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THERMAL STRESSES IN TWO-LAYER COMPOSITE SLAB WITH DISCONTINUOUS AT THE INTERFACE

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ABSTRACT

This paper is concerned with the determination of temperature distribution T_i (y), i=1,2 and thermal stresses σ_xi of the transient thermoelastic problem of two-layer composite slabs with outer surfaces at y=-h_1 and y=h_2 and interface at y=0.Let b is the width of the slab which is the same for both the layers and h_i , i=1,2 is the height of each layer. Initially, both the layers of the slab are kept at T_0 temperature. For time t>0 the outer boundary surfaces y=-h_1 and y= h_2 are kept at zero prescribed temperature. The properties of the solid are uniform with each layer but discontinuous at the interface at y=0. The solution can be determined with prescribed initial and boundary conditions and expressed graphically.

Keywords: Temperature Distribution, Thermal Stresses, Composite, Interface, Slab, Two-Layer

1. INTRODUCTION

Adams and Bert [2] have studied the direct problem of thermoelasticity in a rectangular plate under thermal shock. W. K. Dange et. al. [5,6] have studied the thermoelastic Problem of a Thin rectangular plate and square plate. A. Vattr´ ea. E. Panb [1]has derived Three-dimensional exact solutions for temperature and thermoelastic stresses in multilayered anisotropic plates for advanced boundary-value problems with general boundary conditions . Laxman V Bharambe et. al.[4] have studied the stress and deformations in the elliptical and rectangular plates with and without cutout by using finite element analysis . Huang Sheng et. al. [5] has investigated the stress state of the rectangular domain.This paper aims to evaluate temperature distribution $T_i(y)$, $i = 1,2$ and thermal stresses σ_{xi} of the transient thermoelastic problem of two layer composite slab with outer surface at $y = -h_1$ and $y = h_2$ and interface at $y = 0$.Let b is width of slab which is the same for both the layers and h_i , $i = 1,2$ is height of each layer. Initially both the layers of slab are kept at T_0 temperature. For time $t > 0$ the outer boundary surfaces $y = -h_1$ and $y = h_2$ are kept at zero prescribed temperature . The properties of the solid are uniform with each layer but discontinuous at the interface at $y = 0$. The orthogonal expansion technique is used to determine the solution and assessed graphically.

2. STATEMENT OF THE PROBLEM

Consider two-layer composite slab with outer boundaries at $y = -h_1$ *and* $y = h_2$ and interface at $y = 0$.Let b is the width of the slab which is the same for both the layers and h_i , $i = 1,2$ is the height of each layer. Let temperature distribution $T_i(y)$, $i = 1,2$. Transient boundary value problem *of heat conduction* equation is:

$$\frac{\partial^2 T_i}{\partial y^2} = \frac{1}{\alpha_i}\frac{\partial T_i}{\partial t} \quad , i = 1,2 \qquad (1)$$

Where α_i , $i = 1,2$ is thermal diffusivity of the slab.

Subject to the initial conditions

$$M_t(T_1, 1,0,0) = T_0 \text{ For all } -h_1 \le y \le 0 \qquad (2)$$

$$M_t(T_2, 1,0,0) = T_0 \quad 0 \le y \le h_2 \qquad (3)$$

and boundary conditions

$$M_y(T_1, 1,0, -h_1) = 0 \qquad (4)$$

$$M_y(T_1, 0, k_1, 0) = M_y(T_2, 0, k_2, 0) \text{ at interface at } y = 0 \text{ and } t > 0 \qquad (5)$$

$$M_y(T_1, 0, -k_1, 0) = h(T_1 - T_2) \text{ at interface at } y = 0 \text{ and } t > 0 \qquad (6)$$

$$M_y(T_1, 1,0, h_2) = 0 \qquad (7)$$

Where h is contact conductance at the interface and k_i , $i = 1,2$ are thermal conductivities of the slab.

