SECOND GRADE MHD FLOW AND HEAT TRANSFER OVER A STRETCHING SHEET WITH VISCOUS AND OHMIC DISSIPATIONS

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ABSTRACT

Two dimensional second grade fluid has been considered for analysis. Basic governing equation of velocity and temperature are partial differential equation which is converted to ordinary differential equation by using transformation variable. Employing fifth order Runge-Kutta-Fehlberg method with shooting to solve momentum equation. The results are analysed for the situation when stretching boundary sheet is prescribed by non-isothermal temperature and variable heat flux, varying quadratically with the flow directional coordinate x.

Keywords: Viscoelastic Stretching Surface, Viscous Dissipation, Ohmic Dissipation, Magnetic Field and Normal Electric Field

1. INTRODUCTION

The reason behind opting boundary layer flow problem over a stretching sheet, finds many industrial applications such wind-up roll, glass fiber and paper production, drawing of plastic films, liquid films in condensation process etc. Taking note of this the studies in the subsequent years gained momentum on various aspects of momentum and heat transfer characteristics in a viscoelastic boundary layer second order fluid flow over stretching sheets (Rajagopal et al. [1], Dandapat et al. [2] , Cortell [3], Varjravelu and Rollins [4] and Mahapatra and Gupta [5], Pop and Soundalgekar [6], Abel et al. [7] Very recently Dandapat et al. [8] have carried out stability analysis of such flow and has shown that the magnetic field, one of the controlling forces, has stabilizing effect on the boundary layer flow. some attempts have been made to investigate theoretically the effect of transverse magnetic field on boundary layer flow characteristics (Andresson [9], Char [10] and Lawrence and Rao [11]). Andresson [9] presented a systematic mathematical analysis on MHD flow of second order viscoelastic fluid over an impermeable stretching sheet and showed that magnetic parameter has same effect as that of viscoelastic parameter in flow characteristics. Lawrence and Rao [11] discussed the non-uniqueness of the MHD flow of viscoelastic fluid and discussed some theoretical aspects of the solution of the momentum boundary layer equation. However, all these studies are concerned with flow characteristics only associated with the viscoelastic fluid flow over stretching sheet. CHAMKHA [12] presented an analysis on unsteady state hydromagnetic flow and heat transfer from a non-isothermal stretching sheet in a porous medium. HELMY [13] presented a work on MHD unsteady free convection flow past a vertical porous plate. The electric field has been excluded from all these studies. Moreover, their analyses are confined to the viscous fluid only. Interestingly, CHIAM [14] presented heat transfer analysis taking into consideration the variable thermal conductivity for slightly different kind of problem of stagnation-point flow towards a stretching sheet.

2. RHEOLOGICAL MODEL AND GOVERNING EQUATIONS

COLEMAN and NOLL [15] derived the constitutive equation of second order incompressible viscoelastic fluid flow in the form

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \qquad (2.1)$$

using the postulates of gradually fading memory. Here, $T$is the stress tensor,$p$is the pressure, μ is the dynamic viscosity, $\alpha_1, \alpha_2$are the normal stress moduli. The kinematical tensors $A_1 and A_2$ are define by

$$A_1 = (gradq) + (gradq)^T$$

$$A_2 = \frac{dA_1}{dt} + A_1(gradq) + (gradq)^T . A_1 \qquad (2.2)$$

The symbol q appearing in the equation above stands for velocity. Using some experimental data verification Fosdick and Rajagopal [16] gave the range of values of μ, α_1 andα_2 as

$$\mu \geq 0, \alpha_1 \leq 0, \alpha_1 + \alpha_2 \neq 0. \qquad (2.3)$$

Making use of the model equation (2.1) and following the works of BEARD and WALTERS [17] the unsteady state two-dimensional boundary layer equation for viscoelastic fluid in Cartesian co-ordinate system my be written as (DANDAPAT et al. [8])

