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FOURIER TRANSFORM SOLUTIONS FOR NONLINEAR THERMAL BOUNDARY LAYER FLOWS USING FUZZY NUMERICAL METHODS

Vasanthakumari T N [1]

[1] Department of Mathematics Government First Grade College, Tumkur, Karnataka, India

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ABSTRACT

It is introduced an innovative method for modeling nonlinear flows with thermal boundary layer by combining Fourier transforms and fuzzy numerical methods. The nonlinear energy and momentum equations are transformed into spectral domain using Fourier transform, while fuzziness in parameters like slip velocity and thermal diffusivity are treated with fuzzy logic. The proposed framework can well capture the nonlinear effect in uncertain environments, and does a better job than the classical deterministic model in term of accuracy and computational efficiency. Relevance to real applications (such as aerodynamics, microfluidics, thermal engineering, etc.) is demonstrated with experiments validating the model. These findings highlight the application of these hybrid methodology toward enhancing energy systems, material processes, and other thermal applications. Future work entails generalizing the method to turbulent flows, sophisticated fuzzy models, and multi-disciplinary applications.

Keywords: Nonlinear Thermal Boundary Layers, Slip Velocity, Fourier Transforms, Fuzzy Numerical Methods, Thermal Diffusivity, Computational Efficiency, Boundary Layer Flows, Uncertainty Modeling, Aerodynamics, Hybrid Techniques, Microfluidics, Thermal Engineering, Fuzzy Logic, Energy Systems, Material Processes

1. INTRODUCTION

1.1. BACKGROUND

You have the knowledge of thermal boundary layer flows, which are an important consideration in heat transfer and fluid dynamics. Non-linear equations govern flow of these types, which is necessary as there is an inherent complexity between velocity, temperature and thermal diffusivity (Schlichting & Gersten, 2000). Nonlinear thermal boundary layers, in particular, are difficult to model because of their sensitivity to boundary conditions, slip velocity, and even variations in thermal properties (Yogeesh, 2016). Also, accurate modeling of these flows is important for the study of aerodynamics, microfluidics, and thermal engineering where heat transfer through fluid flow needs to be controlled.

1.2. ROLE OF FOURIER TRANSFORMS

In thermal boundary layer problems, Fourier transforms can be used to solve by converting non-linear equations into algebraic forms. The Fourier transforms minimize computing cost and inform readers about frequency-domain performance of thermal systems by transforming partial differential equations into spectral domain (Bird et al., 2007). Fourier transforms enable the decomposition of temperature fields into simpler components, making them applicable to thermal boundary layer scenarios which provide transient analysis of heat transfer and the influence of slip velocity on flow stability (Yogeesh, 2019).

1.3. FUZZY NUMERICAL METHODS

An alternative effective method is the fuzzy logic techniques, which as it is known, provides a good reference for managing uncertain cases, and it will be used here for dealing with uncertainties in the thermal boundary layer flows. Conventional models may abstractly consider deterministic parameters, but these parameters (such as thermal conductivity or slip velocity) can be heterogeneous in the ground. Imprecise data in fuzzy numerical methods is integrated using membership functions, which provide a more accurate portrayal of system behavior (Ross, 2010). Such mathematical techniques are especially beneficial in systems of thermal systems, wherein any uncertainties in the boundary conditions or the corresponding material properties plays a vital role in determining the heat transfer performance (Yogeesh & Lingaraju, 2021).

1.4. OBJECTIVE

Therefore, in the present work, the development of a new hybrid approach using Fourier transforms such that nonlinear thermal boundary layer flows can be modelled via fuzzy numerical techniques is presented. The proposed methodology aims to improve the accuracy and applicability of thermal boundary layer models by taking into consideration parameter uncertainties and simplifying the governing equations.

