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AN IMPROVED EDAS METHOD BASED ON TRAPEZOIDAL NEUTROSOPHIC NUMBER AND ITS APPLICATION IN GROUP DECISION MAKING

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ABSTRACT

The trapezoidal neutrosophic set is a useful tool for dealing with vague, complex, and uncertain information. In this study, the authors enhanced the original EDAS (Evaluation Based on Distance from Average Solution) method by incorporating trapezoidal neutrosophic numbers (TNNs) to solving a multiple-criteria group decision-making (MCGDM) problem. They calculated the average solution for each criteria using two existing aggregation operators of TNNs. After that, they determined the positive and negative distances of every alternative from the average ideal solution and calculated these appraisal scores for the alternatives. Using these scores, they ranked the alternatives. At last, the authors illustrated the practicality, stability, along with the effectiveness made of the improved EDAS method by analyzing the influence of various parameters.

Keywords: Tnns -Score Function-Trapezoidal Neutrosophic Number Weighted Arithmetic Av- Eraging Operator -Trapezoidal Neutrosophic Number Weighted Geometric Averaging Operator- Multiple-Criteria Group Decision-Making

1. INTRODUCTION

A fuzzy set is a mathematical concept introduced by Lotfi Zadeh [28] in 1965, representing elements with degrees of membership rather than binary inclusion.It allows for partial mem- bership in a set, accommodating uncertainty and vagueness in various applications. In 1983, Atanassov expanded upon Zadeh's work by introducing the intuitionistic fuzzy set, which in- corporates not only membership but also nonmembership, adding a degree of indeterminacy to account for ambiguity in decision-making. Following this, Florentin Smarandache intro- duced the neutrosophic set in the 1990s, aimed at exploring the nature of contradictions and indeterminacies in reasoning. It extends traditional logic by introducing a third truth value, representing indeterminacy, alongside true and false. This allows for a more nuanced approach to dealing with uncertainty and vagueness in various contexts.

Building on the neutrosophic set,The idea of trapezoidal neutrosophic numbers were intro- duced by Florentin Smarandache in 2010. A trapezoidal neutrosophic number stands a exten- sion about the trapezoidal fuzzy number. It is characterized by a trapezoidal shape for its mem- bership function regarding the three elements truth, indeterminacy and falsity. Each of these components is represented by a trapezoidal function, defined by four parameters, allowing for a more nuanced representation of uncertainty and ambiguity in various contexts.This frame- work is particularly useful in decision-making, modeling, and reasoning in scenarios where information is incomplete or contradictory, offering a robust way to capture the complexities of uncertain situations.

When addressing certain Problematic decisions, for instance those involving assessments and forecasts, it can be challenging for represent alternative rankings using precise values, es- pecially when evaluations are gathered via surveys. Utilizing fuzzy sets, intuitionistic fuzzy sets, and trapezoidal neutrosophic sets can greatly facilitate those resolution of these complex decision-making issues. Nonetheless, fuzzy sets and intuitionistic fuzzy sets have specific limi- tations when it comes to neutrosophic set theory. By employing three independent membership functions originating from neutrosophic set theory, respondents as part of surveys can easily express their opinions and choices. Researchers have identified the advantages of neutrosophic sets, including trapezoidal neutrosophic sets, and have integrated them into the multiple-criteria decision-making process to enhance there accuracy and effectiveness of evaluations.

