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INDIA'S CONTRIBUTION IN MATHEMATICS

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ABSTRACT

Mathematics is the Language of the Universe. Mathematics is the Language of the God. Oldest source of knowledge, the four Vedas originated in India. India is a land of knowledge. India has rich contribution in mathematics. Great Indian Mathematicians like Aryabhatta, Brahmagupta, Bhaskara, Mahavira, Madhava , whose contribution in Mathematics made them immortal. This paper explains their contribution in the field of Mathematics. Zero is the greatest gift to the World by India. Bauthayana explained the result of Pythogoras theorem. Pingala's Meruprastara is known as Pascal's triangle.

Keywords: Shunya, Bakshaali Manuscript, Vedic Saṃhitā, Baudhayana's Subla Sutra, Sine Function, Sanskrit

1. INTRODUCTION

Origin of Zero(Shunya)

In India, the zero as a concept probably predated zero as a number by hundreds of years. The Sanskrit word for zero, shunya, meant void or empty. The word is probably derived from shunya which is the past participle of svi, to grow. In one of the early Vedas, Rigveda, occurs another meaning: the sense of lack or deficiency. It is possible that the two different words, were fused to give shunya a single sense of absence or emptiness with the potential for growth

The earliest mention of a symbol for zero occurs in the Chandahsutra of Pingala (fl. 3rd century BC) which discusses a method for calculating the number of arrangements of long and short syllables in a metre containing a certain number of syllables (ie. the number of combinations of two items from a total of n items, repetitions being allowed). The symbol for shunya began as a dot (bindu), found in inscriptions both in India and in Cambodia and Sumatra around the seventh and eighth century and then became a circle (chidra or randra meaning a hole).

Indian system of enumeration was used in Bakshaali manuscript (200-400CE) . Algebraic properties of the number Zero, as a number in itself like all the numbers were first formalized is in Brahmagupta's Brahmasphuta — Siddhanta in 628 CE.

The Hindu Number System

The way we write our numerals today comes from India. Any number however large can be written using only 10 symbols 1,2,3,4,5,6,7,8,9 and 0.

The first surviving reference in which Indian system of enumeration was used in Bakshaali manuscript (200—400 CE). The Hindu Number System was transmitted to Arab world by around 800 CE. It introduced by the great Persian Mathematician AI— Khwarizmi(on the Calculation with Hindu Numerals c. 825 CE) and by great Philosopher AI— Kindi (on the use of Hindu Numerals, c. 830)

Arab transmitted to Europe by around 1100 CE, therefore the European thus mistakenly called it the ‘Arabic Number System’.

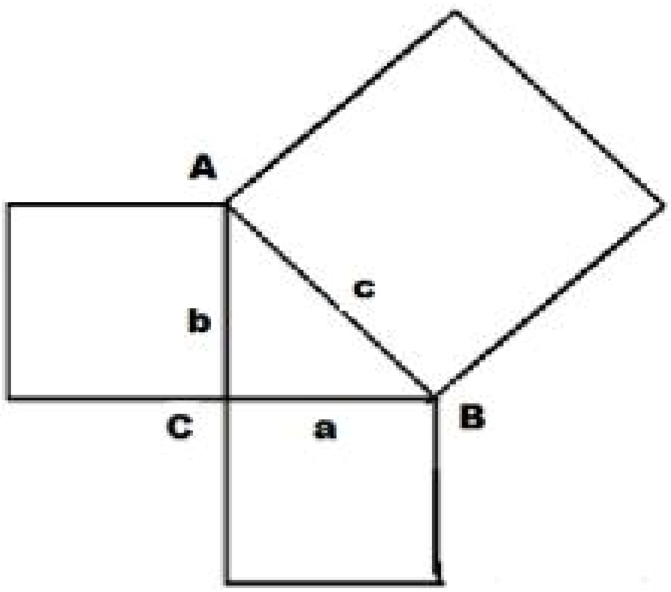
Vedic saṃhitā texts, especially the Ṛgveda and the Taittirīya Saṃhitā recension of the Yajurveda (1200–1000 bce). They have plenty to tell us about the beginnings—as far as we can tell now—of the ‘decimal culture’. The Ṛgveda, in particular, is rich in number names which reflect faithfully the principles governing the formation of based numbers with 1010 as the base.