$$\rho\left[\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k}\right] = \frac{\partial p}{\partial x} - \mu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - k_0 \begin{Bmatrix} \frac{\partial}{\partial t}\left(\frac{\partial^3 u_i}{\partial x_k \partial x_k}\right) + u_m \frac{\partial^3 u_i}{\partial x_k \partial x_k \partial x_k} - \frac{\partial u_i}{\partial x_m} \frac{\partial^2 u_m}{\partial x_k \partial x_k} \\ -2 \frac{\partial^2 u_i}{\partial x_m \partial x_k} \frac{\partial u_m}{\partial x_k} \end{Bmatrix} \qquad (2.4)$$

where q=u_i are the velocity components and μ is the dynamic viscosity of the fluid. This equation has been derived with the assumption that the normal stress is of same order of magnitude as that of the shear stress, in addition to the usual boundary layer approximations. The constant $k_0 = -\frac{\alpha_1}{\rho}$ is the elastic parameter and it takes positive value for negative values of material constant α_1 in case of second order fluid. This equation is valid only for short memory liquids having short relaxation time and small values of elastic parameter k_0 at low shear stress

Governing equations of ANDERSSON [9], and TEMITOPE E. AKINBOBOLA∗, SAMUEL S. OKOYA (18) and those are the modified version of the equation (2.4) and also of the

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.5)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} + k_0 \left\{u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\} - \frac{\sigma}{\rho}(E_0 B_0 + B_0^2 u) \qquad (2.6)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{(uB_0+E_0)^2\sigma}{\rho c_p} \qquad (2.7)$$

3. BOUNDARY CONDITIONS ON VELOCITY

Employ the following boundary conditions on velocity.

$$u=U_w(x)=bx, v=0 aty=0$$

u=0asy→∞. (3.1)

Solution of the momentum boundary layer equation

To solve the governing boundary layer equation (2.6) we use the following transformations.

$$u=bxf_\eta, v=-\sqrt{b\gamma}f(\eta)\eta=\sqrt{\frac{b}{\gamma}}y. \qquad (3.2)$$

Here $f(\eta)$ is the dimensionless stream function and $\eta$is the pseudo-similarity variable. Substitution of the transformations equation (3.2) in the equation (2.6) results in a fourth order non-linear quasi-ordinary differential equation of the followi form.

$$f_\eta^2-ff_{\eta\eta}=f_{\eta\eta\eta}+k_1^*\left[2f_\eta f_{\eta\eta\eta}-ff_{\eta\eta\eta\eta}-f_{\eta\eta}^2\right]-Mn^2\left(E_1+f_\eta\right) \qquad (3.3)$$

where $k_1^*=\frac{k_0 b}{\gamma}$ is the dimensionless viscoelastic parameter, $Mn=\sqrt{\frac{\sigma}{\rho b}}B_0$ is Hartmann number, $E_1=\frac{E_0}{B_0U_w}$) is the local normal electric parameter and the subscript η stands for differentiation with respect to η.

In view of the transformations the boundary conditions on stream function f of equation (3.1) take the following non-dimensional form.

$$f(0)=0, f_\eta(0)=1, \rightleftarrows\rightleftarrows\rightleftarrows f_\eta(\infty)=0 \qquad (3.4)$$

Reasonably argued that in case of boundary layer flow of viscoelastic fluid with short memory, the characteristic time scale associated with the motion is large compared with the relaxation time of the fluid. Thus terms of order k^2,k^3 and higher orders may be neglected and therefore we may seek the solution of the equation (3.3) in the form

$$f=f_0(\eta)+k_1^*f_1(\eta)+0\left({k_1^*}^2\right) \qquad (3.5)$$

Substituting the series expansion of the equation (3.5) in the equation (3.3) and equating the constant terms and the coefficient of k_1^* to zero we deduce the following equations for $f_0(\eta)$ and $f_1(\eta)$.

