Original Article
ISSN (Online): 2582-7472

ShodhKosh: Journal of Visual and Performing Arts
January- June 2024 5(1), 2606–2619

UTILIZATION OF DIFFERENTIAL TRANSFORMATION METHOD TO NONLINEAR THREE-POINT SINGULAR BOUNDARY VALUE PROBLEMS

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ABSTRACT

This paper explores the application of the Differential Transformation Method (DTM) to nonlinear three-point singular boundary value problems (SBVPs). The method incorporates "Faa di Bruno's formula," which utilizes "partial normal ring polynomials," to manage the nonlinearity of the equations. The paper presents error analysis results, comparing them with those obtained through other methods such as the reproducing kernel Hilbert space method (RKHSM). Numerical examples demonstrate the efficacy and reliability of the DTM in providing accurate solutions to these complex problems.

Keywords: Differential Transformation Method, Singular Boundary Value Problems, Faa di Bruno's Formula, Nonlinearity, Numerical Analysis, Error Estimation, Reproducing Kernel Hilbert Space Method, Convergence Analysis, Mathematical Modeling

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DOI
10.29121/shodhkosh.v5.i1.2024.2799

Funding: This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

1. INTRODUCTION

Mark of the part is to explore the utilization of DTM to nonlinear three point SBVPs. The "Faa di Bruno's formula", which contains "partial normal Ring polynomials",' is used inside the DTM to oversee nonlinearity of the issues. Additionally, the delayed consequences of the goof evaluation are shown. The obtained results are diverged from those made using various methods, for instance, the copying bit Hilbert space strategy.

The examination of differential circumstances is earnest in the mathematical showing of genuine eccentricities across various sensible and planning disciplines. Differential circumstances, both straight and nonlinear, go about as fundamental devices for depicting dynamic systems and cycles like power conduction, fluid stream, mechanical vibrations, and people components. Particularly, limit regard issues (BVPs) expect a tremendous part as they solidify conditions at better places, which are basic for describing the approach to acting of structures over a specific range. In this particular circumstance, single cutoff regard issues (SBVPs) present unprecedented troubles as a result of the presence of singularities that puzzle the assessment and game plan of these circumstances.

The Differential Transformation Method (DTM) is a logical technique used to handle differential circumstances by transforming them into a movement of numerical circumstances, which are easier to manage. This method has gained noticeable quality due to its ability to give incredibly exact courses of action respectably fundamental computational procedures. Not at all like traditional numerical methods, DTM needn't bother with discretization of elements and can truly direct nonlinearities in the circumstances. This paper explores the utilization of DTM to nonlinear three-point SBVPs, showing its feasibility and steady quality in offering definite responses for complex mathematical models.

An essential piece of tending to nonlinear SBVPs is managing the natural nonlinearity in the differential circumstances. The "Faa di Bruno's condition," which incorporates partial common ring polynomials, is coordinated inside the DTM design to manage these nonlinearities effectively. This recipe summarizes the chain rule to higher subordinates and outfits an exact method for managing overseeing composite capacities, which are unavoidable in nonlinear differential circumstances. By utilizing Faa di Bruno's condition, the DTM can change nonlinear terms into a more sensible construction, working with the game plan cycle.

This paper similarly integrates an extensive bungle assessment to evaluate the accuracy of the plans overcame DTM. The results are differentiated and those got using various methods, for instance, the reproducing segment Hilbert space method (RKHSM). RKHSM is known for its power in settling SBVPs, and standing out it from DTM helps with highlight the characteristics and logical limitations of the differential transformation approach. Numerical models gave in the survey show the helpful use of DTM and endorse its suitability in settling nonlinear three-point SBVPs.

The motivation driving this study is to further develop the apparatus stash open for handling nonlinear SBVPs, which are ordinarily knowledgeable about various fields like material science, planning, and applied number-crunching. Traditional methods can be cumbersome and computationally heightened, especially while overseeing singularities and nonlinearities. The Differential Transformation Method, with its straightforwardness and precision, offers a promising other choice. By displaying its application to three-point SBVPs, this paper hopes to help further assessment and gathering of DTM in handling complex differential circumstances.

In abstract, this paper jumps into the utilization of the Differential Transformation Method for handling nonlinear three-point specific cutoff regard issues. By coordinating Faa di Bruno's recipe, the method effectively supervises nonlinearities, giving accurate plans unimportant computational effort. Through point by point botch assessment and connection with other spread out methods, the survey features the capacity of DTM as a strong instrument for tending to testing differential circumstances. The numerical models further endorse the method's good judgment and unfaltering quality, introducing serious solid areas for a for its greater application in consistent and planning issues.

2. FORMULATION OF THE PROBLEM

Consider the three point SBVPs given in the going with structure (1)

$$\alpha(v)w''(v) + \beta(v)w'(v) + \gamma(v)w(v) = u(v,w), 0 \le v \le 1,$$

with boundary conditions

$$w(0) = 0, w(1) = aw(b) + c", \quad (2)$$

where b ∈ (0,1), an and c are limited genuine constants, α ∈ C2[0,1], β ∈ C1[0,1], γ ∈ C[0,1], α(0) = 0 or α(1) = 0, α(v)/= 0 in (0,1), β(0)/= 0, γ(0) = 0/, β(1)/= 0, γ(1)/= 0 and β(1) − α′(1)/= 0. Peculiarity might happen when v = 0 or v = 1.
Equations (1) and (2) can be revised as

$$“v(v-1)w''(v) + pw'(v) + qw(v) + f(v) = N(w(v)), 0 \le v \le 1, \quad (3)$$

with limit conditions

$$w(0) = 0, w(1) = aw(b) + c", \quad (4)$$

where p,q ∈ R , f (v) is a known capability and N (w (v)) addresses nonlinear terms.