The Baudhayana— Pythagoras Theorem

The Pythagorean Theorem was known before Pythagoras(c. 500BC). The theorem is explicitly stated for a general triangle in the way we describe it in Baudhayana's Subla Sutra(c.800BC)

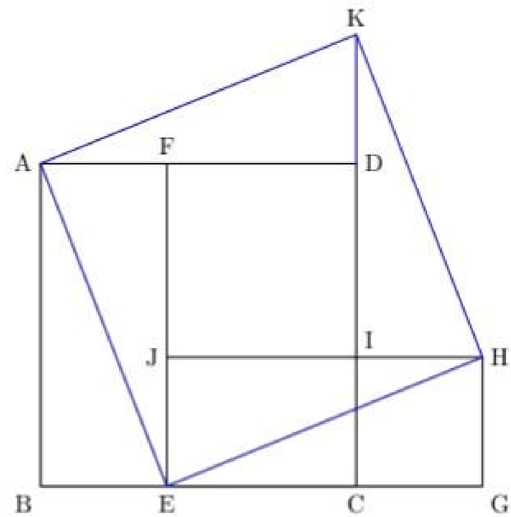
1.45 " दीर्घचतुरश्रस्याक्ष्णया रज्जु: पार्श्र्वमानी तिर्यग् मानी च यत् पृथग् भूते कुरूतस्तदुभयं करोति ॥

“A rope stretched along the diagonal produces an area which the vertical and Horizontal sides make together.”



$$c^2 = a^2 + b^2$$

1.48 “ To combine different squares mark out the rectangle from the larger with a side of the smaller. The diagonal of this rectangle is a side of the sum.”



"To combine different squares [ABCD and ICGH], mark out the rectangle [ABEF] from the larger (square) [ABCD] with a side [BE=CG] of the smaller one [ICGH]. The diagonal [AE] of this rectangle [ABEF] is a side of the sum (of the two squares) [AEHK]."
That the area of AEHK is the sum of the areas of ABCD and ICGH becomes obvious from the described diagram, observing that the triangles ABE and EGH are being replaced, respectively, by the congruent triangles KHI and ADK.

Sine function
The sine function had it's origin in the work ' Aryabhatiya' of Aryabhatta. The first appearance of the sine of an angle appears in the work of the Hindu Aryabhata the Elder (476-550), in about 500, that gives tables of half chords (that are 120 times the sine) based on the Greek half-angle formula and uses the word of jya to describe these quantities [5, 6, 9]. The same sort of table was presented by Brahmagupta (in 628) and a detailed method for constructing table of sine was presented by Bhaskara in 1150.
Sine (jya) and Cosine(Koti-jya) introduced by Aryabhatta (c.SIOCE)
Jya means— half chord which is transliterated from Sanskrit to Arabic as jiba. The word jiba was incorrectly translated to jaib. Meaning of jaib is bosom and so the Latin word Sinus for bosom was used for the half- chord function and Sinus became the word Sine.
Sine (Jyā) –The Sanskrit word Jya means a bow-string and hence the Chord of an arc. The arc of a circle is called Chapa in Sanskrit. Generally, Jya means the straight line of one point to another In a circumference of a circle which is known as 'chord' in English. But in astronomical calculations, half of a chord is Called jya which is called Sine in English. Bhaskaracharya Clearly says in Siddhanta Shiromani –

अर्धज्याग्रे खेचरो मध्यसूत्रात् तिर्यक्संस्था जायते येन तेन।
अर्धज्याभि: कर्म सर्वं ग्रहाणामर्धज्यैर्व ज्याभिधानात्र वेंद्या॥1

That means, the planets moves in their orbits on the tip of Ardhajya. Hence all astronomical calculations are in the base On ardhajya. So in Astronomy, ardhajya is called jya. Jya or sine is the horizontal line drawn from the apex of the Arc towards the base of the arc to the diameter.
Versed Sine (Utkramajyā or Bhujotkramajyā) –Utkruma refers to a concept of reversal, outward movement, or surpassing. Therefore, the term utkramajya essentially translates to "reversed sine." This function earns its name in contrast to krama-jya because its tabulated values are obtained by subtracting elements from the radius in reverse order from the tabulated values of the latter. In simpler terms, it represents the exceeding part of the krama-jya, considered in reverse sequence.Thus, in Sürya Siddhanta it is stated:

प्रोज्झ्योत्क्रमेण व्यासार्धादुत्क्रमज्यार्धपिण्डका: ॥
"The (tabular) versed sines are obtained by subtracting from the radius the (tabular) sines in the reversed order."