$$f_{0\eta}^2-f_0f_{0\eta\eta}=f_{0\eta\eta\eta}-Mn^2\left(E_1+f_{0\eta}\right) \qquad (3.6)$$

$$f_{1\eta\eta\eta}-Mn^2f_{1\eta}-2f_{0\eta}f_{1\eta}+f_0f_{1\eta\eta}+f_1f_{0\eta\eta}=2f_{0\eta}f_{0\eta\eta\eta}-f_0f_{0\eta\eta\eta\eta}-f_{0\eta\eta}^2 \qquad (3.7)$$

Making use of the series expansion of the equation (3.5) in the boundary conditions (3.4) we obtain boundary conditions for $f_0(\eta)$ and $f_1(\eta)$ in the following form.

$$f_0(0)=0, f_{0\eta}(0)=1, f_{0\eta}(\infty)=0 \qquad (3.8)$$

$$f_1(0) = 0, f_{1\eta}(0) = 0, f_{1\eta}(\infty) = 0 \quad (3.9)$$

Now we find the zeroth order stream function equation (3.6) as a third order equation of $f_0(\eta)$ for which three boundary conditions are prescribed by equation (3.8). The first order stream function equation (3.7) is also a third order equation of $f_1(\eta)$ for which three boundary conditions are prescribed by equation (3.9). Since the order of the differential equations (3.6) and (3.7) matches well with the number of boundary conditions prescribed by the equations (3.8) and (3.9) respectively, the equations (3.6) and (3.7) would produce unique solutions.

It is to be mentioned that expansion of $f(\eta)$ in a series (3.5) does not constitute a singular perturbation expansion as no boundary condition is ignored in obtaining two well posed boundary values problem by equations (3.6) and (3.8) and equations (3.7) and (3.9). In fact, in a single perturbation problem, apart from expansion of the form (3.5) an asymptotic expansion is also needed and then matching of the two expansion is done. No such things are being done in the procedure adopted in this paper.

The dimensionless local skin-friction coefficient C_f is expressed as

$$C_f = -\frac{\left(\gamma\frac{\partial u}{\partial y}+k_0\left\{u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}-2\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}\right\}\right)}{(bx)^2} \, at y = 0 \quad (3.10)$$

$$= -\frac{1}{\sqrt{Re_x}} f_{\eta\eta}^{(0)}(1 + 3k_1^*)$$

where $Re_x = \frac{bx^2}{\gamma}$ is the local Reynolds number and $f_{\eta\eta}^{(0)} = f_{0\eta\eta}^{(0)} + k_1^* f_{1\eta\eta}^{(0)}$.

4. SIMILARITY SOLUTION OF THE HEAT TRANSFER EQUATION

Case A: prescribed surface temperature (PST)

To solve the thermal boundary layer equation (2.7) in PST case we take non-isothermal temperature boundary condition as follows.

$$T = T_w = T_\infty + A_0\left(\frac{x}{l}\right)^2 \, at y = 0$$

$$T \to T_\infty as y \to \infty \quad (4.1)$$

Here A_0 is the parameter of temperature distribution on the stretching surface, T_w stands for stretching sheet temperature and T_∞ is the temperature far away from the stretching sheet. We have considered the above form of quadratic power law temperature boundary condition on stretching sheet in order to obtain local similar solution of the equation (2.7).

Similarity equation from the thermal boundary layer equation (2.7) we define dimensionless temperature variable θ(η) of the form:

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \quad (4.2)$$

we arrive at the following form of non-dimensional thermal boundary layer equation.

$$\theta_{\eta\eta} + Pr\left(f\theta_\eta - 2f_\eta\theta\right) = -Pr\,E\left\{f_{\eta\eta}^2 + Mn^2\left(f_\eta^2 + E_1^2 + 2E_1 f_\eta\right)\right\} \quad (4.3)$$

here $Pr = \frac{\gamma}{\alpha}$ is the Prandtl number and $E = \frac{b^2 l^2}{A_0 c_p}$ is the Eckert number.