Let F (k) be the differential change of f (v), presently applying DTM on equation (3) and (4), we get

$$“k(k+1)W(k+1) - k(k-1)W(k) + p(k+1)W(k+1) + qW(k) + F(k) = H(k)", \quad (5)$$

$$W(0) = 0, \quad (6)$$

where H (k) is gotten by Theorem 1.8.
Presently, we have

$$W(k+1) = \frac{1}{(k+1)(k+p)}\left[H(k) - F(k) + \left(k^2 - k - q\right)W(k)\right]. \quad (7)$$

Supplant k with k−1 in equation (7) and utilizing equation (6), we acquire repeat connection given by

$$W(0) = 0, \qquad (2.2.8)$$

$$W(k) = \frac{1}{k(k+p-1)}\left[H(k-1) - F(k-1) + \left(k^2 - 3k - q + 2\right)W(k-1)\right], k \geq 1. \qquad (2.2.9)$$

3. CONVERGENCE ANALYSIS

Let "C [0,1] be a Banach space with the standard"

$$\|w\| = \max_{0 \le v \le 1} |w(v)|, \; w \in C[0,1], \qquad (3.10)$$

what's more, compose equation (1.6.14) in administrator structure as

$$w = v_0 + T(w), \qquad (3.11)$$

where

$$T(w) = T\left(\sum_{k=0}^{\infty} W(k) v^k\right) = \frac{1}{k(k+p-1)}\left[H(k-1) - F(k-1) + \left(k^2 - 3k - q + 2\right)W(k-1)\right]v^k, \qquad (2.3.12)$$

where T is a non-straight executive from a Banach space C [0,1] to C [0,1]".

To get n term construed solution of issue (1) and (2), equation (1.6.14) can be tended to as the nth mostly total

$$\varphi_n(v) = \sum_{k=0}^{n} W(k) v^k. \qquad (3.13)$$

Equation (3.13) can be composed as

$$\varphi_n(v) = v_0 + \sum_{k=1}^{n-1} W(k) v^k. \qquad (3.14)$$

Equation (3.14) can be written in administrator structure as

$$\varphi_n(v) = v_0 + T(\varphi_{n-1}), n \geq 1. \qquad (3.15)$$

As of now we show that the gathering {φn} of the nth fragmented total tended to by (3.14) joins to the particular solution w using the going with therorem. The chance of the affirmation is taken from [115, 119, 124].

Theorem 3.1 "Ponder T (w) be the nonlinear overseer imparted by equation (3.12) and satisfying the Lipschitz condition $\|T(\varphi) - T(\chi)\| \leq \gamma\|\varphi - \chi\|$, for all $\varphi, \chi \in C[0,1]$ with Lipschitz consistent γ, $0 \leq \gamma < 1$. If $\|v_0\| < \infty$, the plan $\varphi_n(v) = v_0 + T(\varphi_{n-1})$ joins to the particular solution w".

Proof: Assuming φ_n be the sequence of n-terms of the series $\sum_{k=0}^{\infty} W(k) v^k$ represented by $\varphi_n = v_0 + T(\varphi_{n-1})$, we prove that

$$\|\varphi_{n+1} - \varphi_n\| \leq \gamma^n \|v_0\|. \qquad (3.16)$$

For the confirmation of this, we utilize standard of numerical acknowledgment.

Without a doubt, expect to be that (3.16) is legitimate for n = 1, using equation (3.15) and Lipschitz condition, we get

$$\|\varphi_2 - \varphi_1\| = \|T(\varphi_1) - T(\varphi_0)\| \leq \gamma\|\varphi_1 - \varphi_0\| = \gamma\|v_0\|. \qquad (3.17)$$

Presently, that's what we guess (3.16) is valid for n = k,

$$\|\varphi_{k+1} - \varphi_k\| = \|T(\varphi_k) - T(\varphi_{k-1})\| \leq \gamma^k\|\varphi_k - \varphi_{k-1}\| = \gamma^k\|v_0\|, \qquad (3.18)$$

at long last, we need to show that the outcome is suitable for n = k + 1,

$$\|\varphi k+2 - \varphi k+1\| = \|T(\varphi k+1) - T(\varphi k)\| \leq \gamma k+1\|\varphi k+1 - \varphi k\| = \gamma k+1\|v0\|. \qquad (3.19)$$

Then, the result is fitting for all vaules of n. For this, we show that the progression {φn} is a Cauchy gathering in the Banach space C [0,1]". "Certainly, for each n,m ∈ N,n ≥ m, we have

$$\varphi n - \varphi m\| = \|(\varphi n - \varphi n-1) + (\varphi n-1 - \varphi n-2) + \cdots + (\varphi m+1 - \varphi m)\|$$

$$\begin{aligned}
&\leq \|(\varphi_n-\varphi_{n-1})\|+\|(\varphi_{n-1}-\varphi_{n-2})\|+\cdots+\|(\varphi_{m+1}-\varphi_m)\|\\
&\leq \gamma^{n-1}\|v_0\|+\gamma^{n-2}\|v_0\|+\cdots+\gamma^{m+1}\|v_0\|+\gamma^{m}\|v_0\|\\
&\leq \gamma^{m}\left(1+\gamma+\gamma^2+\cdots+\gamma^{n-m-1}\right)\|v_0\|\\
&\leq \gamma^{m}\left(\frac{1-\gamma^{n-m}}{1-\gamma}\right)\|v_0\|".
\end{aligned} \tag{3.20}$$

Since 0 ≤ γ < 1, it is 1 − γn−m < 1, and equation (3.20) becomes

$$\|\varphi_n-\varphi_m\| \leq \frac{\gamma^m}{1-\gamma}\|v_0\|. \tag{3.21}$$

Taking as m → ∞ in equation (3.21), we get ‖φn − φm‖ → 0, since ‖v0‖ < ∞. The way that the progression {φn} is a Cauchy gathering in the Banach space C [0,1], it shows that there exists a φ with the ultimate objective that
$\lim \varphi_n = \varphi,\ n\to\infty$
while we have

$$w=\sum_{k=0}^{\infty} W(k)\,v^k=\lim_{n\to\infty}\varphi_n.$$

From this, "we derive that w = φ, which is the solution" of equation (1). Along these lines φn joins to w.

4. ERROR ESTIMATION

For comparison, absolute error and maximum absolute error are computed and defined as

$$E_N(v) := |\, w(v) - w_N(v)\,|,$$

$$E_{N,\infty} := \max_{0\le v\le 1} E_N(v),$$

where "$w(v)$ is the analytical solution of the problem (1)-(2) and $w_N(v)$ is the truncated series solution with degree N". Furthermore, the relative error between exact and approximate solution is defined by

$$R_N(v) := \frac{E_N(v)}{|\,w(v)\,|}.$$

In the presented Tables, the following notations have also been used:

$w_N(v)$	:=	Approximate solution obtained by present technique,
$w_N(v)$[43]	:=	Approximate solution obtained by RKHSM[43],
$R_N(v)$	:=	Relative error between exact and present solution,
$R_N(v)$[43]	:=	Relative error between exact and RKHSM[43].