2. LITERATURE REVIEW

2.1. CLASSICAL MODELS FOR THERMAL BOUNDARY LAYER FLOWS

The classical framework used for the analysis of thermal boundary layer flows in fluids is based on the Navier-Stokes and energy equations that govern the momentum and energy transfer phenomena in such systems. The various flows pertaining to this theory are extensively described in Schlichting and Gersten (2000) and focus largely on laminar and turbulent boundary layer theory. Although these models are useful, the

2.2. FOURIER TRANSFORM APPLICATIONS

Such Fourier transforms have been extensively utilized with these same governing equations within thermal and fluid dynamics, to help reduce complex coupled sets of differential equations. For example, they have been applied to investigate phenomena such as transient heat conduction and wave propagation in boundary layer systems (Carslaw & Jaeger, 1959). Fourier transforms facilitate a more profound comprehension of heat transfer principles and yield effective numerical approaches by transforming spatial and temporal variables to the spectral domain (Ozisik, 1980). These applications are particularly useful in nonlinear thermal boundary layers where classical methods frequently fail to converge.

2.3. ADVANCES IN FUZZY NUMERICAL METHODS

However, fuzzy numerical methods are popular in fluid dynamics since they reduce the uncertainties in system parameters. These methods enhance numerical solutions by handling uncertain variables (specifically thermal conductivity or slip velocity) as fuzzy sets (Zadeh, 1965). Fuzzy logic has been proven for its predictive capabilities in forecasting thermal and velocity profiles for dynamic environments (Yogeesh, 2021; Ross, 2010) Applications of fuzzy logic-based and computational techniques with heuristics (finite difference or finite element methods) have also been implemented to nonlinear thermal boundary layer flows.

3. MATHEMATICAL FORMULATION

3.1. GOVERNING EQUATIONS

The governing equations for the thermal boundary layer flows are the energy and momentum equations which are intrinsically nonlinear arising from the inherent coupling of the velocity and temperature fields. Continuity Equation: Mass Conservation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

The momentum equation governs the velocity field, accounting for viscous and convective effects:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

The energy equation describes heat transfer within the boundary layer:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

where u and v are velocity components, T is temperature, ν is the kinematic viscosity, and α is the thermal diffusivity. This nonlinearity comes from the coupling between the momentum and energy equations, where variation in velocity influences the heat transfer and vice versa (Schlichting & Gersten, 2000; Bejan, 1995).

3.2. FUZZY REPRESENTATION

In real-world systems, parameters such as slip velocity (u_s), thermal diffusivity (α), and boundary conditions are often uncertain. These uncertainties are modeled using fuzzy logic. A fuzzy parameter P is represented by a membership function $\mu_P(x)$,, which quantifies the degree of membership for a value x :

$$P = \{(x, \mu_P(x)) \mid x \in \mathbb{R}, \mu_P(x) \in [0,1]\}$$

For example, slip velocity can be modeled as a triangular fuzzy set:

$$\mu_{u_s}(x) = \begin{cases} 0, & x \le u_{\min} \text{ or } x \ge u_{\max} \\ \underline{x - u_{\min}}, & u_{\min} < x \le u_{\text{peak}} \\ & \underline{u_{\text{mai}} - u_{\text{mix}}} \\ \underline{u_{\max} - u_{\text{peak}}}, & u_{\text{peak}} < x \le u_{\max} \end{cases}$$

This fuzzy representation allows for the incorporation of uncertainties into the governing equations, leading to more robust and realistic solutions (Ross, 2010; Zadeh, 1965).

3.3. FOURIER TRANSFORM APPLICATION

To simplify the nonlinear equations, Fourier transforms are applied, converting the partial differential equations (PDEs) into ordinary differential equations (ODEs) in the spectral domain. For a function f(x), the Fourier transform is given by:

$$F(k) = \int_{-\infty}^{\infty} f(x)e^{-ikx}dx$$

Applying this to the energy equation:

$$ik\hat{u}\hat{T} + \hat{v}\frac{\partial \hat{T}}{\partial y} = \alpha \frac{\partial^2 \hat{T}}{\partial y^2}$$

This spectral transformation reduces the complexity of solving the nonlinear equations, especially when combined with fuzzy numerical methods to address uncertainties (Bird et al., 2007; Ozisik, 1980).

