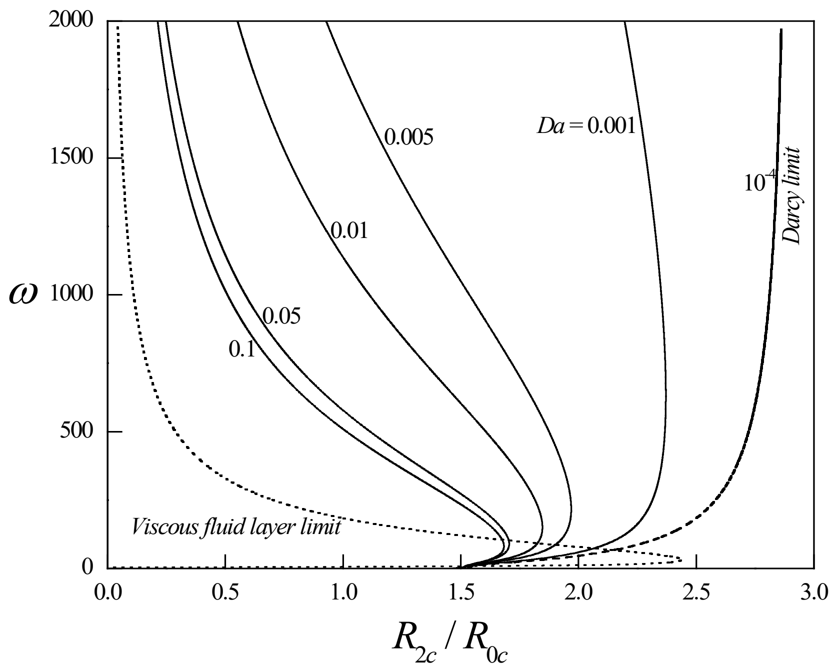

Fig. 9. Variation of R_{2c} with ω for different values of Da for the Brinkman porous layer ($A_1 = A_2 = 1$, $Ra_S = 100$, $Le = 5$, $Pr = 5$, $\chi = 0.6$, $M = 6$).

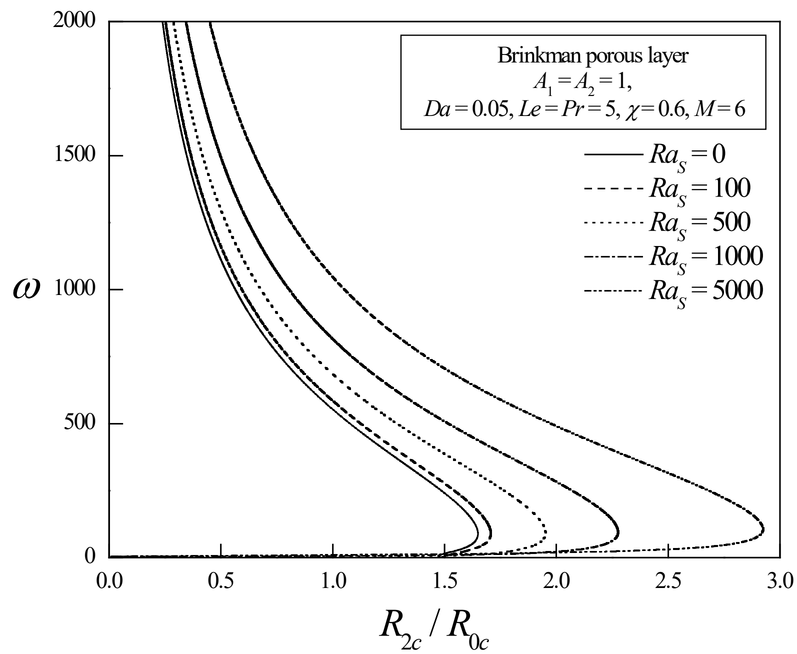

Fig. 10. Variation of R_{2c} with ω for different values of Ra_S for the Brinkman porous layer.