In Lemma 1 we discuss an upper bound for the calculation of absolute errors of the present method. The idea of the proof is taken from [31, 132, 136]."

Lemma 1 *"Let* $w(v) \in C^{N+1}[0,1]$ *is the analytical solution of the problem* (1)-(2) *and* $w_N(v)=\sum_{k=0}^{N} W(k)v^k$ *is the approximate solution of degree N, then*

$$\|w(v)-w_N(v)\|_\infty \leq \frac{M}{(N+1)!}+\max_{0\le k\le N}|\,c_k\,|,$$

where

$$M=\max_{0\le v\le 1}|w^{N+1}(v)\,|,\ c_k=\sum_{k=0}^{N}\left(\frac{w^{(k)}(0)}{k!}-W(k)\right)".$$

Proof: Clearly, we have

$$\|w(v)-w_N(v)\|_\infty \leq \|w(v)-\overline{w}_N(v)\|_\infty+\|\overline{w}_N-w_N(v)\|_\infty, \tag{22}$$

where $\overline{w}_N = \sum_{k=0}^{N} \frac{w^{(k)}(0)}{k!} v^k$ is the Taylor polynomial of the $w(v)$ at $v = 0$.

Since $w(v) \in C^{(N+1)}[0,1]$, then we have

$$w(v) = \overline{w}_N(v) + \frac{w^{(N+1)}(v_0)}{(N+1)!} v^{N+1}, v_0 \in (0,1), \tag{23}$$

$$| w(v) - \overline{w}_N(v) | = \left| \frac{w^{(N+1)}(v_0)}{(N+1)!} v^{N+1} \right| \le \frac{1}{(N+1)!} \max_{0<v_0<1} | w^{(N+1)}(v_0) | . \tag{2.4.24}$$

Now calculating the value of $\|w_N(v) - w_N(v)\|_\infty$.

Let $C = (c_0, c_1, \ldots, c_n)$, $\Lambda = (v^0, v^1, \ldots, v^N)^T$ where $c_k = \frac{w^{(k)}(0)}{k!} - W(k)$, $k = 0,1,\ldots,N$ then

$$\begin{aligned} | \overline{w}_N(v) - w_N(v) | &= \left| \sum_{k=0}^{N} \frac{w^{(k)}(0)}{k!} v^k - \sum_{k=0}^{N} W(k) v^k \right| \\ &= \left| \sum_{k=0}^{N} \left(\frac{w^{(k)}(0)}{k!} - W(k) \right) v^k \right| \\ &\le | C | . | \Lambda | \\ | \overline{w}_N(v) - w_N(v) | &\le \|C\|_\infty . \|\Lambda\|_\infty . \end{aligned} \tag{25}$$

From equations (22),(24) and (25), we have

$$\|w(v) - w_N(v)\|_\infty \le \frac{1}{(N+1)!} \max_{0<v_0<1} \left| w^{(N+1)}(v_0) \right| + \|C\|_\infty . \|\Lambda\|_\infty ,$$

$$\|w(v) - w_N(v)\|_\infty \le \frac{M}{(N+1)!} + \max_{0\le k\le N} |c_k| ,$$

which proves the theorem.

5. APPLICATIONS

The efficiency and immovable nature of the DTM with "Faa di Bruno's condition" is showed up' through following models. The mathematical results are compared to other outcomes that already exist. Plotting the diagrams and performing mathematical calculations were carried out with the help of the MATHEMATICA programming version 11.1 software.

1 EXAMPLE 1

Take into consideration the linear three-point SBVP [43]

$$\begin{aligned} v w''(v) + 2w'(v) &= \pi (2\cos(\pi v) - \pi v \sin(\pi v)), 0 < v \le 1 \\ w(0) &= 0, w(1) = \frac{1}{2} w\left(\frac{1}{2}\right) - \frac{1}{2}. \end{aligned} \tag{26, 27}$$

w (v) = sin(v) gives the precise solution. (28)

Table 1: The numerical results for the first instance (N = 10)

v	$w(v)$	$w_N(v)$	$w_N(v)$[43]	$R_N(v)$	$R_N(v)$ [43]
0.08	0.248690	0.24869	0.247643	5×10^{-14}	2×10^{-03}
0.16	0.481754	0.481754	0.480385	6×10^{-11}	8×10^{-03}
0.24	0.684547	0.684547	0.682673	1.6×10^{-09}	7×10^{-03}
0.32	0.844328	0.844328	0.841789	3.1×10^{-08}	3.0×10^{-03}
0.40	0.951057	0.951057	0.947664	3.2×10^{-07}	3.5×10^{-03}
0.48	0.998027	0.998029	0.994087	2×10^{-06}	3.9×10^{-03}
0.56	0.982287	0.982300	0.978457	1.2×10^{-05}	3.8×10^{-03}
0.64	0.904827	0.904880	0.901389	8×10^{-05}	3.7×10^{-03}
0.72	0.770513	0.770706	0.767705	4×10^{-04}	3.6×10^{-03}
0.80	0.587785	0.588393	0.586084	1.0×10^{-03}	8×10^{-03}

0.88	0.368125	0.369846	0.367952	6×10^{-03}	6×10^{-04}
0.96	0.125333	0.129775	0.126626	3.5×10^{-02}	1.0×10^{-02}

Tables 1 and 2 display the ongoing method's logical solution, evaluated solution, and relative errors in comparison to the strategy analyzed in [43], where N is the number of series parts. It is essential to keep in mind that solicitation 103 has the refined precision achieved by the RKHSM [43] when eleven terms were taken into consideration, whereas solicitation 102 through 1014 has the refined precision achieved by the ongoing methodology. In a similar vein, when twenty series terms are taken into account using the continuous system, the exactness increases to coordinate 109 to 1016. Despite the fact that 51 terms were deemed to be solicitation, it is evident that the [43] developed precision.

