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EXISTENCE OF FIXED POINT VIA SELF-MAPPING OF ENRICHED G_b −CONTRACTION IN G_b −METRIC SPACES

Abid Khan [1], Santosh Kumar Sharma [1], Girraj Kumar Verma [1], Ramakant Bhardwaj [2], Umashankar Sharma [3]

[1] Department of Applied Mathematics, Amity University, Madhya Pradesh, Gwalior (MP), India
[2] Department of Mathematics, Amity University Kolkata, West Bengal, India
[3] Department of Physics, RJIT BSF Tekanpur, MP, India

Corresponding Author
Abid Khan, abid69304@gmail.com

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ABSTRACT

In the present work we show here, we prove some fixed point results on enriched G_b −complete metric space for a novel contraction and produce some enriching fixed point results utilising Banach G_b −contraction enriching G_b −metric spaces. The suggested results compile a few earlier findings based on G_b −metric space.

Keywords: Fixed Point, G −Metric Space, Enriched G_b −Contractions, G −Convergent

1. INTRODUCTION

The field of fixed point theory has advanced rapidly in recent years. On the one hand, the study of new spaces has long been a subject of interest to the mathematical research community. In 1993, Czerwik [4] introduced the concept of b-metric space as a generalization of metric space.

Mustafa and sims [8] presented new metric space generalizations by designating to each $(x, y, z) \in X \times X \times X$ a real number and is named as G −metric space. Subsequently, some authors like Aydi $et. al.$ [3], W. Shatanawi [14] have generalized some results of Mustafa $et. al.$ [10, 11] what's more, concentrated on some decent point results for self-mapping in a total G −metric space. Recently, Some prominent fixed point speculations have been portrayed by a couple of columnists after this fundamental report, including setting of G −metric space ($see, e. g.,$ [12,13,14,15,18]). In 2014, Aghajani et al. [1] used the ideas of the G −metric and b-metric to create additional generalizations of a metric space and created a new form of metric known as G_b −metric space. The class of G_b −metric space is effectively greater than that of G −metric spaces, they further noted. It should be noted that a G −metric space becomes a specific instance of a G_b −metric space. Very recently, Chabo Li and Yunan Cui [13] proved some fixed point results connected with certain contractions are obtained in the setting of enriched rectangular G_b-metric spaces.

When $s = 1$. Further, they showed that every G_b −metric space is equivalent to a b −metric space topologically. To show that fixed point outcomes exist for G_b −metric spaces, many researchers tried to generalize new contractive mappings. ($for\ examples, see$ [5,6,7,18,21]).

To show that fixed point outcomes exist for G_b −metric spaces, recent results in this direction can also be found in ([6,13,17]) and the references therein.

Motivated and inspire by the above describe work; we derive a Banach enriched G_b −contraction metric spaces to generalize results based on enriching G_b −metric space. Also, an example to illustrate the main result is given. Rational type contraction mappings are taken in view of fixed point theory. For this type of contraction recent literature can be seen from [22-29]

2. DEFINITIONS AND PRELIMINARIES

In this section, using the concepts of G_b −metric space which was introduced by Aghajani et al. [1] In this situation, we created a few fresh enriched fixed-point results. In order to get started, let's briefly go over some fundamental concepts and findings for enriching G_b −metric required spaces in the continuation.

Definition 2.1 ([7]) Let X be a nonempty set and $G : X \times X \times X \to [0, \infty)$ satisfies:

($\boldsymbol{G1}$) $G(x, y, z) = 0\ if\ x = y = z,$

($\boldsymbol{G2}$) $G(x, x, y) > 0\ \forall\ x, y \in X\ with\ x \neq y;$

($\boldsymbol{G3}$) $G(x, y, y) \leq G(x, y, z)\ \forall\ x, y, z \in X\ with\ x \neq z$

($\boldsymbol{G4}$) $G(x, y, z) = G(y, z, x) = G(z, x, y) = \cdots (symmetry\ in\ x, y, z);$

($\boldsymbol{G5}$) $G(x, y, z) \leq G(x, c, c) + G(c, y, z)\ \forall x, y, z, c \in X.$

The function G is called a G −metric on X, and the pair (X, G) is called a G −metric space.