Keshavarz Ghorabaee and his associates developed the Evaluation Based on Distance from Average Solution (EDAS) method [8]. Since its introduction, this method is regarded as em- ployed to address various issues across different fields, including ABC inventory classification [8], facility location selection [15], supplier selection [6,23], third-party logistics provider se- lection [3], prioritization of sustainable development goals [14], selection of autonomous ve- hicles [27], evaluation of e-learning materials [12], renewable energy adoption [2], safety risk assessment [10], and industrial robot selection [21]. Several extensions to the EDAS method have also been proposed, including fuzzy EDAS [7], an interval type-2 fuzzy extension of the EDAS method [6], rough EDAS [23], Grey EDAS [22], intuitionistic fuzzy EDAS [13], interval-valued fuzzy EDAS [19], and adaptations of the EDAS method in Minkowski space [27]. Additional extensions involve the EDAS method in q-rung orthopair fuzzy environments [17], as well as those based on interval-valued complex fuzzy soft weighted arithmetic averag- ing (IV-CFSWAA) and interval-valued complex fuzzy soft weighted geometric averaging (IV-CFSWGA) operators [4], and trapezoidal bipolar fuzzy information [18]. Moreover, several EDAS extensions are designed for neutrosophic environments, such as refined single-valued neutrosophic EDAS [24], single-valued complex neutrosophic EDAS [26], single-valued tri- angular neutrosophic EDAS [5], neutrosophic EDAS[1], and various adaptations incorporat- ing multivalued neutrosophic sets, linguistic neutrosophic contexts, and interval-valued neutro-sophic approaches [9][16][25][14][42][20].

This article presents a innovative extension of the EDAS method enabling its application to complex decision-making challenges involving trapezoidal neutrosophic numbers.Thus, the rest of this paper is structured as follows: In Section 2, the authors provide some basic defini- tions related to trapezoidal neutrosophic sets. Section 3 presents the gradual process from the improved EDAS method. In Section 4, the authors analyze and compare the results obtained from the improved EDAS algorithm with other methods using a numerical illustration involv- ing recipients of social welfare assistance showcases the practical application and efficacy of the improved EDAS method. Finally, the results are presented in the last section.

2. PRELIMINARIES

2.1. BASIC CONCEPTS

Definition 2.1.1. Let [11] S be a universal set with a general element in S denoted by s. A neutrosophic set (NS) λ within S is defined by three membership functions a truth-membership function $T\lambda(s)$: $S \rightarrow [0-, 1+[$, an indeterminacy-membership function $I\lambda(s)$: $S \rightarrow]0-, 1+[$, and a falsity-membership function $F\lambda(s)$: $S \rightarrow]0-, 1+[$. The neutrosophic set λ can be expressed as:

$$\lambda = \{(s, (T\lambda(s), I\lambda(s), F\lambda(s))): s \in S\}, \qquad (1)$$

where $T\lambda(s), I\lambda(s), F\lambda(s) \in]0-, 1+[$, and

$$0- \leq T\lambda(s) + I\lambda(s) + F\lambda(s) \leq 3+.$$

Definition 2.1.2. Let S be a universal set with a general element denoted by s. A single-valued neutrosophic set (SVNS) δ in S is defined by three membership functionssuch as the truth- membership function $T\delta(s)$, the indeterminacy-membership function $I\delta(s)$, and the falsity- membership function $F\delta(s)$, where each function maps elements of S to the interval [0, 1]. Specifically, $T\delta(s) : S \to [0, 1]$, $I\delta(s) : S \to [0, 1]$, and $F\delta(s) : S \to [0, 1]$. The single-valued neutrosophic set δ is expressed as:

$$\delta = \{(s, (T\delta(s), I\delta(s), F\delta(s))) : s \in S\}, \qquad (2)$$

where $T\delta(s), I\delta(s), F\delta(s) \in [0, 1]$ and

$$0 \le T\delta(s) + I\delta(s) + F\delta(s) \le 3.$$

Definition 2.1.3. Let S be a universal set. A trapezoidal neutrosophic set N is defined as:

$$N = \{(s, T(s), I(s), F(s)) : s \in S\} \qquad (3)$$

where $T(s) \subseteq [0, 1]$, $I(s) \subseteq [0, 1]$, and $F(s) \subseteq [0, 1]$ are three trapezoidal neutrosophic num- bers. These are given by the functions:

$$T(s) = (\alpha(s), \beta(s), \gamma(s), \mu(s)): S \to [0, 1],$$
$$I(s) = (\lambda(s), \mu(s), \kappa(s), \iota(s)): S \to [0, 1],$$
$$F(s) = (\phi(s), \rho(s), \psi(s), \sigma(s)): S \to [0, 1],$$

Provided that $0 \le \mu(s) + \iota(s) + \sigma(s) \le 3$ for all $s \in S$. Here, T (s), I(s), and F (s) are called the truth-membership function, indeterminacy-membership function, and falsity-membership function of the element s to the set N, respectively.