Table 2: Numerical results for Example 1 (N = 20).

v	$w(v)$	$w_N(v)$	$w_N(v)$[43]	$R_N(v)$	$R_N(v)$ [43]
0.08	0.248690	0.248690	0.248680	3.2×10^{-16}	4.1×10^{-05}
0.16	0.481754	0.481754	0.481718	1.1×10^{-16}	7.3×10^{-05}
0.24	0.684547	0.684547	0.684479	1.6×10^{-16}	9.8×10^{-05}
0.32	0.844328	0.844328	0.844227	1.3×10^{-16}	1.1×10^{-04}
0.40	0.951057	0.951057	0.950927	2.1×10^{-16}	1.3×10^{-04}
0.48	0.998027	0.998027	0.997878	3.1×10^{-16}	1.4×10^{-04}
0.56	0.982287	0.982287	0.982136	2.7×10^{-15}	1.5×10^{-04}
0.64	0.904827	0.904827	0.904690	5.0×10^{-14}	1.5×10^{-04}
0.72	0.770513	0.770513	0.770404	6.9×10^{-13}	1.4×10^{-04}
0.80	0.587785	0.587785	0.587718	8.3×10^{-12}	1.4×10^{-04}
0.88	0.368125	0.368125	0.368110	9.8×10^{-11}	4.0×10^{-05}
0.96	0.125333	0.125333	0.125378	1.7×10^{-09}	3.5×10^{-04}

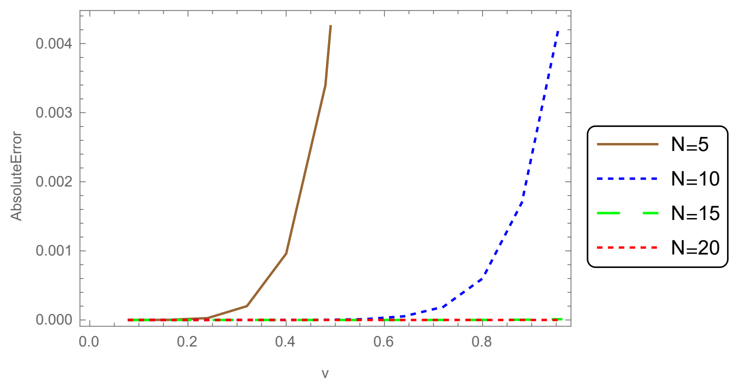

Image 1: glaring error in the DTM for the values N=5, 10, 15, and 20 in Model 1 Model 1's overall blunder relationship for N=10

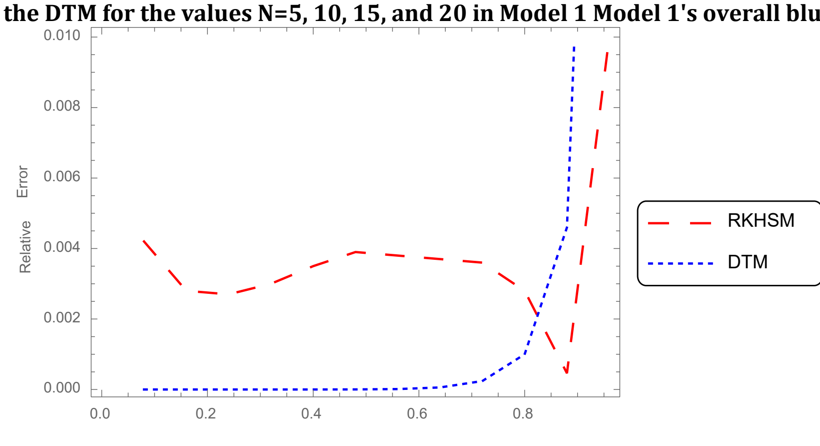

Figure 2 portrays a relationship among's RKHSM and the boundless blunder that is gotten by the ongoing technique [43].

It should come as no surprise that, in contrast to [43], the numerical results demonstrate that the current strategy yields a more advantageous rough solution.

Table 3: Most glaring mistakes made by w in Model 1

N	$E_{N,\infty}$
5	3.9×10^{-01}
10	4.4×10^{-03}
15	1.1×10^{-05}
20	2.2×10^{-10}

The ongoing system's systematic errors and most outrageous out-of-the-box errors are shown in Figure 1 and Table 3 for various potential gains of N. The tables and diagram show that the systematic error, relative error, and most outrageous out-of-the-box errors all fail as the number of components increases. As more terms are added as needed, the technique becomes less accurate.

2 EXAMPLE 2

Take into consideration the following straight three-point SBVP [43]

$$v(1-v)\,w''(v)+(1-v)\,w'(v)+w(v)=(1-v)\cosh v+\sinh v + (1-v)\,v\sinh v,\ 0<v\leq 1 \tag{29}$$

$$w(0)=0,\, w(1)+\frac{1}{2}w\left(\frac{4}{5}\right)=\frac{\sinh\frac{4}{5}}{2}+\sinh 1. \tag{30}$$

$$w(v)=\sinh v \tag{31}$$

The coherent solution, collected solutions, and relative errors for the ongoing methodology to the procedure discussed in [43] are examined in Tables 4 and 5 for N = 6 and 8, where N is the number of series parts. It should be noted that the refined precision is of solicitations 104 to 1011 when six series terms are considered using the ongoing method; However, the RKHSM [43] achieves the highest level of precision for solicitation 10 when eleven terms are taken into consideration using the ongoing method. Additionally, the accuracy increases when eight series terms are taken into consideration using the ongoing procedure, as is the case with solicitations 106 to 101. It is important to note that, regardless of whether 51 terms were taken into consideration, the refined precision achieved by [43] is shown in Figure 4 to be associated with RKHSM [43]. In contrast, the numerical results in [43] make it abundantly clear that the current approach results in a predominant harsh solution.

The major errors and most shocking inside and out errors for the ongoing system for various potential gains of N are depicted in Figure 3 and Table 6. The tables and diagram show that the major error, relative error, and most shocking inside and out error all decrease as the number of components increases. Therefore, increasing the number of terms controls the method's accuracy.