Temperature boundary conditions of the equation (4.1) take the following non-dimensional form.

$$\theta(0) = 1, \theta(\infty) = 0 \quad (4.4)$$

The solution of the equation (4.3) subject to the boundary conditions of the equation (4.4) is obtained numerically by applying the method of fifth order Runge-Kutta-Fehlberg method with shooting and outline of the numerical solution procedure is described in the next section.

Case B: prescribed boundary heat flux (PHF)

To solve the thermal boundary layer equation (2.7) in PHF case we consider variable heat flux boundary condition of the following form:

$$k\left(\frac{\partial T}{\partial y}\right)_w = A_1\left(\frac{x}{l}\right)^2$$

T→T_∞ (4.5)

Here A_1 is the parameter of temperature distribution on the stretching surface and T_∞ is the temperature far away from the stretching sheet. We have considered the above form of quadratic power law heat flux boundary condition on stretching sheet in order to obtain local similar solution of the equation (2.7).

As we look for local similar equation from the thermal boundary layer equation (2.7) we define dimensionless temperature variable $g(\eta)$ of the form:

$$g(\eta) = \frac{T - T_\infty}{A_1\left(\frac{x}{l}\right)^2 \frac{1}{k}\sqrt{\frac{\gamma}{b}}} \quad (4.6)$$

Making use of the equation (4.6) in the dimensional energy equation (2.7) we arrive at the following form of non-dimensional thermal boundary layer equation in PHF case.

$$g_{\eta\eta} + Pr\left(f g_\eta - 2 f_\eta g\right) = -Pr\, E\left\{f_{\eta\eta}^2 + Mn^2\left(f_\eta^2 + E_1^2 + 2E_1 f_\eta\right)\right\} \quad (4.7)$$

where $Pr = \frac{\gamma}{\alpha}$ is the Prandtl number and $E = \frac{b^2 l^2 k}{A_1 c_p \sqrt{\frac{\gamma}{b}}}$ is the Eckert number in PHF case.

In view of the transformation equation (4.6) the temperature boundary conditions of the equation (4.5) take the following non-dimensional form.

$$g_\eta(0) = -1, g(\infty) = 0. \quad (4.8)$$

The solution of the equation (4.7) subject to the boundary conditions of the equation (4.8) is obtained numerically by applying the fifth order Runge-Kutta-Fehlberg integration scheme with shooting and outline of the numerical solution procedure is described in the next section.

5. NUMERICAL SOLUTION

The equations (3.6)-(3.7) are highly nonlinear ordinary differential equations and (4.3) is a non-homogeneous quasi-ordinary differential equation with variable coefficient. In order to solve these equations numerically we follow most efficient numerical shooting technique with fifth order Runge-Kutta-Fehlberg integration scheme. In this method it is most important to choose the appropriate finite values of $\eta \to \infty$. To select η_∞ for the boundary value problem of Eqs. (3.6) and (3.8) we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $f_{0\eta\eta}(0)$. The solution process is repeated with another large values of η_∞ until two successive values of $f_{0\eta\eta}(0)$ differ only after desired significant digit. The last value of η_∞ is chosen as appropriate finite value of the limit $\eta \to \infty$ for that particular set of parameters to solve the unknown $f_0(\eta)$. Similar procedure is applied to obtain the finite value of η_∞ for the problems of Eqs. (3.7) and (3.9) and Eqs. (4.3)-(4.4) involving unknowns $f_1(\eta)$ and $\theta(\eta)$ respectively. For different set of parameters the appropriate finite values of η_∞ are different. The coupled boundary value problems of (i) Eqs. (3.6) and (3.8), (ii) Eqs. (3.7) and (3.9) and (iii) Eqs. (4.3) and (4.4) or Eqs. (4.7-4.8) are solved numerically following the method of superposition (Na [32]). In this method the third order non-linear equations (3.6) and (3.7) and second order equation (4.3) have been reduced to a system of eight simultaneous ordinary differential equations for which five initial conditions (Eqs. (3.8), (3.9) and (4.4) or (4.8)) are prescribed. In order to convert this system into a system of initial value problem we employ numerical shooting technique with fifth order Runge-Kutta- Fehlberg integration scheme where three infinity boundary conditions have been utilized to generate three more initial conditions. After knowing all the eight initial conditions we solve this system of simultaneous equations by employing fifth order Runge-Kutta-Fehlberg integration scheme applicable to the system of eight simultaneous first order differential equations. For better approximation of the solutions we employ Newton's linear interpolation.