Definition 2.2. ([9]) Let X be a nonempty set and $s \geq 1$ be a given real number. Suppose that $G_b : X \times X \times X \to [0, \infty)$ be a function satisfying the following properties

(G_b1) $G_b(x, y, z) = 0\ if\ x = y = z,$

(G_b2) $0 < G_b(x, x, y), \forall\ x, y \in X, with\ x \neq y,$

(G_b3) $G_b(x, x, y) \leq G_b(x, y, z), \forall\ x, y, z \in X, with\ z \neq y,$

(G_b4) $G_b(x, y, z) = G_b(p(x, z, y)), where\ p$ is a permutation of x, y, z (symmetry)

(G_5) $G_b(x, y, z) \leq s(G_b(x, c, c)) + G_b(c, y, z), \forall\ x, y, z, c \in X, (rectangle\ inequality)$

Then the function G_b is called a generalized b −metric, or G_b −metric on X, and the pair (X, G) is called a G_b −metric space.

Obviously, the class of G_b −metric spaces is really bigger than that of G −metric spaces given in [1]. Indeed; each G −metric space is a G_b −metric space with $s = 1$.

Example 2.3. Let R be the set of all real numbers. Define $G_b: R \times R \times R \to R^+$ by

$$G_b(x, y, z) = |x - y| + |y - z| + |z - x|, \forall\ x, y, z \in X.$$

Then it is clear that (R, G_b) is enriched G_b −metric space for G_b −contractions.

Lemma 2.4. Let (X, G_b) be a G_b −metric space.

1) A sequence $\{x_n\}_{n=0}^{\infty}$ is G_b −converges if and only if $G_b(x_n, x, x) \to 0$, as $n \to \infty$. More precisely, for every given $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $n > N$, $G_b(x, x_n, x_n) < \varepsilon$ or $G_b(x_n, x, x) < \varepsilon$.
2) A sequence $\{x_n\}_{n=0}^{\infty}$ in (X, G_b) is named as G_b −cauchy sequence if for all $\in > 0$, there exists $N \in \mathbb{N}$ such that $G_b(x_n, x_m, x_l) < \varepsilon$ for all $n, m, l \geq N$.
3) A space X is said to be G_b −converges if for every G_b −cauchy sequence in X is G_b −convergent.

Definition 2.5. ([9]) Let (X, G_b) be a G_b −metric space, let (x_n) be a sequence of points of X, we say that (x_n) is G_b −convergent to x

if $\lim_{n,m\to\infty} G_b(x, x_n, x_m) = 0$; *that is, for any* $\epsilon > 0$ there exist $n_0 \in N$ such that $G_b(x, x_n, x_m) < \epsilon, \forall\, n, m \geq n_0$. we refer to x as the limit of the sequence (x_n) and write $x_n \to x$.

Definition 2.6. ([6]) Let (X, G_b) be a G_b −metric space, a sequence (x_n) is called G_b −cauchy if given $\epsilon > 0$, there exist $n_0 \in N$ such that $G_b(x_n, x_m, x_l) < \epsilon, \forall\, n, m, l \geq n_0$ if $G_b(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.

Proposition 2.7. ([1]). Let (X, G_b) be a G_b − metric space. Then for each $x, y, z, c \in X$ it follows that:

$$G_b(x, y, z) = 0 \; if \; x = y = z,$$

$$G_b(x, y, z) \leq s\big(G_b(x, x, y) + G_b(x, x, z)\big),$$

$$G_b(x, y, y) \leq 2sG_b(y, x, x),$$

$$G_b(x, y, z) \leq s\big(G_b(x, c, z) + G_b(c, y, z)\big).$$

Lemma 2.8. Let (X, G_b) be a G_b −metric space for enriching G_b −contraction. Then

$$G_b(x, x, y) \leq 2G_b(x, y, y) \;\; \forall\, x, y \in X.$$

Proposition 2.9. ([1]) In a G_b −metric space (X, G_b), the following are equivalent.