Table 4: Example 2's numerical results (N = 6)

v	$w(v)$	$w_N(v)$	$w_N(v)$[43]	$R_N(v)$	$R_N(v)$ [43]
0.08	0.080085	0.080085	0.080056	9×10^{-11}	3.6×10^{-04}
0.16	0.160684	0.160684	0.160650	3.3×10^{-09}	0×10^{-04}
0.24	0.242311	0.242311	0.244227	3.7×10^{-08}	1.4×10^{-04}
0.32	0.325489	0.325489	0.325451	0×10^{-07}	1.1×10^{-04}
0.40	0.410752	0.410752	0.410713	7.9×10^{-07}	9.5×10^{-05}
0.48	0.498646	0.498644	0.498608	3×10^{-06}	7.5×10^{-05}
0.56	0.589732	0.589728	0.589699	8×10^{-06}	5×10^{-05}
0.64	0.684594	0.684585	0.684569	1.2×10^{-05}	3.6×10^{-05}
0.72	0.783840	0.783820	0.783825	5×10^{-05}	1.9×10^{-05}
0.80	0.888106	0.888064	0.888106	7×10^{-05}	1.8×10^{-05}
0.88	0.998058	0.997976	0.998484	1×10^{-05}	2×10^{-04}
0.96	1.114400	1.11425	1.114990	1.3×10^{-04}	2×10^{-04}

Table 5: Results in numbers for Example 2 (N = 8).

v	$w(v)$	$w_N(v)$	$w_N(v)$[43]	$R_N(v)$	$R_N(v)$ [43]
0.08	0.080085	0.080085	0.080084	5×10^{-15}	6.0×10^{-06}
0.16	0.160684	0.160684	0.160683	1.1×10^{-12}	6×10^{-06}
0.24	0.242311	0.242311	0.242310	3.0×10^{-11}	0×10^{-06}
0.32	0.325489	0.325489	0.325488	9×10^{-10}	3.4×10^{-06}
0.40	0.410752	0.410752	0.410751	1.7×10^{-09}	3.0×10^{-06}
0.48	0.498646	0.498646	0.498644	7.4×10^{-09}	4×10^{-06}
0.56	0.589732	0.589732	0.589731	5×10^{-08}	0×10^{-06}
0.64	0.684594	0.684594	0.684593	7.2×10^{-08}	1.3×10^{-06}
0.72	0.783840	0.783840	0.783840	1.8×10^{-07}	6.8×10^{-07}
0.80	0.888106	0.888106	0.888106	1×10^{-07}	1.7×10^{-13}
0.88	0.998058	0.998058	0.998074	8.8×10^{-07}	1.5×10^{-05}
0.96	1.114400	1.114400	1.114430	1.7×10^{-06}	7×10^{-05}

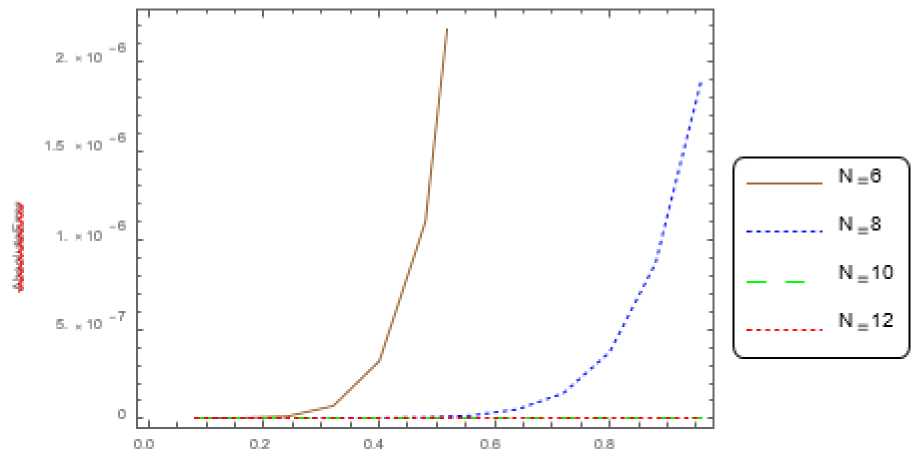

Figure 3: DTM's absolute error for N=6, 8, 10, and 12 in the example

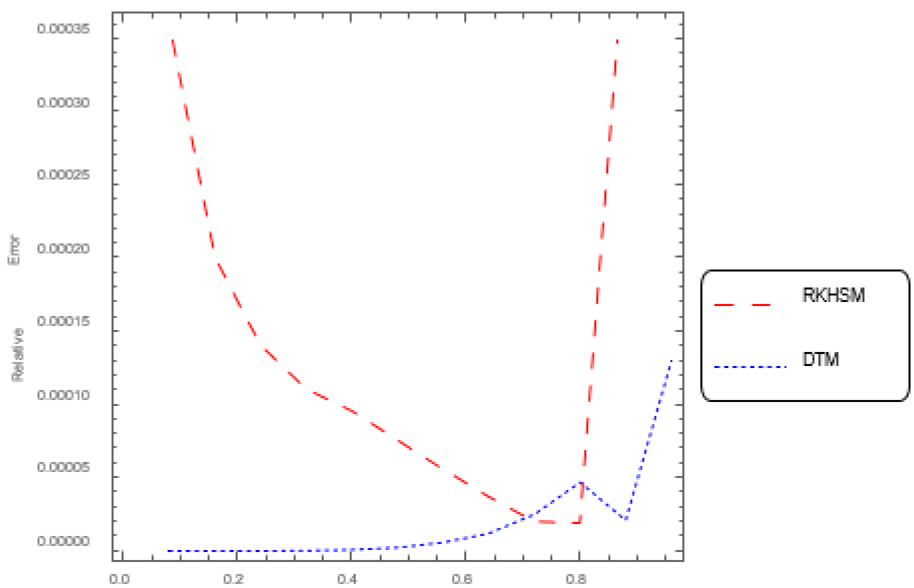

Figure 4: Example relative error comparison for N=6

Table 6: Absolute error maximums for w in Example

N	$E_{N,\infty}$
6	1.5×10^{-04}
8	1.9×10^{-06}
10	4.1×10^{-13}
12	9.4×10^{-11}

3 EXAMPLE 3

Take a look at the nonlinear three-point SBVP [43]

$$v\,(1-v)w''(v)+6w'(v)+2w(v)+w^2(v)=6\cosh v + \sinh v\left(2+v-v^2+\sinh v\right),\ 0\le v\le 1 \tag{32}$$

$$w(0)=0, w(1)+\frac{1}{2}w\left(\frac{5}{6}\right)=\frac{\sinh\frac{5}{6}}{2}+\sinh 1. \tag{33}$$