6. RESULTS AND DISCUSSION

We have depicted numerical results in the forms

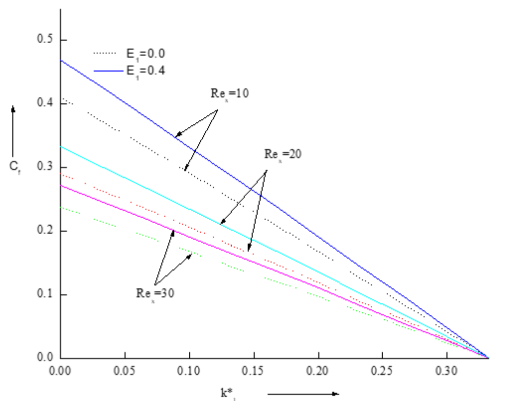

Fig.3. Graph of local skin-firiction parameter C_f vs. viscoelastic parameter k^*_1 for different values of local Reynolds Re_x and local electric parameter E_1 when M_n=0.8.

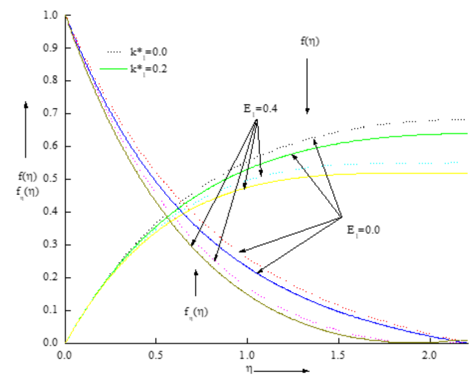

Fig.2. Graph of stream function $f(\eta)$ and velocity function $f_\eta(\eta)$ vs. η for different values of viscoelastic parameter k^*_1 and local electric parameter E_1 when Mn=0.8.

Fig. 2. is plotted to represent graphically the velocity profile $f_\eta(\eta)$ and stream function $f(\eta)$ for various values of viscoelastic parameter k_1^* and local normal electric parameter E_1. Analysis of the graphical behavior of the stream function $f(\eta)$ shows that the effect of local normal electric parameter E_1 is to shift the streamlines towards boundary stretching boundary in both the cases of viscous and viscoelastic fluids flows. The magnitude shifting of streamlines asymptotically increases with the distance from the stretching sheet. This is quite consistent with the boundary layer behaviour. The effect of local normal electric parameter E_1 on velocity is to decrease its value throughout the boundary layer, more significantly little away from the stretching sheet in the both cases of viscous and viscoelastic fluid flows. This is because Lorentz force arising due to electric field acts as a decelerating force and thereby increases frictional resistance. The combined effect of local normal electric parameter E_1 and viscoelastic parameter k_1^* is to reduce velocity significantly little away from the boundary layer sheet.

Fig. 3 demonstrates the graphs of skin-friction coefficient C_f vs. viscoelastic parameter k_1^*, for different values of local Reynolds number Re_x and local normal electric parameter E_1. Analysis of the graphs reveals the fact that the skin-friction coefficient C_f would decrease with the increase of local Reynolds number Re_x. Whereas, increase of the values of local normal electric parameter E_1 would increase the values of skin-friction parameter C_f. This is because the Lorentz force arising due to electric field coupled with the reduced magnitude of viscous force decelerates the flow in the down stream direction. It is interesting to observe that skin-friction coefficient C_f decreases linearly with the increase of viscoelastic parameter k_1^*. Separation of boundary layer occurs for the value of viscoelastic parameter $k_1^* = \frac{1}{3}$ which is independent of the values of local Reynolds number Re_x, Hartmann number Mn and local normal electric parameter E_1.