Thermal stresses are given by

$$\sigma_{\ xi}(y) = -\alpha_{t_i} E_i T_i\,(y) + \frac{2bE_i}{D}\{2[\aleph_1 + \aleph_2][E_2 {h_2}^3 + E_1 {h_1}^3] - 3[\aleph_3 + \aleph_4][E_2 {h_2}^2 + E_1 {h_1}^2]\} + \frac{6E_i}{D}\{2[\aleph_3 + \aleph_4][E_2\, h_2 b + E_1 h_1 b] - [\aleph_1 + \aleph_2][E_2 {h_2}^2 b + E_1 {h_1}^2 b]\} \quad i = 1,2 \quad (8)$$

$$\text{Where } D = b^2\left(E_2 {h_2}^2 - E_1 {h_1}^2\right)^2 + 4\, b^2 h_1 h_2 E_1 E_2 (h_1 + h_2)^2 \qquad (9)$$

$$\aleph_1 = \int_{-h_1}^{0} \alpha_{t_1} E_1\, T_1\,(y) dy \qquad (10)$$

$$\aleph_2 = \int_{0}^{h_2} \alpha_{t_2} E_2\, T_2\,(y) dy \qquad (11)$$

$$\aleph_3 = \int_{-h_1}^{0} \alpha_{t_1} E_1\, y T_1\,(y) dy \qquad (12)$$

$$\aleph_4 = \int_{0}^{h_2} \alpha_{t_2} E_2\, y T_2\,(y) dy \qquad (13)$$

Since external forces do not act on the slab then

$$\int_{-h_1}^{0} \alpha_{t_1} E_1\, \sigma_{\ x1} dy + \int_{0}^{h_2} \alpha_{t_2} E_2\, \sigma_{\ x2} dy = 0 \qquad (14)$$

And

$$\int_{-h_1}^{0} \alpha_{t_1} E_1 y\, \sigma_{\ x1} dy + \int_{0}^{h_2} \alpha_{t_2} E_2\, y\sigma_{\ x2} dy = 0 \qquad (15)$$

Where α_{t_i} $i = 1,2$ *are* thermal expansion coefficient, E_i, i= 1,2 are Young's modulus

3. DETERMINATION OF TEMPERATURE FIELD

temperature with prescribed boundary and initial conditions are given by

$$T_i(y,t) = \sum_{n=1}^{\infty} A_n Y_{in}(y) \exp\left(-\alpha_i\, {\beta_{in}}^2 t\right) \quad , i = 1,2 \qquad (16)$$

Where $Y_{in}(y)$ is eigenfunctions which are chosen in the form

$$Y_{1n}(y) = sin\beta_{1n}\, y + D_{1n}\, cos\beta_{1n}\, y \qquad (17)$$

$$Y_{2n}(y) = C_{2n}\, sin\beta_{2n}\, y + D_{2n}\, cos\beta_{2n}\, y \ (18)$$

Where D_{1n} , C_{2n} , D_{2n} are unknown coefficients to be determined from given boundary conditions (4), (5) *and* (7) as follows

$$D_{1n} = \tan\, \beta_{1n} h_1 \qquad (19)$$

$$C_{2n} = \sqrt{\frac{\alpha_1}{\alpha_2}\frac{k_1}{k_2}} \qquad (20)$$

$$D_{2n} = -C_{2n} \tan\, \beta_{2n} h_2 \qquad (21)$$

β_{1n} *and* β_{2n} *are* eigenvalues of eigenfunctions $Y_{1n}(y)$ and $Y_{2n}(y)$ respectively. Equation (6) will be used to determine the relationship between eigenvalues β_{1n} and β_{2n}. As time is the same on either side of the interface due to their no storage of energy at the interface then

$$\beta_{2n} = \sqrt{\frac{\alpha_1}{\alpha_2}}\ \beta_{1n} \qquad (22)$$

And

$$\beta_{1n} = \frac{-h}{k_1}\sqrt{\frac{\alpha_1}{\alpha_2}}\ \left[\tan\ \beta_{1n}h_1 + \frac{k_1}{k_2}\sqrt{\frac{\alpha_2}{\alpha_2}}\tan\ \beta_{2n}h_2 cos\beta_{2n}\, y\right] \qquad (23)$$

Thus temperature distribution in the two layers Slab with contact resistance at the interface is given in the form

$$T_1(y,t) = \sum_{n=1}^{\infty} A_n[sin\beta_{1n}\, y + \tan\ \beta_{1n}h_1\, cos\beta_{1n}\, y]\exp\left(-\alpha_1\, {\beta_{1n}}^2 t\right) \qquad (24)$$

$$T_2(y,t) = \sum_{n=1}^{\infty} A_n\ \left[\sqrt{\frac{\alpha_1}{\alpha_2}}\frac{k_1}{k_2} sin\beta_{2n}\, y + \tan\ \beta_{1n}h_1\, cos\beta_{2n}\, y\right]\exp\left(-\alpha_2\, {\beta_{2n}}^2 t\right) \ (25)$$