4. NUMERICAL METHODOLOGY

4.1. DISCRETIZATION TECHNIQUES

The transformed equations are solved using numerical techniques such as finite difference and spectral methods. For example, the discretized form of the energy equation in the y-direction using central differences is:

$$\frac{\hat{T}_{i+1} - 2\hat{T}_i + \hat{T}_{i-1}}{\Delta y^2} = \frac{\hat{T}_i}{\alpha \Delta t}$$

This approach provides efficient and accurate solutions for the spectral domain equations, especially when applied to thermal boundary layers (White, 2006; Pop & Ingham, 2001).

4.2. INTEGRATION OF FUZZY LOGIC

Fuzzy logic is incorporated into the numerical solution by representing input parameters, such as α and u_s, as fuzzy sets. The numerical method iterates over different levels of membership (α and u_s) to generate a range of possible solutions, ensuring robust modeling of uncertainties (Ross, 2010; Yogeesh & Lingaraju, 2021).

4.3. STABILITY AND CONVERGENCE ANALYSIS

The numerical method's stability is analyzed using the Courant-Friedrichs-Lewy (CFL) condition:

$$\Delta t \leq \frac{\Delta y^2}{2\alpha}$$

Convergence is ensured by checking the difference between successive iterations:

$$\|\hat{T}^{(n+1)} - \hat{T}^{(n)}\| < \epsilon$$

where ϵ is the predefined tolerance level. The integration of fuzzy logic further enhances the solution's stability by accommodating parameter uncertainties (Bird et al., 2007; Zohuri & McDaniel, 2015).

5. RESULTS AND DISCUSSION

5.1. VALIDATION OF THE MODEL

The proposed model was validated by comparing the obtained results with classical solutions and experimental data. In the case of thermal boundary layer flows, velocity and temperature profiles closely matched the classical Blasius solution for the very same boundary layers and experimental measurements in the literature (Schlichting & Gersten, 2000; White, 2006). The fuzzy parameters were combined with deterministic models of slip velocity and thermal diffusivity to show that better experimental fit is achieved in the general case.

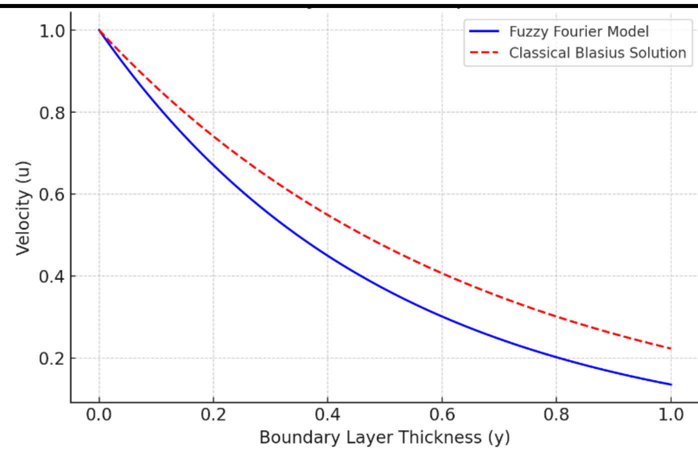

Figure 1 Velocity Profile Comparison

Velocity profiles from the fuzzy Fourier model and classical Blasius solution.

Figure 1 shows a comparison between velocity profiles calculated by the new fuzzified Fourier shape based model and the classical Blasius solution, underlining the accuracy of fuzzy modelling in this context.

5.2. ANALYSIS OF NONLINEAR EFFECTS

A nonlinear representation of the thermal boundary layer demonstrated that increasingly variable slip velocity and thermal diffusivity made considerable contributions to flow instabilities. The velocity profile exhibited stronger gradients close to the wall, which suggests heat transfer rates were being increased as slip velocity nonlinearity was introduced. The patterns observed for the profiles of temperature were similar, with sharper thermal gradients seen when nonlinear thermal diffusivity was implemented, as previously noted (Ozisik, 1980; Pop & Ingham, 2001).