The sequence (a_n) is G_b −cauchy sequence.

For every $\epsilon > 0$, there exists $n_0 \in N$ such that $G_b(a_n, a_m, a_m) < \epsilon, \forall\, n, m \geq n_0$.

Proposition 2.10. ([1]). Let (X, G_b) be a G_b − metric space. Then, the following statements are equivalent:

(a_n) is G_b −convergent to a.

$$G_b(a_n, a_n, a) \to 0, as\; n \to \infty.$$

$$G_b(a_n, a, a) \to 0, as\; n \to \infty.$$

$$G_b(a_m, a_n, a) \to 0, as\; n, m \to \infty.$$

3. MAIN RESULTS

Theorem 3.1. Let (X, G_b) be G_b −metric space for enriching G_b −contraction and let $T: X \to (X)$ satisfy the following conditions

For each $x \in X, T(x), T(y), T(z) \in G_b(X)$,

$$G_b(T(x), T(y), T(z)) \leq \alpha_1\{G_b(x, y, z)\} + \alpha_2\{G_b(x, T(x), T(x)\}$$
$$+\alpha_3\{G_b(y, T(y), T(y)\} + \alpha_4\{G_b(z, T(z), T(z)\}$$
$$+\alpha_5 \left\{\frac{G_b(x, T(z), T(z)}{(1 - G_b(z, T(x), T(x) + G_b(z, T(y), T(y)}\right\}$$
$$+\phi max\{G_b\big((x, T(x), T(x)\big), G_b\big(G_b(y, T(y), T(y)\big), G_b\big((z, T(z), T(z)\big)\} \qquad (3.1)$$

For all $x, y, z \in X$, $\alpha_i \geq 0$ *where* $i = 0,1,2,3$ ....with $\left(\sum_{i=0}^{5} \alpha_i\right) + 2\phi < \frac{1}{2}$. Then there exists $p \in X$ such that $p \in T(x) \cap \phi(x)$ and T , ϕ is enriching G_b −continuous at p.

Proof. Let $x_o \in X, T(x_o)$ is a non-empty subset of X. We can choose that $x_1 \in T(x_o)$, for this x_1 by the same reason mentioned above $T(x_1)$ is non-empty closed subset of X.

Since $x_1 \in T(x_o)$ and $T(x_1)$ of X, there exist $x_2 \in T(x_1), x_2 \in T(x_0)$ such that

$$G_b(x_1, x_2, x_2)` \leq G_b(T(x_o), T(x_1), T(x_1)$$
$$+\phi max\{G_b\big(x_1, T(x_0), T(x_0)\big), G_b(x_1, T(x_1), T(x_1)), G_b(T(x_0), T(x_1), x_1)\}$$

$$G_b(x_1, x_2, x_2) \leq G_b(T(x_o), T(x_0), T(x_2) \quad + \phi max\{G_b\big(x_0, T(x_0), T(x_1)\big), G_b(x_1, T(x_0), T(x_0)), G_b(T(x_0), T(x_1), x_0)\}$$
$$\leq \alpha_1[G_b(x_o, x_1, x_1)] + \alpha_2[G_b(x_1, T(x_0), x_1)] + \alpha_3[G_b(x_1, T(x_o), T(x_0)] + \alpha_4[G_b(x_1, T(x_o), T(x_0)]$$
$$+ \alpha_5 \left\{\frac{G_b(x_0, T(x_1), T(x_1))}{(1 - G_b(x_1, T(x_0), T(x_1))}\right\}$$

$$+\phi max\{G_b\big(x_0,T(x_0),T(x_1)\big),G_b(x_1,T(x_0),T(x_0)),G_b(T(x_0),T(x_1),x_0)\}$$
$$\le \alpha_1[G_b(x_o,x_1,x_1)]+\alpha_2[G_b(x_1,T(x_0),x_1)]+\alpha_3[G_b(x_1,T(x_o),T(x_0)]\ +\alpha_4[G_b(x_1,T(x_o),T(x_0)]$$
$$+\alpha_5\left\{\frac{G_b(x_0,T(x_1),T(x_1))}{(1-G_b(x_1,T(x_0),T(x_1))}\right\}$$
$$+\phi max\{G_b\big(x_0,T(x_1),T(x_1)\big),G_b(x_1,T(x_0),T(x_0)),G_b(T(x_1),T(x_1),x_0)\}$$

