

A STUDY ON VARIATION OF SURFACE GRAVITY OF BOSONIC FIELD AND FERMIONIC FIELD OF SPINNING BLACK HOLES

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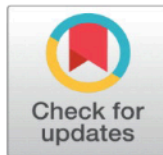
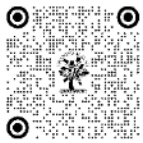
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ABSTRACT

The present paper discusses the variation of surface gravity of Bosonic and Fermionic Fields of spinning black holes with different spin parameters of these categories of black holes for co-rotation as well as counter rotation and concludes that the Bosonic, Fermionic and Other fields of specific rotations follows the same character as well as nature.

Keywords: E Bosonic Field, Fermionic Field and Spinning Parameter

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1. INTRODUCTION

The surface of gravity of any objects as per Newtonian Mechanics is quite different from the surface gravity of black holes given by Stephen Hawking using the quantum field theory concerned with Killing horizon representing the geometrical construction denoted by null vectors and provides the connection between the space time geometry and emission of radiation from the black holes (Bardeen et al. 1970, Hawking 1975, Wald 2001). Mahto and Kumari discussed the surface gravity of black holes in terms of mass and angular momentum during the study of entropy change of black hole w.r.t. change in mass (Mahto & Kumari 2018). Mahto et al. studied the surface gravity of black holes in terms of mass and spin parameter during the calculation of entropy change of bosonic fields of black hole w.r.t. change in mass (Mahto et al. 2020). The same work for the Fermionic fields of black holes has also done to conclude the approximately the same results (Mahto et al. 2020). The relativistic surface gravity of black holes of quarter and three quarter spin parameters in XRBs has been studied to conclude that the surface gravity becomes maximum for the lower values of mass of black holes (Chandan M et al. 2023). The same work for maximum spin parameters (Niharika et al. 2023) and half spin

parameters(Niharika et al. 2023) have done to obtain the same result and conclusions. The present paper discusses the variation of surface gravity of Bosonic and Fermionic Fields of Spinning black holes with different spin parameters of these categories of black holes for co-rotation as well as counter rotation.

2. THEORETICAL DISCUSSION

The first law of black hole mechanics gives the mathematical model for the change in mass with corresponding change in its angular momentum, entropy and charge. For a given mass(M), angular momentum(J), entropy(S) and charge(Q), this law can be represented by the following equation(Narayan 2005).

$$\delta M = \frac{\kappa}{2\pi} \delta S + \Omega \delta J - \nu \delta Q \quad \dots\dots\dots(1)$$

Where $\nu = \phi$ is the electric potential difference, Ω be the angular velocity and κ is the surface gravity of black holes directly connected to the absolute temperature of black holes to be focused in the present work.

Where(Smarr 1972).

$$\Omega = \frac{4\pi L}{MA} \quad \dots\dots\dots(2)$$

$$\phi = \frac{1}{M} \left[\frac{Q}{2} + \frac{2\pi Q^3}{A} \right] \quad \dots\dots\dots(3)$$

The surface gravity for the Kerr Newman BHs is given by the following equation(http://en.wikipedia.org/wiki/surface_gravity, 2011, Mahto et al. 2012):

$$\kappa = \frac{\sqrt{M^2 - Q^2 - J^2 / M^2}}{2M^2 - Q^2 + 2M\sqrt{M^2 - Q^2 - J^2 / M^2}} \quad \dots\dots\dots(4)$$

The final solution of above equation for uncharged black holes, Q=0 with the help of proper mathematical operations and approximation as requires, we get the following equation(Mahto& Kumari 2018).

$$\kappa = \frac{1}{4M} \left[1 - \frac{J^2}{4M^4} \right] \quad \dots\dots\dots(5)$$

The angular momentum, spin parameters and the mass of black holes are related by the following equation as(Mahto 2020):

$$J = a^* M^2 \quad \dots\dots\dots(6)$$

Using the above equation in the equation (6), we get

$$\frac{1}{\kappa} = 4M \left[1 + \frac{a^{*2}}{2} \right] \quad \dots\dots\dots(7)$$

The equation(7) gives a relation between the surface gravity, mass and spin parameters of black holes. This equation shows that the surface gravity of black holes depends on mass of black holes (M) and spin parameters of black holes (a^*). Using the spin parameters of Bosonic Field ($a^* = \dots -3, -2, -1, 0, +1, +2, +3 \dots$) Tayal 1990 &Yash 2022), Fermionic Field ($a^* = \dots -5/2, -3/2, -1/2, +1/2, +3/2, +5/2, \dots$) Tayal 1990 &Yash 2022) and other fields of the spin parameters ($a^* = -1/4, -3/4, +1/4$ and $+3/4$) (Yash 2022) on the equation (8), the relativistic surface gravity is calculated and listed in the table (1).

