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A STUDY ON RECENT NEW RESULTS ON SOME GRAPH VALUED FUNCTIONS

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ABSTRACT

In this paper, the result on some valued function(digraph operator), namely the block line cut vertex digraph BLC(D) of a digraph D is defined, and the problem of reconstructing a digraph from its block line cut vertex digraph is presented. Outer planarity, maximal outer planarity, and minimally non-outer planarity properties of these digraphs are discussed.

Keywords: Planar And Nonplanar Graphs, Cutvertex, Line Graph, Wheel Graph, Total Blict Graph

1. INTRODUCTION

All graphs considered here are finite, undirected and without loops or multiple edges. The edges, cut vertices and blocks of a graph G are called its members. Two blocks B_i and B_j are adjacent if they have common cutvertex.

Definition 1.1 The Edge degree of an edge uv in G is the number of the edges adjacent to edge uvor deg u + deg v □ 2. A Block vertex is a vertex in TBn (G) corresponding to a block of G.

Definition 1.2 A graph is said to be Planar if it can be embedded in a plane so that no two edgesintersect. Otherwise, the graph is nonplanar.

A maximal planar graph is one to which no edge can be added without losing planarity. Theconcept of outerplanar graphs was studied by Tang [27]. A planar graph is said to be outerplanar ifit can be embedded in a plane so that all its vertices lie on the same region. Otherwise the graph is nonouterplanar. An outerplanar graph G is maximal outerplanar if no edge can be added without losing outerplanarity. Chartrand and Harary [2] obtained a characterization of outerplanar graphs in terms of forbidden subgraphs.

Definition 1.3 The concept of non-zero inner vertex number of a planar graph was introduced byKulli [11]. A nonnegative integer r such that any plane embedding of a planar graph G has at least r vertices not lying on the boundary of the exterior region of G is called the inner vertex number ofG, denoted as i(G) and this indicates that G has r inner vertices. In general, the planar graphs having i(G) = r, r > 0, are called r-nonouterplanar graphs. In particular, zero nonouterplanar graphs are outerplanar graphs. 1-nonouterplanar graphs will be called minimally nonouter planar graphs. For these graphs i(G) = 1. This concept has been extensively studied by Kulli [11] and others.

Definition 1.4 The Line graph of a graph G, denoted L (G), is the graph whose vertices are the edges of G, with two vertices of L(G) adjacent whenever the corresponding edges of G are adjacent. The concept of the Line graph of a given graph is so natural that it has been independently discovered by many authors giving different name.

Definition 1.5 The crossing number C(G) of a graph G is the minimum number of pair wise intersections (or crossings) of its edges when G is drawn in the plane. Obviously, C(G) = 0 if and only if G is planar. If C(G) =1, then G is said to have crossing number one

Definition 1.6 A vertex v of G is called a cut vertex if its removal produces a disconnected graph. That is, G-v has at least two components.

Definition 1.7 A Wheel graph W_n is a graph with n vertices formed by connecting a single vertex to all vertices of an (n-1) cycle.

All undefined terms may be referred to Harary [8].

We need the following theorems for the proof of our further results.

Theorem 1.1[8]: If G is a graph (V,E) whose vertices have degree d_i, then Line graph L(G) has E vertices and E_L edges, where $E_L = -E + \frac{1}{2}\sum_i d_i^2$

Theorem 1.2[25]: The line graph L(G) of a graph G is planar if and only if G is planar, the degreeof each vertex of G is atmost 4 and every vertex of degree 4 is a cutvertex.

Theorem 1.3[4]: The Line graph L(G) of graph G is outerplanar if and only if the degree of eachvertex of G is atmost 3 and every vertex of degree 3 is a cutvertex.

Theorem 1.4[12]: The Line graph of G has crossing number one if and only if G is planar and (i)or (ii) holds.

(i) The maximum degree Δ(G) is 4 and there is a unique non-cutvertex of degree 4.

(i) The maximum degree Δ(G) is 5, every vertex of degree 4 is a cut vertex, there is aunique vertex of degree 5 and it has atmost 3 edges in any block.

THEOREM 1.5[8]: A GRAPH G(V,E) IS PLANAR IF AND ONLY IF $|E| \leq 3|V| - 6$.

Theorem 1.6[8]: If G is a nontrivial connected graph with $|V|$ … $L^n(G)$ is Hamiltonian for all $n \geq |V| - 3$.

MAIN RESULTS

Definition 2.1 Total Blict graph TBn (G) of a graph G is the graph whose vertex set is the union of the set of edges, set of cut vertices and set of blocks of G in which two vertices are adjacent if and only if the corresponding members of G are adjacent or incident except the adjacency of cut vertices. In Figure 1.1, a graph G and its Total Blict graph TBn (G) are shown.