Which implies that,

$$G_b(x_1,x_2,x_2)\le\frac{\sum_{i=1}^{2}a_i+\phi}{1-\left(\sum_{i=3}^{5}a_i\right)-\phi}G_b(x_o,x_1,x_1)$$

$$G_b(x_1,x_2,x_2)\le G_b(x_o,x_1,x_1)+\phi max\{G_b\big(x_1,T(x_0),T(x_0)\big),G_b(x_1,T(x_0),T(x_1)),G_b\big(x_0,T(x_0),T(x_0)\big)\}$$

Thus for this x_2, $T(x_2)$ is a non-empty of X.

Since $x_2\in T(x_1)$ and $T(x_1)$ and $T(x_2)$ of X, there exist $x_3\in T(x_2)$ such that

$$G_b(x_2,x_3,x_3)\le G_b(T(x_2),T(x_1),T(x_1))$$
$$+\phi max\{G_b\big(x_2,T(x_3),T(x_3)\big),G_b\big(x_1,T(x_1),T(x_1)\big)\big(G_b\big(x_1,T(x_2),T(x_3)\big)\}^2$$
$$\le \alpha_1[G_b(x_o,x_1,x_1)]+\alpha_2[G_b(x_1,T(x_0),x_1)]+\alpha_3[G_b(x_1,T(x_o),T(x_0)]\ +\alpha_4[G_b(x_1,T(x_o),T(x_0)]$$
$$+\alpha_5\left\{\frac{G_b(x_0,T(x_1),T(x_1))}{(1-G_b(x_1,T(x_0),T(x_1))}\right\}$$
$$+\phi max\{G_b(x_1,T(x_2),T(x_2),G_b(x_1,T(x_2),T(x_3)),G_b(T(x_1),T(x_3),x_3)\}^2$$

Which implies that

$$G_b(x_2,x_3,x_3)\le\frac{\sum_{i=1}^{5}a_i+\phi}{1-\left(\sum_{i=1}^{5}a_i\right)-\phi}G_b(x_1,x_2,x_2)$$
$$+\phi max\{G_b(x_1,T(x_2),T(x_2),G_b(x_1,T(x_2),T(x_3)),G_b(x_1,T(x_3),T(x_3))\}^2$$
$$\le G_b(x_2,x_3,x_3)+\phi max\{G_b(x_2,x_3,x_3)G_b(x_1,(x_2),T(x_3)),G_b(x_1,(x_3),T(x_3))\}^2$$
$$\le G_b(x_2,x_3,x_3)+\phi max\{G_b(x_2,x_3,x_3)G_b(x_1,(x_2),T(x_3)),G_b(x_1,(x_3),T(x_3))\}^2$$
$$+\phi max\{G_b(x_2,x_3,x_3)G_b(x_1,T(x_2),T(x_3)),G_b(x_3,(x_2),T(x_1))\}^2$$
$$G_b(x_2,x_3,x_3)\le {G_b}^2(x_o,x_1,x_1)$$
$$+2\phi max\{G_b(x_1,T(x_2),T(x_2),G_b(x_3,T(x_2),T(x_2)),G_b(T(x_3),T(x_2),x_1)\}^2$$