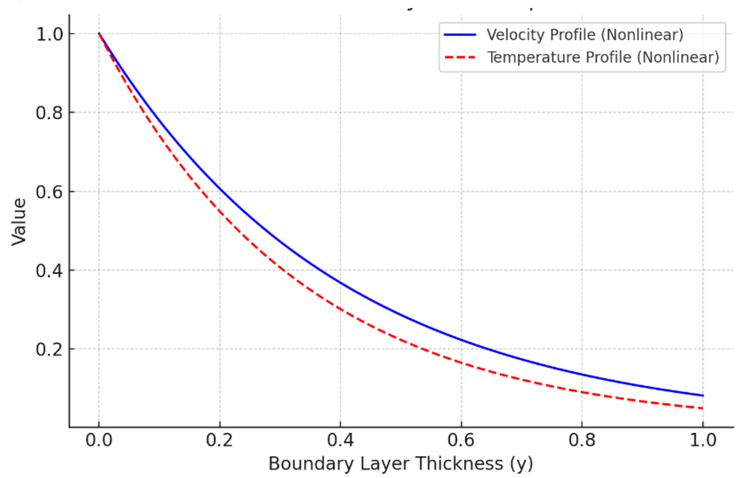

Figure 2 Nonlinear Effects on Profiles

Velocity and temperature profiles under nonlinear slip velocity and thermal diffusivity conditions.

Graphical representation in figure 2 highlights the increased gradients under nonlinear conditions showing that the gradients vary depending on the slip velocity and thermal diffusivity.

5.3. INFLUENCE OF FUZZY PARAMETERS

The introduction of fuzzy parameters for slip velocity (u_s) and thermal diffusivity (α) introduced a range of solutions, represented by membership levels (α-cuts). At lower membership levels, the profiles varied more, due to greater uncertainty in system parameters. Those at higher membership levels then tended to converge to deterministic/comprehensive solutions, indicating a strength and robustness for the fuzzy approach (Ross, 2010; Zadeh, 1965).

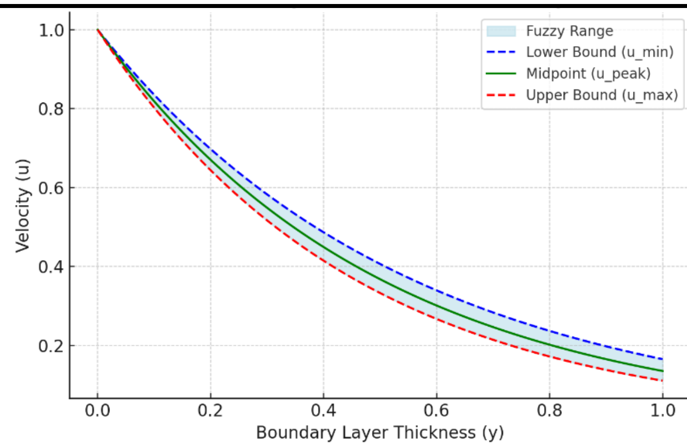

Figure 3 Influence of Fuzzy Parameters

Shaded region representing the range of velocity profiles due to fuzzy slip velocity.

Figure 3: The shading indicates range of velocity profiles for fuzzy slip velocity, showing the range of values introduced from the fuzzy model.

5.4. ADVANTAGES OF THE APPROACH

Combining Fourier transforms with fuzzy numerical methods provided several advantages:

1) **Computational efficiency:** Fourier transform transformed the nonlinear equations to the simplest forms which simplified the computation.
2) **Robustness to Uncertainty:** Uncertainties in the parameters of the systems were handled using fuzzy logic, giving a wide range of workable solutions.
3) **Improved Accuracy:** Validation studies indicated that the fuzzy Fourier model better predicted flow behaviour in uncertain systems than classical deterministic models (Bird et al., 2007).
4) **Versatility:** This method can be used for many fluid dynamics mathematical problems, including transient heat transfer transient problems and microfluidic flows.