Where

$$A_n = \frac{4k_2T_0}{k_1}\frac{cos\,\beta_{1n}\,h_1 sin\beta_{1n}\,h_1 + cos\beta_{2n}\,h_2 + sin\beta_{2n}\,h_2 - 3}{2\beta_{1n}\,h_1 sec^2\beta_{1n}\,h_1 + sin\beta_{1n}\,h_1(tan^2\beta_{1n}\,h_1 - 1) - 5tan\beta_{1n}\,h_1 sin\beta_{1n}\,h_1}$$

$$+2\beta_{1n}\,h_2(1 + tan\beta_{2n}\,h_2 sin\beta_{2n}\,h_2) + \sqrt{\frac{\alpha_2}{\alpha_1}} sin\beta_{2n}\,h_2(tan\beta_{2n}\,h_2 sin\beta_{2n}\,h_2 - 1)$$

$$-\sqrt{\frac{\alpha_2}{\alpha_1}} sin^2\beta_{2n}\,h_2 \qquad (26)$$

Thermal stresses are given by

$$\sigma\ _{x1}(y) = -\alpha_{t_1}E_1\sum_{n=1}^{\infty} A_n[sin\beta_{1n}\,y + \tan\ \beta_{1n}h_1\,cos\beta_{1n}\,y]\exp\left(-\alpha_1\,{\beta_{1n}}^2 t\right) + \sum_{n=1}^{\infty}\left\{\frac{2b^2E_1}{D}\left\{\frac{2\alpha_{t_1}E_1}{\beta_{1n}}\ [cos\,\beta_{1n}\,h_1 - 1 + tan\beta_{1n}\,h_1 sin\beta_{1n}\,h_1] + \alpha_{t_2}E_2\left[\frac{1}{\beta_{2n}}\ [1 - cos\,\beta_{2n}\,h_2\ + tan\beta_{2n}\,h_2 sin\beta_{2n}\,h_2]\right]\left[E_2{h_2}^3 + E_1{h_1}^3\right] - 3\alpha_{t_1}E_1\left[\frac{-1}{\beta_{1n}}h_1 cos\,\beta_{1n}\,h_1 + \frac{1}{{\beta_{1n}}^2}sin\beta_{1n}\,h_1 + tan\beta_{1n}\,h_1\left[\frac{-h_1 sin\beta_{1n}\,h_1}{\beta_{1n}} + \frac{1}{{\beta_{1n}}^2}[1 - cos\,\beta_{1n}\,h_1]\right] + \alpha_{t_2}E_2\left[\frac{-h_2 cos\,\beta_{2n}\,h_2}{\beta_{2n}} + \frac{1}{{\beta_{1n}}^2}sin\beta_{2n}\,h_2 + tan\beta_{2n}\,h_2\left[\frac{h_2 sin\beta_{2n}\,h_2}{\beta_{2n}}\right] + \frac{1}{{\beta_{2n}}^2}[cos\,\beta_{2n}\,h_2 - 1]\right]\right]\left[E_2{h_2}^2 + E_1{h_1}^2\right] + \frac{6b^2E_1}{D}\left\{2\alpha_{t_1}E_1\left[\frac{-1}{\beta_{1n}}h_1 cos\,\beta_{1n}\,h_1 + \frac{1}{{\beta_{1n}}^2}sin\beta_{1n}\,h_1 + tan\beta_{1n}\,h_1\left[\frac{-sin\beta_{1n}\,h_1}{\beta_{1n}} + \frac{1}{{\beta_{1n}}^2}[1 - cos\,\beta_{1n}\,h_1][1 - cos\,\beta_{1n}\,h_1]\right]\right] + \alpha_2E_2\left[\frac{-h_2 cos\,\beta_{2n}\,h_2}{\beta_{2n}} + \frac{1}{{\beta_{22n}}^2}sin\beta_{2n}\,h_2 + tan\beta_{2n}\,h_2\left[\frac{h_2 sin\beta_{2n}\,h_2}{\beta_{2n}} + \frac{1}{{\beta_{1n}}^2}[cos\,\beta_{2n}\,h_2 - 1]\right][E_2\ h_2 b + E_1 h_1 b]\right] - \left\{\alpha_{t_1}E_1\frac{1}{\beta_{1n}}\ [cos\,\beta_{1n}\,h_1 - 1 + tan\beta_{1n}\,h_1 sin\beta_{1n}\,h_1] + \alpha_2E_2\left[\frac{1}{\beta_{2n}}\ [1 - cos\,\beta_{2n}\,h_2\ + tan\beta_{2n}\,h_2 sin\beta_{2n}\,h_2]\right]\right\}\left[E_2{h_2}^2 b + E_1{h_1}^2 b\right]\right\}\right\} \qquad (27)$$