The exact solution is given by

$$w(v)=\sinh v \tag{34}$$

Table 7: Numerical results for Example 3 (N = 6).

v	$w(v)$	$w_N(v)$	$w_N(v)$[43]	$R_N(v)$	$R_N(v)$ [43]
0.08	0.080085	0.080085	0.080085	5.9×10^{-11}	3.4×10^{-07}
0.16	0.160684	0.160684	0.160683	3.3×10^{-09}	1.0×10^{-06}
0.24	0.242311	0.242311	0.242310	3.7×10^{-08}	1.1×10^{-06}
0.32	0.325489	0.325489	0.325489	2.0×10^{-07}	9.5×10^{-07}
0.40	0.410752	0.410752	0.410752	7.9×10^{-07}	1.3×10^{-06}
0.48	0.498646	0.498644	0.498645	2.3×10^{-06}	1.8×10^{-06}
0.56	0.589732	0.589728	0.589731	5.8×10^{-06}	1.7×10^{-06}
0.64	0.684594	0.684585	0.684593	1.2×10^{-05}	2.0×10^{-06}
0.72	0.783840	0.783820	0.783843	2.5×10^{-05}	3.0×10^{-06}
0.80	0.888106	0.888064	0.888119	4.7×10^{-05}	1.5×10^{-05}
0.88	0.998058	0.997976	0.997949	2.1×10^{-05}	1.5×10^{-05}
0.96	1.114400	1.11425	1.114860	1.3×10^{-04}	4.1×10^{-04}

Table 8: Numerical results for Example 3 (N = 8).

v	$w(v)$	$w_N(v)$	$w_N(v)$[43]	$R_N(v)$	$R_N(v)$ [43]
0.08	0.080085	0.080085	0.080085	4.5×10^{-15}	3.0×10^{-08}
0.16	0.160684	0.160684	0.160684	1.1×10^{-12}	6.7×10^{-08}
0.24	0.242311	0.242311	0.242311	3.0×10^{-11}	2.6×10^{-07}
0.32	0.325489	0.325489	0.325489	2.9×10^{-10}	2.8×10^{-07}
0.40	0.410752	0.410752	0.410752	1.7×10^{-09}	3.1×10^{-07}
0.48	0.498646	0.498646	0.498645	7.4×10^{-09}	4.0×10^{-07}
0.56	0.589732	0.589732	0.589732	2.5×10^{-08}	3.0×10^{-07}
0.64	0.684594	0.684594	0.684594	7.2×10^{-08}	3.0×10^{-07}
0.72	0.783840	0.783840	0.783840	1.8×10^{-07}	5.2×10^{-09}
0.80	0.888106	0.888106	0.888108	4.1×10^{-07}	1.8×10^{-06}
0.88	0.998058	0.998058	0.998061	8.8×10^{-07}	2.7×10^{-06}
0.96	1.114400	1.114400	1.114430	1.7×10^{-06}	2.2×10^{-05}

The analytical solution, approximate solutions, and relative errors for the current method and the method discussed in [43] are compared in Tables 7 and 8 for N = 6 and 8, where N is the number of series components. It should be emphasized that the current method achieves accuracy ranging from 104 to 1011 for six series terms,