3. Table.1:

S. No.	Spin parameters (a^*)	Surface gravity (κ)	$\left(\frac{1}{\kappa} \right)$ (in terms of M)	Types Fields
1.	0	$1/4M$	4.00	Bosonic Field of Black Holes
2.	-1	$1/8M$	6.00	
3.	-2	$1/12M$	12.00	

4.	-3	$1/22M$	22.00	Fermionic Field of Black Holes
5.	+1	$1/8M$	6.00	
6.	+2	$1/12M$	12.00	
7.	+3	$1/22M$	22.00	
8.	-1/2	$7/32M$	4.50	
9.	-3/2	$2/17M$	8.5	
10.	-5/2	$2/33M$	16.5	
11.	+1/2	$7/32M$	4.50	
12.	+3/2	$2/17M$	8.5	Other Field of Black Holes
13.	+5/2	$2/33M$	16.5	
14.	-1/4	$31/128M$	4.10	
15.	-3/4	$23/128M$	5.12	
16.	+1/4	$31/128M$	4.10	
17.	+3/4	$23/128M$	5.12	

4. Figure 1:

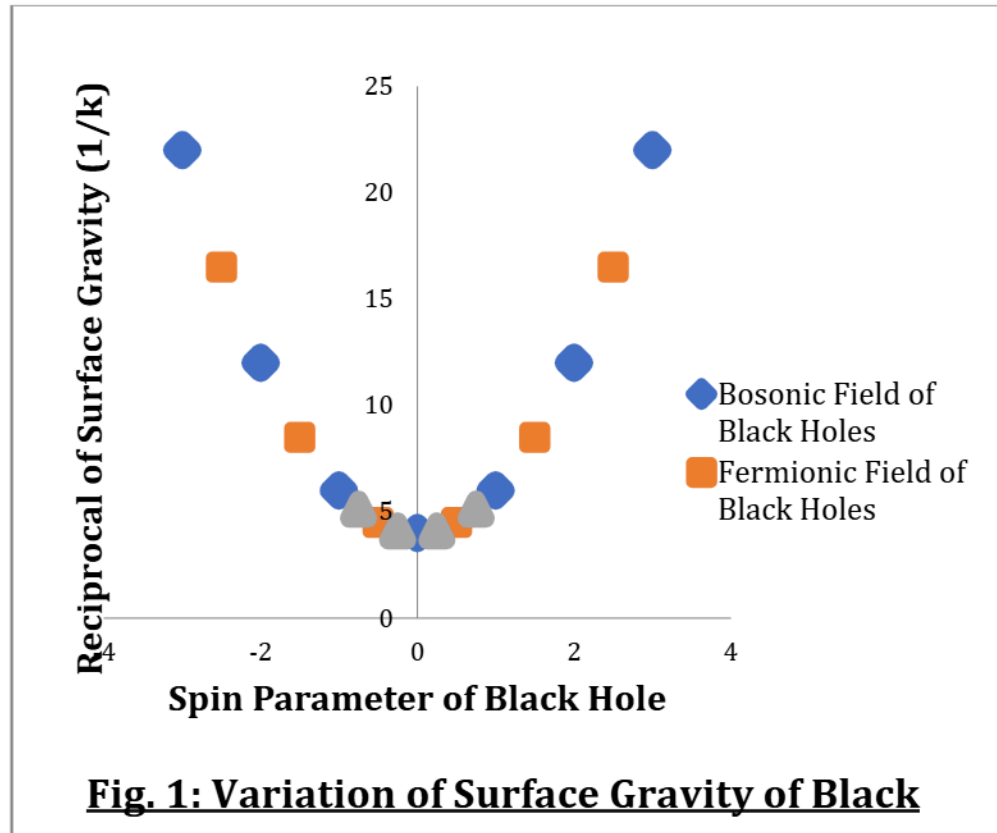


Fig: 1: Variation of Surface Gravity of Bosonic, Fermionic and other fields of black holes.

5. RESULT AND DISCUSSIONS

The present work discusses the relativistic surface gravity of Bosonic, Fermionic and other expected fields of black holes regarding their spin parameters. In the present work, we have used the model for the relativistic surface gravity represented by the equation (8). This equation shows that the surface gravity of black holes depends on:

- (1) Mass of black holes (M).
- (2) Spin parameters of black holes (a^*).

For the different values of spin parameters of Bosonic Field of black holes ($a^* = \dots -3, -2, -1, 0, +1, +2, +3 \dots$) and Fermionic Field of black holes ($a^* = \dots -5/2, -3/2, -1/2, +1/2, +3/2, +5/2, \dots$) and other fields ($a^* = -1/4, -3/4, +1/4$ and $+3/4$) are applied on this model. The positive sign shows for co-rotation of black holes and the negative sign shows for counter rotation. From the model, it is clear that the surface gravity remains the same for both co-rotation and counter rotation for a particular value of spin parameters. We also apply for other spin parameters ($a^* = -1/4, -3/4, +1/4$ and $+3/4$) showing quarter rotation and three quarter rotation.

Here the graph is plotted between the spin parameters (a^*) on the X-axis and the reciprocal of the surface gravity on the Y axis. The variation of the relativistic surface gravity with the spin parameters is shown in the figure 1. The figure 1 consists of three plots in which the first for Bosonic fields denoted by blue colour of square box, second for fermionic fields denoted by red colour of square box and third one for other fields denoted by green colour. From the observations of graph of figure 1 for each case, we see that the reciprocal of the surface gravity increases with increasing the values of the spin parameters, which means that the surface gravity decreases. It is also clear that the variation of the surface gravity for all these fields follows the same pattern. These all variations follow the parabolic nature for co-rotation and counter rotation spin. From above discussions, it may be concluded that all the fields of black holes like Bosonic, Fermionic and Other fields of specific rotations follows the same character as well as nature.

6. CONCLUSIONS

The reciprocal of the surface gravity increases with increasing the values of the spin parameters and all the fields of black holes like Bosonic, Fermionic and Other fields of specific rotations follows the same character as well as nature.

CONFLICT OF INTERESTS

None.

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