Similarly this process continue and we get a sequence $\{x_n\}$ such that $x_{n+1}\in T(x_n)$ or $x_{n+2}\in T(x_n)$ and

$$G_b(x_{n+1},x_n,x_n)\le$$
$$\phi max\{G_b\big(x_{n+1},T(x_n),T(x_n)\big),G_b\begin{pmatrix}x_{n+1},T(x_{n+1}),T(x_n),\\G_b\big(x_{n+1},T(x_n),T(x_n)\big)\end{pmatrix}^n$$
$$(x_{n+1},x_n,x_n)$$
$$+n\phi max\{G_b\big(x_{n+1},T(x_n),T(x_n)\big),G_b\begin{pmatrix}x_{n+2},T(x_n),T(x_n),\\G_b\big(x_{n+2},T(x_{n+2}),T(x_{n+1})\big)\end{pmatrix}^n$$

Suppose $0\ll u$ be given, choose that, a natural number N_1 such that

$$\phi max\{G_b\big(x_{n+1},T(x_n),T(x_n)\big),G_b\begin{pmatrix}x_{n+2},T(x_{n+1}),T(x_n),\\G_b\big(x_n,x_{n+1},T(x_{n+2})\big)\end{pmatrix}^n G_b(x_{n+1},x_n,x_n)$$
$$+n\phi max\{G_b(T(x_{n+1}),T(x_n),x_n),G_b(x_{n+1},x_{n+1},x_n),G_b(x_{n+1},x_n,x_n)\}^n\ll u\ \forall\ n\ge N_1$$
$$\Rightarrow\ G_b(x_{n+1},x_n,x_n)\ll u.$$

$\therefore\{x_n\}$ is a Cauchy sequence in (X,G_b) is a enriched $G_b-metric\ space$ $\exists\ p\in X$ such that $x_n\to p$. So choose a natural number N_2 such that

$$G_b(x_n,p,p) \leq \frac{u\left(1-\left(\sum_{i=1}^{5} a_i\right)-2\phi\right)}{2v\left(1+\left(\sum_{i=1}^{5} a_i\right)+\phi\right)} G_b(p,x_n,x_n)$$

Again by the same argument we will find

$$G_b(x_{n-1},p,p) \leq \frac{u\left(1-\left(\sum_{i=1}^{5} a_i\right)-2\phi\right)}{2v\left(\sum_{i=1}^{5} a_i-\phi\right)} G_b(p,x_{n-1},x_{n-1}) \forall\, n \geq N_2$$

Thus, we have

$$G_b(T(p),p,p) \leq G_b(p,x_n,x_n) + G_b(x_n,T(p),T(p) + G_b(p,T(x_n),T(x_n))$$

$$\leq G_b(p,x_n,x_n) + G_b\big(p,T(x_{n-1}),T(x_{n-1})\big), G_b\big(T(x_{n-1}),T(x_{n-1}),T(p)\big)$$

$$\leq G_b(p,x_n,x_n) + \alpha_1[G_b(x_{n-1},p,p)]$$

$$+\alpha_2[G_b(p,T(p),T(x_{n-1})] + \alpha_3[G_b(p,T(x_{n-1}),p)]$$

$$+\alpha_4\big[G_b\big(p,p,T(P)\big)\big] + \alpha_5\left\{\frac{G_b(x_n,T(p),T(x_{n-1}))}{(1-G_b(p,T(x_{n-1}),T(p))}\right\}$$

$$\leq G_b(p,x_n,x_n) + \alpha_1[G_b(x_n,p,p)] + \alpha_2[G_b(p,T(p),T(x_n)] + \alpha_3[G_b(p,T(x_n),p)]$$

$$+\alpha_4\big[G_b\big(p,p,T(P)\big)\big] + \alpha_5\left\{\frac{G_b(x_n,T(p),T(x_n))}{(1-G_b(p,T(x_n),T(p))}\right\}$$

$$\leq G_b(p,p,p) + \alpha_1[G_b(p,p,p)] + \alpha_2[G_b(p,T(p),T(p)] + \alpha_3[G_b(p,T(p),p)]$$

$$+\alpha_4\big[G_b\big(p,p,T(P)\big)\big] + \alpha_5\left\{\frac{G_b(p,T(p),T(p))}{(1-G_b(p,T(p),T(p))}\right\}$$