Temperature profiles for various values of viscoelastic parameter k_1^*, Hartmann number Mn, Prandtl number Pr, local normal electric parameter E_1 and Eckert number E are depicted in the tables

Table 1

η	k_1^*	Pr	Mn	E	E_1	$\theta(\eta)$
0	0.0	3	0.0	0.0	0.0	0.99
	0.2					0.99
	0.0	5				0.987
	0.2					0.987
	0.0	5	0.8			0.987
	0.2					0.987
	0.0	3				0.99
	0.2					0.99
0.5	0.0	3	0.0			0.293
	0.2					0.299
	0.0	5				0.174
	0.2					0.174
	0.0	5	0.8			0.189
	0.2					0.497
	0.0	3				0.3
	0.2					0.31
1	0.0	3	0.0			0.077
	0.2					0.081
	0.0	5				0.027
	0.2					0.027
	0.0	5	0.8			0.035
	0.2					0.04
	0.0	3				0.091
	0.2					0.099

Table 2

η	k_1^*	Pr	Mn	E	E_1	$\theta(\eta)$
0	0.0	3	0.0	1	0.0	1
	0.2					1
	0.0	5				1
	0.2					
	0.0	5	0.8			1
	0.2					
	0.0	3				1
	0.2					1
0.5	0.0	3	0.0			0.449
	0.2					0.462
	0.0	5				0.375
	0.2					0.39
	0.0	5	0.8			0.51
	0.2					0.529
	0.0	3				0.565
	0.2					0584
1	0.0	3	0.0			0.201
	0.2					0.205
	0.0	5				0.153
	0.2					0.154
	0.0	5	0.8			0.2
	0.2					0.202
	0.0	3				0.261
	0.2					0.27

Table 1 depicts of non-dimensional temperature profile $\theta(\eta)$ for different combination of the values of viscoelastic parameter k_1^*, Hartmann number Mn and Prandtl number Pr. From this graph, in absence of viscous dissipation and electric field, it is seen that the effect of Prandtl number Pr is to decrease temperature in the boundary layer region in PST case. However, the effect of increasing values of Hartmann number magnetic Mn is to increase temperature in the boundary layer region. The boundary layer fluid would attain maximum temperature at any point if the fluid flow is viscoelastic with low Prandtl number and in presence of a transverse magnetic field.

Table 2 depicts the temperature profile in the situation when viscous dissipation is accounted. From table, in presence of viscous dissipation, we analyse that there would be higher temperature throughout the boundary layer in the case of viscoelastic fluid in comparison with the viscous fluid. It is seen that the effect of Prandtl number is to decrease temperature throughout the boundary layer in both the cases of viscous and viscoelastic fluid in absence and also in presence of magnetic field. The combined effect of increasing values of Prandtl number Pr and Hartmann number Mn is to increase temperature near the stretching sheet largely and decrease the same away from the stretching sheet in absence of normal electric field.

When both viscous dissipation and electric field are accounted in heat transfer,

Table3

η	k_1^*	Pr	Mn	E	E_1	$\theta(\eta)$
0	0.0	3	0.0	1	0.2	1
	0.2					1
	0.0	5				1
	0.2					
	0.0	5	0.8			1
	0.2					
	0.0	3				1
	0.2					1
0.5	0.0	3	0.0			0.449
	0.2					0.469
	0.0	5				0.375
	0.2					0.392
	0.0	5	0.8			0.637
	0.2					0.659
	0.0	3				0.671
	0.2					0.692
1	0.0	3	0.0			0.201
	0.2					0.201
	0.0	5				0.153
	0.2					0.157
	0.0	5	0.8			0.29
	0.2					0.295
	0.0	3				0.35
	0.2					0.362