Definition 2.1.4. Let $n1 = ((\alpha1(s), \beta1(s), \gamma1(s), \mu1(s)), (\lambda1(s), \mu1(s), \kappa1(s), \iota1(s)), (\phi1(s), \rho1(s), \psi1(s), \sigma1(s)))$ and $n2 = ((\alpha2(s), \beta2(s), \gamma2(s), \mu2(s)), (\lambda2(s), \mu2(s), \kappa2(s), \iota2(s)), (\phi2(s), \rho2(s), \psi2(s), \sigma2(s)))$ be two trapezoidal neutrosophic numbers. The operational rules are de- fined as follows:

1) **The sum of n1 and n2 is given by:**

$$n1 \oplus n2 = ((\alpha1(s) + \alpha2(s) - \alpha1(s)\alpha2(s), \beta1(s) + \beta2(s) - \beta1(s)\beta2(s), \gamma1(s) + \gamma2(s) - \gamma1(s)\gamma2(s), \mu1(s) + \mu2(s) - \mu1(s)\mu2(s)), (\lambda1(s)\lambda2(s), \mu1(s)\mu2(s), \kappa1(s)\kappa2(s), \iota1(s)\iota2(s)), (\phi1(s)\phi2(s), \rho1(s)\rho2(s), \psi1(s)\psi2(s), \sigma1(s)\sigma2(s)))$$

2) **The product of n1 and n2 is given by:**

$$n1 \otimes n2 = ((\alpha1(s)\alpha2(s), \beta1(s)\beta2(s), \gamma1(s)\gamma2(s), \mu1(s)\mu2(s)), (\lambda1(s) + \lambda2(s) - \lambda1(s)\lambda2(s), \mu1(s) + \mu2(s) - \mu1(s)\mu2(s), \kappa1(s) + \kappa2(s) - \kappa1(s)\kappa2(s), \iota1(s) + \iota2(s) - \iota1(s)\iota2(s)), (\phi1(s) + \phi2(s) - \phi1(s)\phi2(s), \rho1(s) + \rho2(s) - \rho1(s)\rho2(s), \psi1(s) + \psi2(s) - \psi1(s)\psi2(s), \sigma1(s) + \sigma2(s) - \sigma1(s)\sigma2(s)))$$

3) **For λ > 0, the operation λn1 is defined as:**

$$\lambda n1 = ((1-(1-\alpha1(s))\lambda, 1-(1-\beta1(s))\lambda, 1-(1-\gamma1(s))\lambda, 1-(1-\mu1(s))\lambda), (\lambda1(s)\lambda, \mu1(s)\lambda, \kappa1(s)\lambda, \iota1(s)\lambda), (\phi1(s)\lambda, \rho1(s)\lambda, \psi1(s)\lambda, \sigma1(s)\lambda))$$

4) For λ ≥ 0, the operation n1λ is defined as:

n1λ = ((α1(s)λ, β1(s)λ, γ1(s)λ, μ1(s)λ), (1 − (1 − λ1(s)) λ, 1 − (1 − μ1(s)) λ, 1 − (1 − κ1(s)) λ, 1−(1−ι1(s)) λ), (1−(1−ϕ1(s)) λ, 1−(1−ρ1(s)) λ, 1−(1−ψ1(s)) λ, 1−(1−σ1(s)) λ))

Definition 2.1.5. Let N = ((α(s), β(s), γ(s), μ(s)), (λ(s), μ(s), κ(s), ι(s)), (ϕ(s), ρ(s), ψ(s), σ(s))) be a trapezoidal neutrosophic number. The score function for this trapezoidal neutro- sophic number can be defined as:

$$S(n) = \frac{1}{3}\left(2 + \frac{\alpha(s)+\beta(s)+\gamma(s)+\mu(s)}{4} - \frac{\lambda(s)+\mu(s)+\kappa(s)+\iota(s)}{4} - \frac{\phi(s)+\rho(s)+\psi(s)+\sigma(s)}{4}\right), \quad S(n) \in [0, 1] \quad \text{(a)}$$