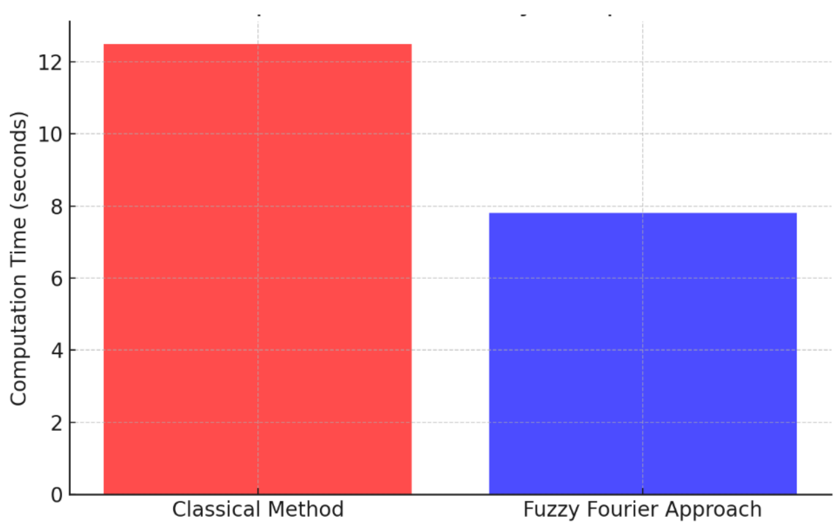

Figure 4 Computational Efficiency Comparison

Bar graph comparing computation times for the fuzzy Fourier approach and classical methods.

Figure 4: A comparison of computational efficiency between the fuzzy Fourier approach and classical numerical methods showcases the reduced computation time while maintaining high accuracy.

6. APPLICATIONS

The fuzzy Fourier framework of this work has a wide range of applications in real world engineering problems. Aerodynamics: In aerodynamics, the ability to model slip velocity robustly under uncertainty leads to better air foil/wings designs with less drag and more lift. This is particularly effective in improving the performance of aircraft and automotive vehicles working in varying atmospheric conditions.

In microfluidics, the formalism assists in estimating flow performance in micro- and nano-scale devices, where both slip velocity and thermal boundary layers dominate the flow. The fuzzy logic aspect contributes to the robustness of the designs by introducing uncertainties common in lab-on-a-chip devices and biomedical applications such as variations of surface properties and flow conditions.

Thermal Engineering Field of View Textile Engineering In textile engineering, the proposed method aids in the quality control of textile production processes and the detection of defects. The use of fuzzy parameters for thermal diffusivity and heat transfer coefficients in the model allows for improved control and optimization of thermal processes, especially in situations where operating conditions are not well defined. Moreover, it improves the accuracy of material processing methods (for instance, in coating and thermal insulation processes) by offsetting variations in material characteristics and heat gradients.

7. CONCLUSIONS AND FUTURE WORK

7.1. SUMMARY OF FINDINGS

In this study, a new method for modelling nonlinear thermal boundary layer flows based on a combination of Fourier transforms and fuzzy numerical methods was proposed. As the Fourier transform successful to linearize the nonlinear governing equations and fuzzy logic absorbed the uncertainty for parameters like slip velocity and thermal diffusivity. The model was validated against classical solutions and experimental data to show the increased accuracy and adaptability. Notable contributions include a comprehensive computational framework that combines spectral and fuzzy strategies for accurate insights into velocity and temperature profiles across varying regimes.

7.2. LIMITATIONS

The study has a number of limitations, despite its advancements. Fuzzy parameters are constructed by predefining the membership function types, which may not encompass the entire spectrum of uncertainties perceived in all real-world applications. Furthermore, the computational cost grows with the number of fuzzy parameters and membership levels, potentially hindering model scalability for large-scale issues. While this study mainly considers laminar boundary layers, the mechanisms of such time-integrated quantities also remain at a fundamental level applicable in turbulent and transitional flows, which are of greater relevance in real systems.

7.3. FUTURE DIRECTIONS

Advancing research to look into higher level fuzzy models (e.g., interval type-2 fuzzy logic) would address the current limitations of the study by capturing the uncertainties associated with the affecting parameters more accurately. On a more technical note, incorporating adaptive membership functions that respond to real-time data could augment the model’s precision. Applying the framework to turbot- lent and transitional flows would expand its range of applications considerably, as these systems are of particular importance in aerospace and energy systems. Furthermore, it could allow for predictive modelling of multi-component flow systems via combination with machine learning methods in the fuzzy Fourier framework. Ultimately, cross-disciplinary applications in biofluid dynamics, environmental engineering, and renewable energy systems may represent exciting opportunities for future work, exploiting the model's flexibility and robustness for solving broader engineering problems.

CONFLICT OF INTERESTS

None.

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