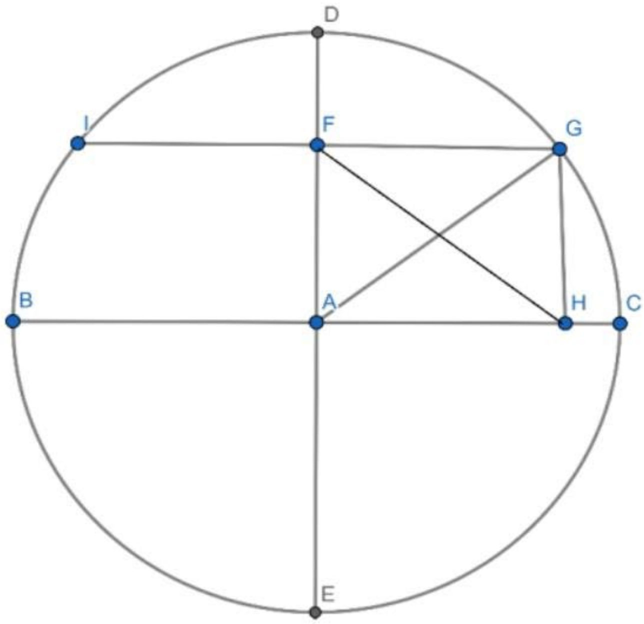


Figure 1: Sine functions in a circle

In Figure 1,
$\widehat{DG}$ = Arc
IG = Chord
FG = Sine
GH = Cosine
FD = Versed Sine
HC = Co-versed Sine
AD = AG = AC = Radius
AFGH Parallelogram. Opposite bases of parallelogram ar equal.
Hence, HF = AG

$$\sqrt{FG^2 + GH^2} = HF = AG$$

$$\sqrt{AG^2 - GH^2} = FG$$

$$\sqrt{AG^2 - FG^2} = GH$$

AD – GH = AD – AF = FD = Versed Sine
AC – FG = AC – AH = HC= Co-versed Sine

Radius − Cosineθ = versed sine

Versed Co-Sine (Kotyutkramajyā) –
The segment of the radius from the point to which the tip of the cosine touches the radius to the arc is called co-versed sine. Radius – sineθ = Co-versed Sine

Formula to find sin 18° and sin 72° –
Samrat Jagannatha mentioned a method of finding sine 18° and sine 72° in Siddhanta Samrat. This method is detailed in the following verse:

त्रिज्यार्धवर्गादिषुत्रिषुसङ्गुणाच्च मूलं खरामांशकजीवयोनम् ।
तिर्धकं स्याद्धृस्याद्धृतिभागजीवा ततत्कोटिजीवा दिव्नगांशकाम् ।।

$$\text{Sin}18^0 = \frac{\sqrt{\left(\left(\frac{R}{2}\right)^2\right) X\,5} - \text{Sin}\,30^0}{2} =$$

This formula provides a method for calculating the sine of 18 degrees. It begins by taking the radius of a circle, dividing it by 2, and then squaring the result. Next, this squared value is multiplied by 5. After finding the square root of this product, the sine of 30 degrees is subtracted from it. Finally, the resulting value is divided by 2 to obtain the sine of 18 degrees. The cosine of 18 degrees is equal to the sine of 72 degrees.

cos 18° = sin 72°

Bhaskaracharya also states two formulas to find Sin $\frac{\theta}{2}$
first formula is:

क्रमोत्क्रमज्याकृतियोगमूद्दलं तदर्धांशकशिञ्जिनी स्यात्।।

$$\frac{\sqrt{Sin^2\,\theta + verSine^2\theta}}{2} = Sin\frac{\theta}{2}$$

This formula illustrates a trigonometric identity relating the sine (sin) and versine (R – cos) functions. It states that the square root of the sum of the squares of the sine and versine of an angle (θ), divided by 2, equals the sine of half that angle $\frac{\theta}{2}$. In simpler terms, if you take the sine and versine of an angle, square them, add the results, find the square root of that sum, and then divide by 2, you will obtain the sine of half the angle. This identity is a consequence of trigonometric properties and is often used in trigonometric calculations and
proofs.

The second formula is:
टत्रज्योत्क्रमज्याटनहतेदधलस्य मूलं तदर्धांशकशिञ्जिनी वर्ा ।

$$\sqrt{\frac{R\ X\ verSin\theta}{2}} = Sin\frac{\theta}{2}$$

Mathematical foundations of sine functions, highlighting the rich heritage and enduring contributions of ancient Indian mathematics to modern trigonometry.

Negative numbers

In India , negative numbers did not appear until about 620 CE in the work of Brahmagupta (598 – 670) who used the ideas of 'fortunes' and 'debts' for positive and negative. By this time a system based on place-value was established in India, with zero being used in the Indian number system. Brahmagupta used a special sign for negatives and stated the rules for dealing with positive and negative quantities as follows:
A debt minus zero Is a debt.
A fortune minus zero is a fortune.
Zero minus zero is a zero.
A debt subtracted from zero is a fortune.
A fortune subtracted from zero is a debt.
The product of zero multiplied by a debt or fortune is zero.