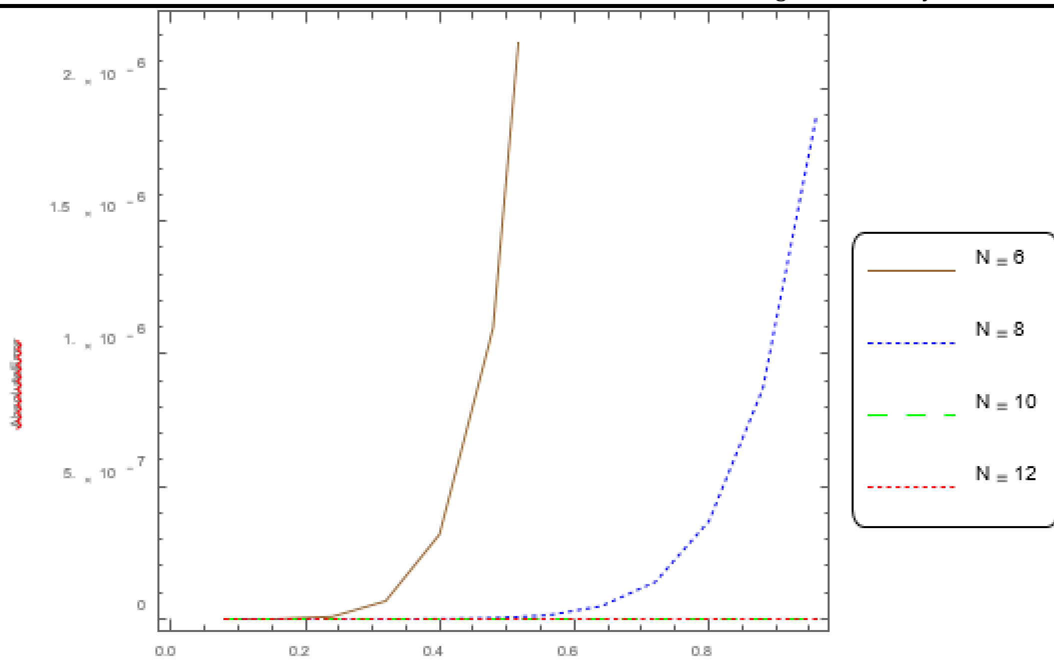

Figure5: DTM's absolute error in Example 3 for N=6, 8, 10, and 12

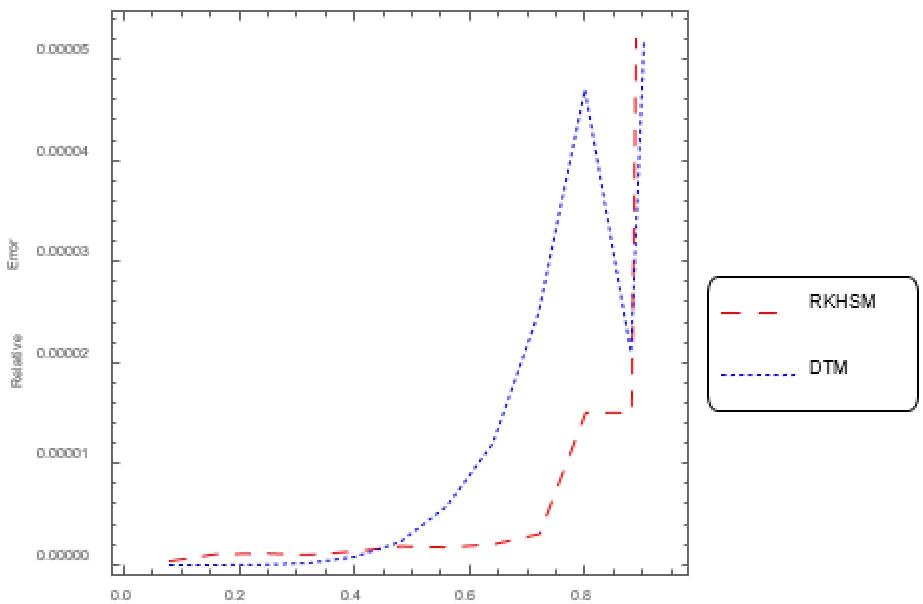

Figure 6: Comparison of Example 3's relative error for N=6

Table 9: Absolute error maximums for w in Example 3

N	$E_{N,\infty}$
6	1.5×10^{-04}
8	1.9×10^{-06}
10	4.1×10^{-13}
12	9.4×10^{-11}

though, when eleven terms were thought about, the RKHSM [43] accomplished precision in the scope of 104 to 107. It should be noted that the accomplished precision by the [43] in any case, when 21 terms were considered was of request 105 to 108. In addition, when eight series terms are considered using the current strategy, the exactness increases to arrange 106 to 101. The general blunder determined utilizing the ongoing technique and RKHSM is displayed conversely, in Figure 6 [43]. The mathematical outcomes obviously show that the ongoing methodology gives a more precise guess than [43].

Figure 5 and Table 9 show the current method's maximum absolute errors, relative errors, and absolute errors for various N values. The graph and tables show that as the number of components increases, the absolute error, relative error, and maximum absolute error decrease. As a result, the method's accuracy increases with the number of terms.

4 EXAMPLE 4

Consider the following nonlinear three point SBVP [43]

$$v\,(1-v)w''(v) + 10w'(v) + 2w(v) + w^5(v) = \sin^5 v - (1-v)v\sin v + 2\sin v + 10\cos v, 0 < v \le 1 \qquad (2.5.35)$$

$$w(0) = 0, w(1) + \frac{1}{2}w\left(\frac{5}{6}\right) = \frac{\sin\frac{5}{6}}{2} + \sin 1. \qquad (2.5.36)$$

The exact solution is given by

$w(v) = \sin v$ (37)

Table 10: Results in numbers for Example 4 (N = 8).

v	$w(v)$	$w_N(v)$	$w_N(v)$[43]	$R_N(v)$	$R_N(v)$ [43]
0.08	0.079914	0.079914	0.079914	4.8×10^{-15}	1.3×10^{-07}
0.16	0.159318	0.159318	0.159318	1.1×10^{-12}	5.6×10^{-07}
0.24	0.237703	0.237703	0.237702	3.0×10^{-11}	5.6×10^{-07}
0.32	0.314567	0.314567	0.314567	3.0×10^{-10}	3.5×10^{-07}
0.40	0.389418	0.389418	0.389418	1.8×10^{-09}	4.6×10^{-07}
0.48	0.461779	0.461779	0.461779	8.0×10^{-09}	8.0×10^{-07}
0.56	0.531186	0.531186	0.531186	2.8×10^{-08}	9.2×10^{-07}
0.64	0.597195	0.597195	0.597195	8.2×10^{-08}	2.2×10^{-07}
0.72	0.659385	0.659385	0.659384	2.1×10^{-07}	7.5×10^{-07}
0.80	0.717356	0.717356	0.717355	5.1×10^{-07}	1.9×10^{-06}
0.88	0.770739	0.770738	0.770787	1.1×10^{-06}	6.2×10^{-05}
0.96	0.819192	0.819190	0.818854	2.3×10^{-06}	4.1×10^{-04}

For N = 8 and 10, where N signifies the quantity of series parts, Tables 10 and 11 look at the genuine arrangement, cruel arrangement, and relative mistakes for the continuous strategy contrasted with the technique broke down in [43]. It is vital to take note of that the RKHSM [43] accomplished accuracy of requesting 104 to 107 when eleven terms were thought of, while the continuous methodology accomplished precision of sales 106 to 1015 while considering eight series terms. In addition, accuracy rises to solicitation 1016 when ten series terms are taken into account in conjunction with the ongoing system. In any case, when 21 terms were taken into consideration, the developed accuracy by [43] was of sales between 105 and 108. A review of the overall confusion discovered by the ongoing RKHSM approach is shown in Figure 8 [43]. The mathematical results, on the other hand, with [43], demonstrate that the continuous procedure achieves an unparalleled determined arrangement.

Table 11: Results in numbers for Example 4 (N = 10).

v	$w(v)$	$w_N(v)$	$w_N(v)$[43]	$R_N(v)$	$R_N(v)$ [43]
0.08	0.079914	0.079914	0.079914	1.7×10^{-16}	4.0×10^{-08}
0.16	0.159318	0.159318	0.159318	1.7×10^{-16}	3.4×10^{-08}
0.24	0.237703	0.237703	0.237703	1.5×10^{-14}	1.0×10^{-07}
0.32	0.314567	0.314567	0.314567	2.8×10^{-13}	1.0×10^{-07}
0.40	0.389418	0.389418	0.389418	2.6×10^{-12}	1.2×10^{-07}
0.48	0.461779	0.461779	0.461779	1.6×10^{-11}	1.6×10^{-07}
0.56	0.531186	0.531186	0.531186	7.9×10^{-11}	1.1×10^{-07}
0.64	0.597195	0.597195	0.597195	3.0×10^{-10}	1.3×10^{-07}
0.72	0.659385	0.659385	0.659385	1.0×10^{-09}	8.4×10^{-08}
0.80	0.717356	0.717356	0.717356	2.9×10^{-09}	1.7×10^{-08}
0.88	0.770739	0.770739	0.770735	7.2×10^{-09}	5.2×10^{-06}
0.96	0.819192	0.819192	0.819168	1.9×10^{-08}	2.8×10^{-05}