When a greater value of S(n) indicates a greater trapezoidal neutrosophic number n. Particu- larly, when β(s) = γ(s), μ(s) = κ(s), and ρ(s) = ψ(s) hold in the trapezoidal neutrosophic number n, Equation (a) simplifies to the following score function for the triangular neutro- sophic number:

$$S(n) = \frac{1}{3}\left(2 + \frac{\alpha(s)+2\beta(s)+\mu(s)}{4} - \frac{\lambda(s)+2\mu(s)+\iota(s)}{4} - \frac{\phi(s)+2\rho(s)+\sigma(s)}{4}\right), \quad S(n) \in [0, 1] \quad \text{(b)}$$

which represents a special case of Equation (a).

Definition 2.1.6. Let N = ((α(s), β(s), γ(s), μ(s)), (λ(s), μ(s), κ(s), ι(s)), (ϕ(s), ρ(s), ψ(s), σ(s))) be a trapezoidal neutrosophic number. The normalized trapezoidal neutrosophic number (TNN) can be expressed as:

$$N = \begin{cases} ((\alpha(s), \beta(s), \gamma(s), \mu(s)), (\lambda(s), \mu(s), \kappa(s), \iota(s)), (\phi(s), \rho(s), \psi(s), \sigma(s))) & \text{for C-benefit} \\ ((1-\alpha(s), 1-\beta(s), 1-\gamma(s), 1-\mu(s)), (1-\lambda(s), 1-\mu(s), 1-\kappa(s), 1-\iota(s))\ (1-\phi(s), 1-\rho(s), 1-\psi(s), 1-\sigma(s))) & \text{for C-cost} \end{cases} \quad (4)$$

Definition 2.1.7. Let N = ((α(s), β(s), γ(s), μ(s)), (λ(s), μ(s), κ(s), ι(s)), (ϕ(s), ρ(s), ψ(s), σ(s))) (s = 1, 2, . . . , p) is a set of trapezoidal neutrosophic numbers (TNNs). The weighted arithmetic averaging (WAA) operator for trapezoidal neutrosophic numbers (TNNWAA) is de- fined as follows.:

$$\text{TNNWAA}(N_1, N_2, N_3, \ldots, N_p) = \sum_{s=1}^{p} \varphi_s N_s$$
$$= \left\langle \begin{array}{l} \left[1 - \prod_{s=1}^{p}(1-\alpha(s))^{\varphi_s}, 1 - \prod_{s=1}^{p}(1-\beta(s))^{\varphi_s}, 1 - \prod_{s=1}^{p}(1-\gamma(s))^{\varphi_s}, 1 - \prod_{s=1}^{p}(1-\mu(s))^{\varphi_s}\right], \\ \left[\prod_{s=1}^{p}(\lambda_s)^{\varphi_s}, \prod_{s=1}^{p}(\mu_s)^{\varphi_s}, \prod_{s=1}^{p}(\kappa_s)^{\varphi_s}, \prod_{s=1}^{p}(\iota_s)^{\varphi_s}\right], \\ \left[\prod_{s=1}^{p}(\phi_s)^{\varphi_s}, \prod_{s=1}^{p}(\rho_s)^{\varphi_s}, \prod_{s=1}^{p}(\psi_s)^{\varphi_s}, \prod_{s=1}^{p}(\sigma_s)^{\varphi_s}\right] \end{array} \right\rangle \quad (5)$$

Definition 2.1.8. Let N = ((α(s), β(s), γ(s), μ(s)), (λ(s), μ(s), κ(s), ι(s)), (ϕ(s), ρ(s), ψ(s), σ(s))) (s = 1, 2, . . ., p) Constitute a group of trapezoidal neutrosophic numbers (TNNs). The weighted geometric averaging (WGA) operator for trapezoidal neutrosophic numbers (TNNWGA) is defined as follows.::

$$\text{TNNWGA}\,(N_1, N_2, N_3, \ldots, N_p) = \prod_{s=1}^{p} (N_s)^{\varphi_s}$$