Remark 2.1: For any graph G, L (G) □ TBn (G).
Remark 2.2: For any cycle Cv, V ≥ 3, i [TBn (G)] ≥ 1.
In particular i [TBn (C3)] = 1.
Remark 2.3: For every non-separable graph G, TBn (G) is a block.
Remark 2.4: Every bridge in G forms a pendant edge in TBn (G).
Remark 2.5: For any non-separable graph G an edge 'a' with edge degree odd corresponds to the vertex 'a' in TBn (G) whose vertex degree is even and vice versa.
Remark 2.6: For any separable graph G an edge 'a' incident to the cutvertex corresponds to the vertex 'a' of odd degree in TBn (G).
Remark 2.7: For any graph G, TBn (G) is a bridgeless graph.

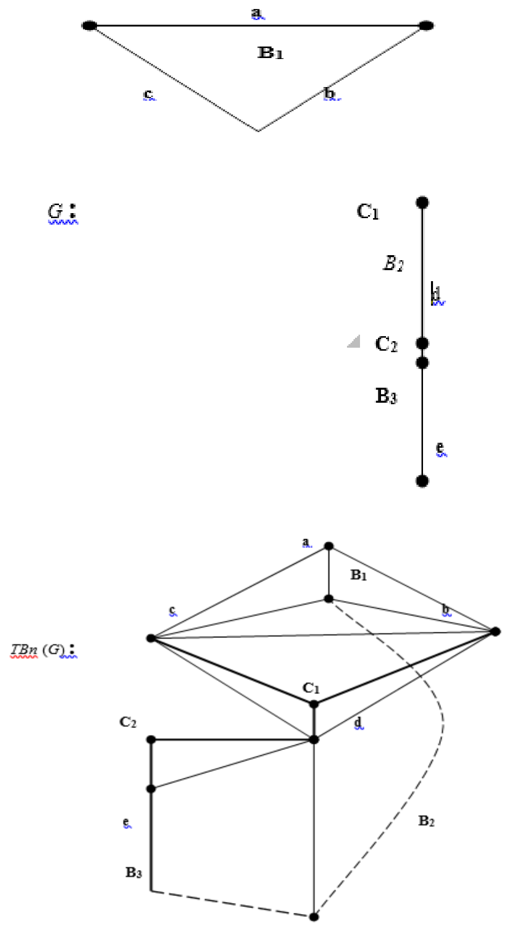

Figure 1.1

Theorem 2.1: For any nontrivial connected (V, E) graph G whose vertices have degree d_i, C is the number of the cutvertices in G, Bk be the number of blocks then TBn (G) has (E + Bk + C) vertices and $\frac{1}{2}\sum_{i=1}^{V} d_i + \sum_{j=1}^{C}\deg C_j + \sum_{i,j=1, i\neq j}^{k} B_{i,j}$ edges, where C_j is the j cutvertex. $B_{i,j}$ denotes that B_i is adjacent to B_j.

Proof: By the definition of TBn (G), the number of vertices is (E + B_k + C). For the number of edges, since $L(G) \subset TBn(G)$, by Theorem 1.1[8], $-E + \frac{1}{2}\sum d_i^2$ edges are contributed to TBn (n).

By definition, every block vertex is adjacent to vertices corresponding to edges from which it is formed in G. This gives E edges to TBn (G). Every cutvertex is adjacent to the vertices corresponding to the edges incident to it in G. This adds $\sum_{j=1}^{c} \deg C_j$ edges to TBn (G). These blocks B_i adjacent to B_j for $i \neq j$ gives $\sum_{i,j=1, i\neq j}^{k} B_{i,j}$ edges this adds to the total number of edges to TBn (G).

Hence the number of edges in TBn (G) is given by

$$E[TBn(G)] = -E + \frac{1}{2}\sum_{i=1}^{v} d_i + E + \sum_{j=1}^{c}\deg C_j + \sum_{i,j=1, i\neq j}^{k} B_{i,j}$$

$$E[TBn(G)] = \frac{1}{2}\sum_{i=1}^{v} d_i + \sum_{j=1}^{c}\deg C_j + \sum_{i,j=1, i\neq j}^{k} B_{i,j}$$

In the following theorem we establish the planarity of TBn (G).

Theorem 2.2: The Total Blict graph TBn (G) of graph G is planar if and only if Δ(G) ≤ 3 andevery vertex of degree 3 is a cut vertex.

Proof: Suppose TBn (G) is planar. Assume Δ(G) ≤ 3. Let v be a vertex of degree 4 in G, we havethe following cases.

Case 1: If v is a non cutvertex, then the number of edges incident to v forms $< K_4 >$ as a subgraph in L (G). By definition of TBn (G) the block vertex is adjacent to all the vertices of $< K_4 >$ which gives $< K_4 > \subset$ TBn (G) which is non planar, a contradiction.

Case 2: If v is a cutvertex then the number of edges incident to v forms < K4 > as a subgraph in L

(G). By the definition, the cutvertex V is adjacent to each vertex of <K4 > gives <K4 > ⊂ TBn (G)., a contradiction for planarity of TBn (G).