$$\sigma\ _{x2}(y) = -\alpha_{t_2}E_2\sum_{n=1}^{\infty} A_n\ \left[\sqrt{\frac{\alpha_1}{\alpha_2}}\frac{k_1}{k_2}sin\beta_{2n}\,y + \tan\ \beta_{1n}h_1\,cos\beta_{2n}\,y\right]\exp\left(-\alpha_2\,{\beta_{2n}}^2 t\right) + \sum_{n=1}^{\infty}\left\{\frac{2b^2E_1}{D}\left\{\frac{2\alpha_{t_1}E_1}{\beta_{1n}}\ [cos\,\beta_{1n}\,h_1 - 1 + tan\beta_{1n}\,h_1 sin\beta_{1n}\,h_1] + \alpha_{t_2}E_2\left[\frac{1}{\beta_{2n}}\ [1 - cos\,\beta_{2n}\,h_2\ + tan\beta_{2n}\,h_2 sin\beta_{2n}\,h_2]\right]\left[E_2{h_2}^3 + E_1{h_1}^3\right] - 3\alpha_{t_1}E_1\left[\frac{-1}{\beta_{1n}}h_1 cos\,\beta_{1n}\,h_1 + \frac{1}{{\beta_{1n}}^2}sin\beta_{1n}\,h_1 + tan\beta_{1n}\,h_1\left[\frac{-h_1 sin\beta_{1n}\,h_1}{\beta_{1n}} + \frac{1}{{\beta_{1n}}^2}[1 - cos\,\beta_{1n}\,h_1]\right] + \alpha_{t_2}E_2\left[\frac{-h_2 cos\,\beta_{2n}\,h_2}{\beta_{2n}} + \frac{1}{{\beta_{1n}}^2}sin\beta_{2n}\,h_2 + tan\beta_{2n}\,h_2\left[\frac{h_2 sin\beta_{2n}\,h_2}{\beta_{2n}}\right] + \frac{1}{{\beta_{2n}}^2}[cos\,\beta_{2n}\,h_2 - 1]\right]\right]\left[E_2{h_2}^2 + E_1{h_1}^2\right]\right\} + \frac{6b^2E_1}{D}\left\{2\alpha_{t_1}E_1\left[\frac{-1}{\beta_{1n}}h_1 cos\,\beta_{1n}\,h_1 + \frac{1}{{\beta_{1n}}^2}sin\beta_{1n}\,h_1 + tan\beta_{1n}\,h_1\left[\frac{-sin\beta_{1n}\,h_1}{\beta_{1n}} + \frac{1}{{\beta_{1n}}^2}[1 - cos\,\beta_{1n}\,h_1][1 - cos\,\beta_{1n}\,h_1]\right]\right] + \alpha_2E_2\left[\frac{-h_2 cos\,\beta_{2n}\,h_2}{\beta_{2n}} + \frac{1}{{\beta_{22n}}^2}sin\beta_{2n}\,h_2 +\right.\right.$$

$$\tan\beta_{2n} h_2 \left[\frac{h_2 \sin\beta_{2n} h_2}{\beta_{2n}} + \frac{1}{{\beta_{1n}}^2}[\cos \beta_{2n} h_2 - 1]\right][E_2 h_2 b + E_1 h_1 b]\Bigg] - \Bigg\{\alpha_{t_1} E_1 \frac{1}{\beta_{1n}}[\cos \beta_{1n} h_1 - 1 + \tan\beta_{1n} h_1 \sin\beta_{1n} h_1] +$$

$$\alpha_2 E_2 \left[\frac{1}{\beta_{2n}}[1 - \cos \beta_{2n} h_2 + \tan\beta_{2n} h_2 \sin\beta_{2n} h_2]\right]\Bigg\}[E_2 {h_2}^2 b + E_1 {h_1}^2 b]\Bigg\}\Bigg\} \qquad (28)$$

4. NUMERICAL CALCULATIONS

Numerical calculations are carried out for the copper slab with properties
thermal expansion coefficient : $\alpha_{t_1} = \alpha_{t_2} = 16.5 \times 10^{-6} K^{-1}$
thermal diffusivity $\alpha_1 = \alpha_2 = 112.34 \times 10^{-6} m^2 s^{-1}$
outer boundaries : $h_1 = 1\ inch\ \ h_2 = 2\ inch$
thermal conductivities $k_1 = k_2 = 1$
Young's modulus=$E_1 = E_2$ =120 Gpa
substituting all these values in the equations (24) ,(25), (27)& (28)and illustrated graphically