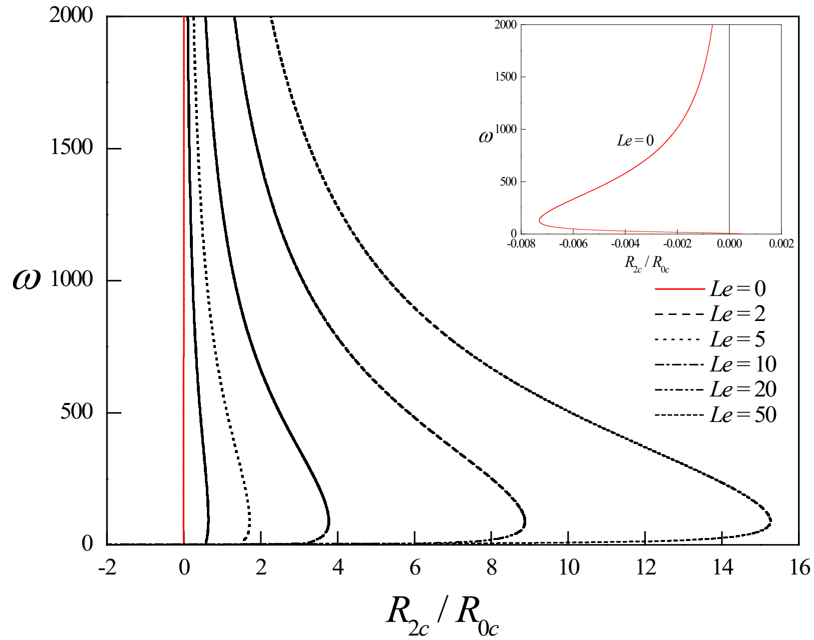

Fig. 11. Variation of R_{2c} with ω for different values of Le for the Brinkman porous layer ($A_1 = A_2 = 1$, $Ra_S = 100$, $Da = 0.05$, $Pr = 5$, $\chi = 0.6$, $M = 6$).

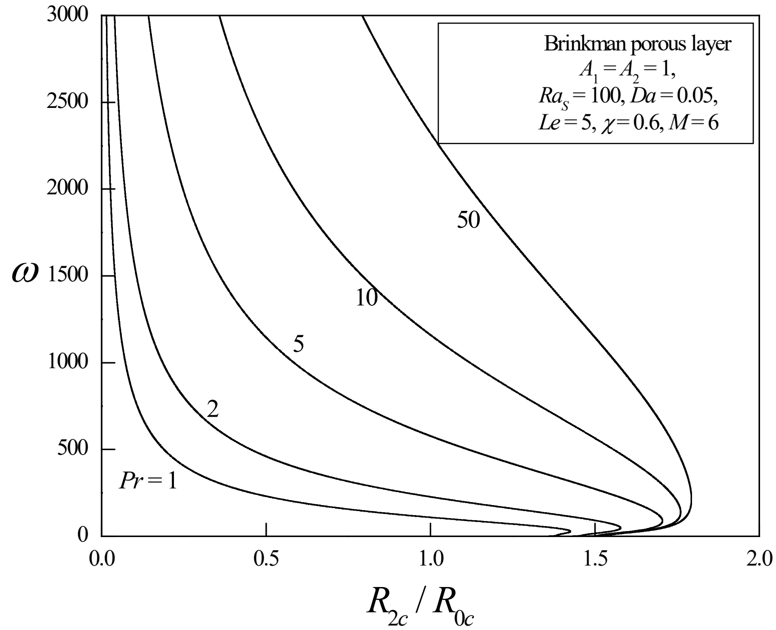

Fig. 12. Variation of R_{2c} with ω for different values of Pr for the Brinkman porous layer.