$$G_b(T(p),p,p) \leq \frac{\left(\sum_{i=1}^{4} a_i\right)+\alpha_5}{\left(1-\left(\sum_{i=1}^{4} a_i\right)+\alpha_5\right)} G_b(x_{n-1},p,p) + \frac{(1+(\alpha_2+\alpha_3)}{\left(1-\left(\left(\sum_{i=1}^{4} a_i\right)\right)\right.}$$

$$G_b(x_n,p,p) \forall\, n \geq N_2.$$

$G_b(T(p),p,p) \ll \frac{u}{v}$ for all $v \geq 1$, we get $\frac{u}{v} - G_b(T(p),p,p) \in P$ and as $n \to \infty$, we get $\frac{u}{v} \to 0$ and P is enriched $G_b -$ *convergent to* $G_b(T(p),p,p) \in P$ but $G_b(T(p),p,p) \in P$. Therefore $G_b(T(p),p,p) = 0$ and so $p \in T(p)$. Similarly it can be established that $p \in \phi(p)$. Hence $p \in T(p) \cap \phi(p)$.

As an application of theorem 3.1, we have the following corollary.

Corollary 3.2. Let (X, G_b) be a complete G_b −metric space for enriching G_b −contraction and let $T: X \to (X)$ satisfy for some $m \in N$:

For each $x \in X, T^m(x), T^m(y), T^m(z) \in G_b(X)$,

$$G_b(T^m(x),T^m(y),T^m(z)) \leq \alpha_1\{G_b(x,y,z)\} + \alpha_2\{G_b(x,T^m(x),T^m(x)\}$$

$$+\alpha_3\{G_b(y,T^m(y),T^m(y)\} + \alpha_4\{G_b(z,T^m(z),T^m(z)\}$$

$$+\alpha_5\left\{\frac{G_b(x,T^m(z),T^m(z)}{(1-G_b(z,T^m(x),T^m(x)+G_b(z,T^m(y),T^m(y))}\right\}$$

$$+\phi^m max\{G_b\big((x,T^m(x),T^m(x)\big), G_b\big(G_b(y,T^m(y),T^m(y)\big), G_b\big((z,T^m(z),T^m(z)\big)\} \quad (3.1)$$

For all $x,y,z \in X$, $\alpha_i \geq 0$ *where* $i = 0,1,2,3$ ....with $\left(\sum_{i=0}^{5} \alpha_i\right) + 2\phi^m < \frac{1}{2}$. Then there exists $p \in X$ such that $p \in T^m(x) \cap \phi^m(x)$ and T^m , ϕ^m is enriched G_b −continuous at p.

Proof. From Theorem 3.1, we see that T^m has a unique fixed point (say p) in X and T^m is enriched G_b −continuous at p. since

$$T^m(p) = T^m\big(T^m(p)\big) = T^{m+1}(p) = T^m\big(T^m(p)\big).$$

We have that $T(p)$ is also a fixed point for T^m. By uniqueness of p. we get $T(p) = p$.

4. RESULTS

To show that existence of fixed point results using self-mappings for enriching G_b −metric spaces, recent results in this direction can also be found in ([1,6,9,17,18]) and the references therein. Motivated and inspire by the above describe work; we derive Enrichied G_b −contraction metric spaces to generalize results based on enriching G_b −metric space. We hope that our new results can be applied to fields such as nonlinear analysis, functional Analysis, mathematical physics, and other related fields in the future.

5. CONCLUSION

we have proved some existence of fixed point results via using self-mappings enriched−G_b −contraction on G_b −metric spaces and also using this technique to prove some fixed point results on enriching G_b −complete metric space for a new contraction. Some example provided to discuss and strengthen primary finding. In summary, our results are original, meaningful, and valuable in the context of the existing literature. We hope that the outcomes of this manuscript will be helpful to understand the literature of fixed point theory.

CONFLICT OF INTERESTS

None.

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