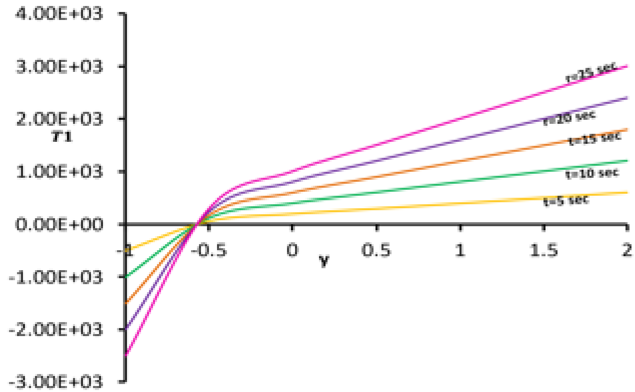

Figure 1 Graph of T_1 versus y for different values of t

Figure 1 shows the graph of $\boldsymbol{T_1}$ versus y for different values of t . It is observed that $\boldsymbol{T_1}$ goes on increasing.

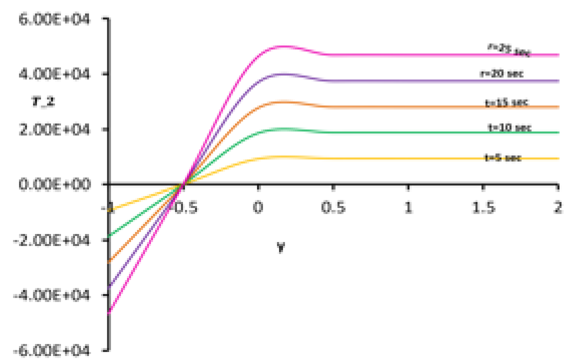

Figure 2 Graph of T_2 versus y for different values of t

Figure 2 shows the graph of $\boldsymbol{T_2}$ versus y for different values of t. It is observed that $\boldsymbol{T_2}$ goes on increasing from $y= -1\ to\ y = 0$ and then $\boldsymbol{T_2}$ is constant for different values of t.

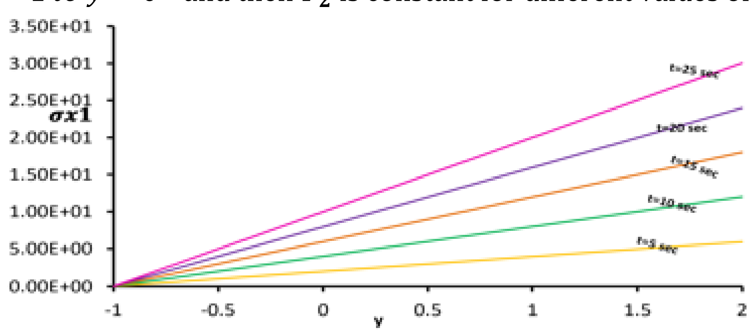

Figure 3 Graph of σ_{x1} versus y for different values of t

Figure 3 shows the graph of σ_{x1} versus y for different values of t . It is observed that σ_{x1} goes on increasing from y= -1 *to* $y = 2$.

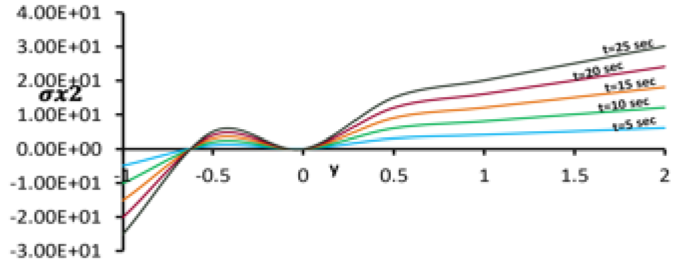

Figure 4 Graph of σ_{x2} versus y for different values of t

Figure 4 shows the graph of σ_{x2} versus y for different values of t . It is observed that σ_{x2} develops tensile stresses from $y = -0.5\ to y = 0$ then goes on increasing from y= -1 *to* $y = -0.5$ and $y = 0$ *to* $y = 2$