Table 4

η	k_1^*	Pr	Mn	E	E_1	$\theta(\eta)$
0	0.0	3	0.8	1	0.0	0.997
	0.3					0.997
	0.0	5			0.0	0.996
	0.3					0.997
	0.0	5			0.4	1.011
	0.3					1.012
	0.0	3				
	0.3					
0.5	0.0	3			0.0	0.565
	0.3					0.584
	0.0	5			0.0	0.51
	0.3					0.529
	0.0	5			0.4	1.41
	0.3					1..445
	0.0	3				
	0.3					
1	0.0	3			0.0	0.261
	0.3					0.27
	0.0	5			0.0	0.2
	0.3					0.2
	0.0	5			0.4	1.096
	0.3					1.13
	0.0	3				
	0.3					

Table 3 represents the variation of temperature with the change of Hartmann number Mn, Prandtl number Pr and viscoelastic parameter k_1^*. These table as tabulated for the same set of data as that of table 2 except for local normal electric field parameter E_1=0.2. From these graphs interestingly we observe that, in presence of magnetic field, the effect of increasing values of Prandtl number Pr is to increase temperature near the stretching sheet very slightly and decrease the same largely away from the stretching sheet. The combined effect of decreasing values of Prandtl number Pr and increasing values of Hartmann number Mn is to increase temperature throughout the boundary layer in presence of local normal electric field. This is owing to the reason that local normal electric field acts as a heat source near the boundary sheet with reduction of magnitude with distance from the sheet.

Table 4 represents temperature profile for different values of local normal electric parameterE_1, Prandtl number Pr and viscoelastic parameter k_1^* when the effects of viscous dissipation and magnetic field are accounted. Analysis of the figure shows that temperature would attain maximum value on the boundary sheet when the electric field is not accounted and then heat transfer will take place from boundary wall to the adjacent fluid layer. When the local normal electric field is accounted in the heat transfer process then heat transfer will take place in reverse direction from adjacent fluid layer to the boundary sheet and temperature would attain maximum value in the boundary layer region little away from the stretching sheet. This is because Ohmic dissipation generated heat in the fluid layer near stretching sheet.

Same process is done for the PHF case. The Resulted similar qualitative effects of Hartmann number Mn, Prandtl number Pr, viscoelastic parameter k_1^*, Eckert number E and local normal electric parameter E_1 on temperature profile, but with different magnitudes. However, interestingly we notice that boundary sheet would always attain higher temperature in presence of magnetic and electric fields.

The effect of viscoelastic parameter k_1^* , in absence of local normal electric parameter E_1, is to reduce the rate of heat transfer. Whereas, effect of viscoelastic parameter k_1^*, in presence of local electric parameter E_1, is also to decrease the rate of heat transfer across the stretching sheet for small values of local normal electric parameterE_1. For higher values of Prandtl number Pr and local normal electric parameter E_1, the effect of increasing values of viscoelastic parameter k_1^* is to reverse the direction of heat transfer across the stretching sheet.

7. CONCLUSION

1) The combined effect of viscoelastic parameter k_1^* and local normal electric parameter E_1 is seen to decrease the boundary layer velocity throughout the boundary layer but more significantly little away from the stretching sheet.
2) The skin-friction coefficient C_f decreases with the increase of local Reynolds number Re_x and viscoelastic parameter k_1^* and it increases with the increase of local normal electric parameter E_1 and Hartmann number Mn.
3) When the electric field is accounted in the heat transfer process then heat transfer will take place from adjacent fluid layer to the boundary sheet and temperature would attain maximum value in the boundary layer region little away from the stretching sheet.
4) In presence of local normal electric parameter E_1, the effect of increasing values of Prandtl number Pr is to decrease the rate of heat transfer for smaller values of local normal electric parameter, but for larger values of local normal E_1 the direction of heat transfer would be changed in case of viscoelastic fluid.

CONFLICT OF INTERESTS

None.

ACKNOWLEDGMENTS

None.

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