The product of zero multiplied by zero is zero.
The product or quotient of two fortunes is one fortune.
The product or quotient of two debts is one fortune.
The product or quotient of a debt and a fortune is a debt.
The product or quotient of a fortune and a debt is a debt.
The algebraic properties of negative numbers were first formalized and described in Brahmagupta's Brahmas-phitasiddhanta in 628CE. The introduction of zero and the negative numbers to our number system is indeed one of the great leaps forward in the history of mathematics.

Brahmagupta and Quadratic equations.
The first explicit statement of the solution for general quadratic equation

$ax^2 + bx + c = 0$ was given by Brahmagupta in his Brahmas-phitasiddhanta in 628 CE. He wrote "To the negative of the constant coefficient multiplied by four times the coefficient of the square, add the square of the coefficient of the middle term, the square root of this minus the coefficient of the middle term being divided by twice the coefficient of the square, is the value."

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Brahmagupta together with Bhaskra gave full solution in integer to the Brahmagupta—Pell Equation, which is a quadratic equation of the form $x^2 - n\,y^2 = \pm 1$and of great importance.

Virahanka's numbers
Virahanka's numbers which are known as Fibonacci numbers. These numbers were discovered in an artistic poetry context which really illustrates the fine line between science and art. The question raised by ancient poets was: How many rhythms can one construct of exactly say 8 beats consisting of short and long syllables?

Poets in ancient India came up with a very ingenious solution to obtaining an answer to this question, which is now part of a huge mathematical and artistic theory.

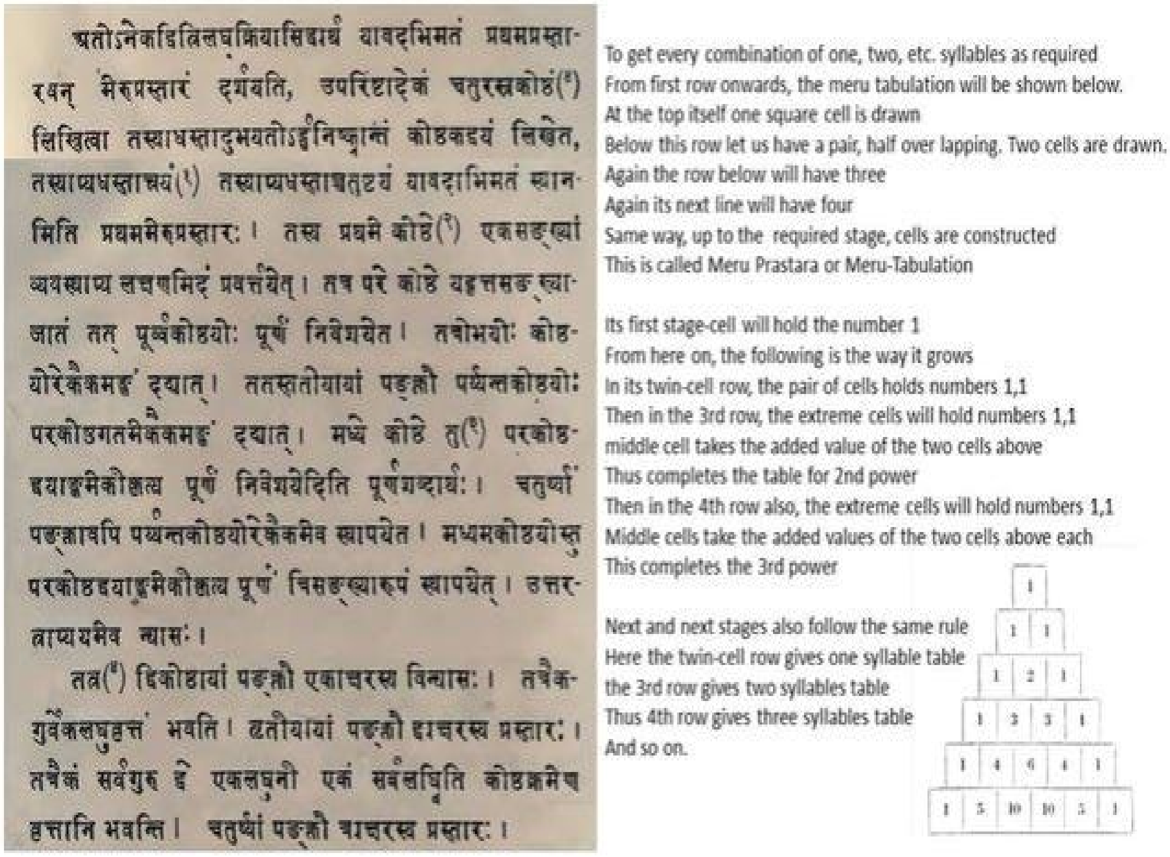
Virahanka's elegant solution (c.700CE)
Write down the numbers 1 and 2 then each subsequence number is obtained by adding up the two previous numbers. The nth number gives the number of rhythms having n beats.
1,2,3,5,8,13,21,34,55,……..
So the answer of the problem was 34,as the 8^{th} term of the sequence is 34.
Yielding the Virahanka numbers these numbers are known as Fibonacci numbers after Italian mathematician (c.1200). The numbers were also studied earlier by Gopala(1135) and Hemachandra (1150) again in the context of poetry.