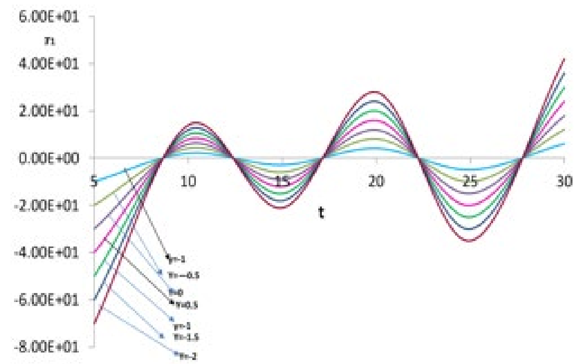

Figure 5: Graph of T_1 versus t for different values of y

Figure 5 shows the graph of $\boldsymbol{T_1}$ versus t for different values of y It is observed that $\boldsymbol{T_1}$ develops tensile stresses from $t = 7$ sec *to* $t = 12$ *and* $t = 17$ sec *to* $t = 22$ sec *and* compressive stress from $t = 12$ sec *to* $t = 17$ *sec and* $t =$ 22 sec $t = 28$ sec *then* $\boldsymbol{T_1}$ goes on increasing from t= 5*sec to* $t = 8$ *sec* and t= 28*sec to* $t = 30$ *sec*

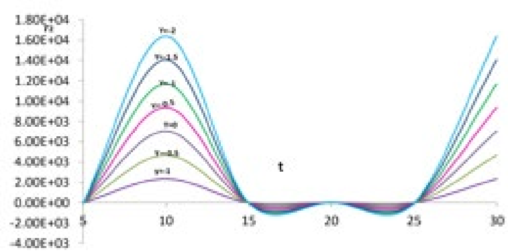

Figure 6 Graph of T_2 versus t for different values of y

Figure 6 shows the graph of $\boldsymbol{T_2}$ versus t for different values of y . It is observed that $\boldsymbol{T_2}$ develops tensile stresses from $t = 5$ sec *to* $t =$ 15 . *It is approximately equal to zero from* $t = 15$ sec *to* $t = 25$ sec *and then* $\boldsymbol{T_2}$ goes on increasing from t= 25 *sec to* $t = 30$ *sec* .

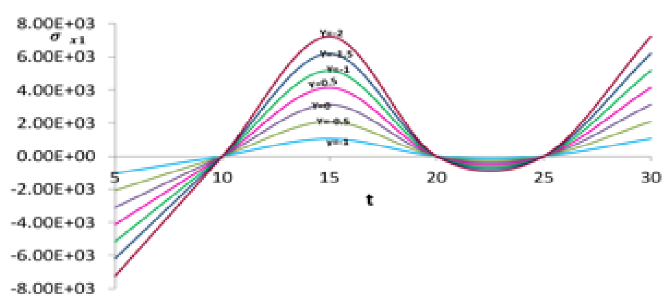

Figure 7 Graph of σ_{x1} versus t for different values of y

Figure 7 shows the graph of σ_{x1} versus t for different values of y . It is observed that σ_{x1} develops tensile stresses from $t = 10 \sec to \; t = 20$. *It is approximately equal to zero from* $t = 20 \sec to \; t = 25$ sec *and then* σ_{x1} goes on increasing from t= $25 sec \; to \; t = 30 \; sec$.

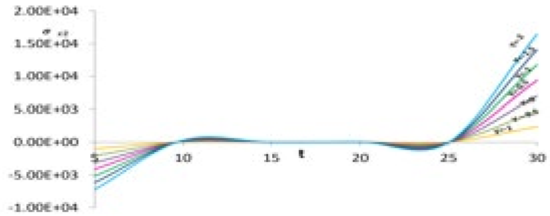

Figure 8 Graph of σ_{x2} versus t for different values of y

Figure 8 shows the graph of σ_{x2} versus t for different values of y . It is observed that σ_{x2} *is approximately equal to zero from* $t = 10 \sec to \; t = 25$ sec *and then* σ_{x2} goes on increasing from t= $5 sec \; to \; t = 10 \; sec$ and t= $25 sec \; to \; t = 30 \; sec$.

5. CONCLUSION

In this paper one treated the transient thermoelectric problem of the two-layer composite slab with prescribed initial and boundary conditions. Temperature distribution $T_i(y)$, $i = 1,2$ and thermal stresses σ_{xi} , $i = 1,2$ have been determined by using the orthogonal expansion technique and illustrated graphically.

CONFLICT OF INTERESTS

None.

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