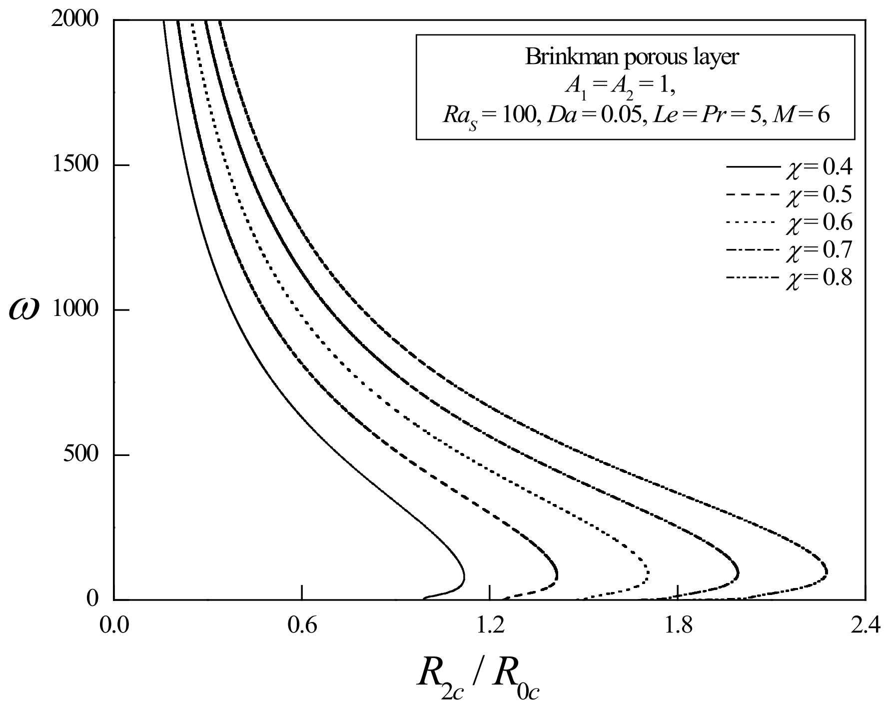

Fig. 13. Variation of R_{2c} with ω for different values of χ for the Brinkman porous layer.

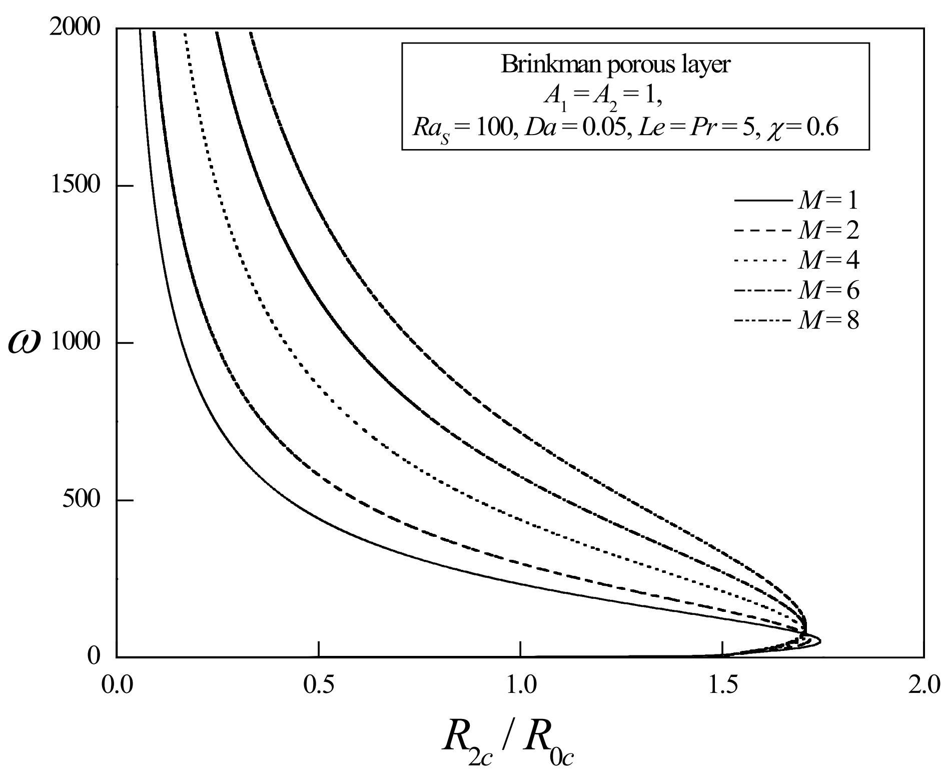

Fig. 14. Variation of R_{2c} with ω for different values of M for the Brinkman porous layer.