Binomial Coefficient
The mathematician Mahavira (c.850) gave the direct formula for all the binomial Coefficients namely

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

In India Pingala's Meruprastara is called Pascal's Triangle.

मेरु प्रस्तार (From हलायुध commentary on छन्दशास्त्र, by पिङ्गल)


अतोऽनेकद्वित्रिलघुक्रियासिद्ध्यर्थं यावदभिमतं प्रथमप्रस्तारवत् मेरुप्रस्तारं दर्शयति, उपरिष्टादेकं चतुरस्रकोष्ठं(१) लिखित्वा तस्याधस्तादुभयतोऽर्द्धनिष्क्रान्तं कोष्ठकद्वयं लिखेत्, तस्याप्यधस्तात्त्रयं(२) तस्याप्यधस्ताच्चतुष्टयं यावदभिमतं स्थानमिति प्रथममेरुप्रस्तारः। तस्य प्रथमे कोष्ठे(३) एकसङ्ख्यां व्यवस्थाप्य लक्षणमिदं प्रवर्त्तयेत्। तत्र परे कोष्ठे यद्वृत्तसङ्ख्याजातं तत् पूर्वकोष्ठयोः पूर्णं निवेशयेत्। तत्रोभयोः कोष्ठयोरेकैकमङ्कं दद्यात्। ततस्तृतीयायां पङ्क्तौ पर्यन्तकोष्ठयोः परकोष्ठगतमेकैकमङ्कं दद्यात्। मध्ये कोष्ठे तु(४) परकोष्ठद्वयाङ्कमेकीकृत्य पूर्णं निवेशयेदिति पूर्णशब्दार्थः। चतुर्थ्यां पङ्क्तावपि पर्यन्तकोष्ठयोरेकैकमेव स्थापयेत्। मध्यमकोष्ठयोस्तु परकोष्ठद्वयाङ्कमेकीकृत्य पूर्णं त्रिसङ्ख्यारूपं स्थापयेत्। उत्तरत्राप्ययमेव न्यासः।

तत्र(५) द्विकोष्ठायां पङ्क्तौ एकाक्षरस्य विन्यासः। तत्रैकगुर्वेकलघुवृत्तं भवति। तृतीयायां पङ्क्तौ द्व्यक्षरस्य प्रस्तारः। तत्रैकं सर्वगुरु द्वे एकलघुनी एकं सर्वलघ्विति कोष्ठक्रमेण वृत्तानि भवन्ति। चतुर्थ्यां पङ्क्तौ त्र्यक्षरस्य प्रस्तारः।

Madhava's exact formula for π

The infinite series for pi is mostly today known as Leibniz formula for π. But many few people know that this series was already discovered in India by Madhava (c. 1340–1425 AD) of Sangamagrama, 300 years before Leibniz or Gregory. Although none of the Madhava's works have survived but most of the series attributed to him can be found in the books of his students and at many places these authors have clearly stated that these series are “As told by Madhava”.

The unique thing about the series given by Madhava is that he gave this series in form of a beautiful verse.

Infinite series for pi — as given in Kriyakramakari :

व्यासे वारिधिनिहते रूपहृते व्याससागराभिहते।
त्रिशरादिविषमसङ्ख्याभक्तमृणं स्वं पृथक् क्रमात् कुर्यात्॥

This translates to

“The diameter multiplied by four and divided by unity (is found and stored). Again, the products of the diameter and four are divided by the odd numbers like 3,5 etc., and the results are subtracted and added in order (to earlier stored result).”

This in other words is:

First exact formula for a π was given by Madhav(c. 1400) who showed that

$$\pi = 4(\,1 - \frac{1}{3} + : \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots \ldots \ldots ..)$$

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None.

CONFLICT OF INTEREST

None.

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