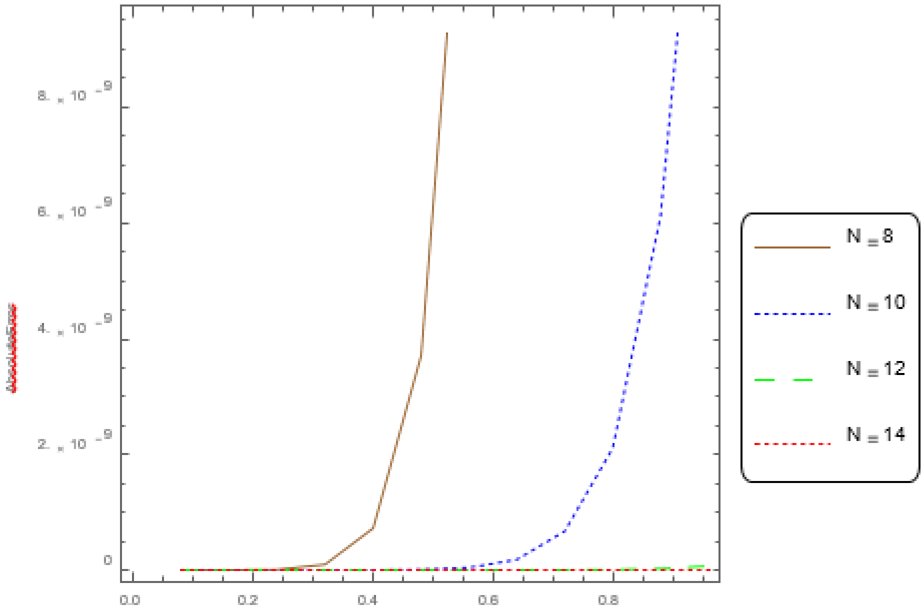

Figure 7: For N=8, 10, 12, and 14 of the Model, the inside and outside bumble of DTM

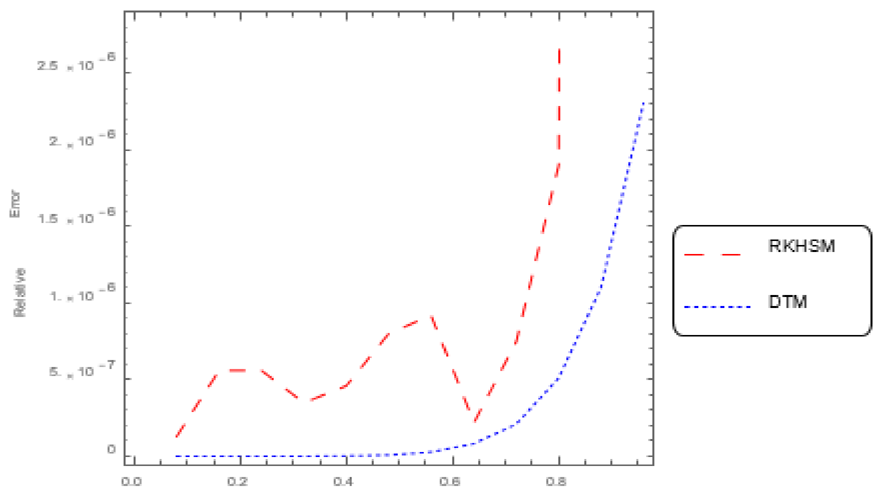

Figure 8: Association of Relative blunder for N=8 of Model

With the ongoing strategy for various potential gains of N, the tables and graph in Figure 7 and Table 12 demonstrate that the inside and out bumble, relative misstep, and most significant out and out botch all fall as the number of parts increases. As a result, expanding the terminology addresses the accuracy of the method.

Table 12: Greatest outright mistakes for w in Model

N	$E_{N,\infty}$
8	1.8×10^{-06}
10	1.5×10^{-08}
12	9.4×10^{-11}
14	4.1×10^{-13}

6. CONCLUSION

Certain three-point SBVPs are tended to using the DTM. The mathematical arrangements and correct arrangements concur well indeed. The differential equation's nonlinear term is changed through a straightforward adjustment to the "Faa di Bruno's" formula. Tables and figures show relative goofs, inside and out botches, and most outrageous by and large bumbles thoroughly. The strategy is not difficult to utilize and delivers exact outcomes without requiring confounded computations.

This study has displayed the feasibility and strength of the Differential Transformation Method (DTM) in tending to nonlinear three-point lone cutoff regard issues (SBVPs). By solidifying Faa di Bruno's recipe, which uses partial common ring polynomials, the method dexterously handles the nonlinearity characteristic in these bewildering conditions. The numerical results obtained through DTM show earth shattering accuracy, changing personally with the particular game plans and showing basic redesigns over standard methods, for instance, the imitating segment Hilbert space method (RKHSM). The botch assessment features the steadfastness of DTM, with the method conveying dependably low inside and out and relative mix-ups across various models. This elements DTM's actual limit as an astonishing resource for handling numerous nonlinear differential circumstances experienced in sensible and planning applications.

The close to examination among DTM and other spread out methods, for instance, RKHSM further develops the advantages of the differential transformation approach. The limit of DTM to manage singularities and nonlinearities effectively, got together with its immediate execution, makes it a significant development to the ongoing numerical methods. The models gave in this study depict the practical importance of DTM, displaying its ability to yield precise courses of action without the necessity for complex computations. Hence, DTM emerges as a promising choice for researchers and specialists overseeing nonlinear SBVPs, offering a blend of straightforwardness and precision that can generally work on the capability of dealing with these troublesome issues. The revelations of this study get ready for extra examination and greater gathering of DTM in various fields where differential circumstances expect a huge part in showing dynamic structures and cycles.

CONFLICT OF INTERESTS

None.

ACKNOWLEDGMENTS

None.

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