$$= \left\langle \begin{array}{c} \left[\prod_{s=1}^{p} (\alpha_s)^{\varphi_s}, \prod_{s=1}^{p} (\beta_s)^{\varphi_s}, \prod_{s=1}^{p} (\gamma_s)^{\varphi_s}, \prod_{s=1}^{p} (\mu_s)^{\varphi_s}\right], \\ \left[1 - \prod_{s=1}^{p} (1 - \lambda(s)^{\varphi_s}), 1 - \prod_{s=1}^{p} (1 - \mu(s)^{\varphi_s}), 1 - \prod_{s=1}^{p} (1 - \kappa(s)^{\varphi_s}), 1 - \prod_{s=1}^{p} (1 - \iota(s)^{\varphi_s})\right], \\ \left[1 - \prod_{s=1}^{p} (1 - \phi(s)^{\varphi_s}), 1 - \prod_{s=1}^{p} (1 - \rho(s)^{\varphi_s}), 1 - \prod_{s=1}^{p} (1 - \psi(s)^{\varphi_s}), 1 - \prod_{s=1}^{p} (1 - \sigma(s)^{\varphi_s})\right] \end{array} \right\rangle \quad (6)$$

3. THE ENHANCED EDAS METHOD UTILIZING TNNS

This segment illustrates an enhanced MCGDM approach through this integration of the original EDAS method with TNNs. Consider a group of e decision-makers denoted as {DM 1, DM 2, . . ., DM e}, each assigned weights represented by the decision maker matrix v = (v1, v2,

v3, . . ., ve), where vk ∈ [0, 1] and �e vk = 1. Their task is to evaluate p alternatives (U1, U2,

. . ., Up) based on q criteria (C1, C2, . . ., Cq), with the criteria weights captured in the matrix

W = (w1, w2, . . ., wq), satisfying wj ∈ [0, 1] and �q wj = 1. The algorithm for the im-

proved EDAS method can be outlined as follows.

3.1. ALGORITHM

Step A: Create the linguistic evaluation matrix for decision maker DM_k and represent it as M_k, where M_k = [mk]p×q, with i taking values from 1 to p {i = 1, 2, . . . , p} and j from 1 to q {j = 1, 2, . . . , q}. Each entry [mk]p×q in this matrix is associated with trapezoidal neutro- sophic information, specifically (αijk, βijk, γijk, μijk, λijk, μijk, κijk, ιijk, φijk, ρijk, ψijk, σijk). These values capture the trapezoidal neutrosophic information of alternative Ui with respect to criterion C_j, as assessed by decision maker DM_k.

Step B: Standardize these assessed TNN matrix M_k, denoted as M_k = [mk]p×q, into M^k,

represented as [m˜ $_k$]p×q. Here, the entries in Mk, m˜ ijk, are defined as follows: For benefit criterion Cj: m˜ ijk = mijk.

For cost criterion Cj: m˜ ijk = (1 − αijk, 1 − βijk, 1 − γijk, 1 − μijk, 1 − λijk, 1 − μijk, 1 −

κijk, 1 − ιijk, 1 − φijk, 1 − ρijk, 1 − ψijk, 1 − σijk).

This standardization process maintains the original information while adapting it for the specific type of criterion being considered, whether benefit or cost.

Step C: Utilizing the standardized TNN decision-making matrix M^k, represented as [m˜ ^{k}]p×q,

and the weight matrix of decision makers **v** = (v1, v2, v3, . . . , ve), the authors will compute the overall values a_{ij} from m˜ k . The authors can achieve this by applying either equation (7) regarding the TNNWAA operator or equation (8) for the TNNWGA operator. Consequently, these au- thors obtain these aggregated decision-making matrix Ak, represented as [a_{ij}]p×q, in which each

entry aij is defined as a TNN, specifically aij = (αijk, βijk, γijk, μijk, λijk, μijk, κijk, ιijk, φijk, ρijk, ψijk, σijk).

$$a_{ij} = \left\langle \begin{array}{c} \left[1 - \prod_{k=1}^{e}(1-\alpha_{ij})^{\upsilon_k},\ 1 - \prod_{k=1}^{e}(1-\beta_{ij})^{\upsilon_k},\ 1 - \prod_{k=1}^{e}(1-\gamma_{ij})^{\upsilon_k},\ 1 - \prod_{k=1}^{e}(1-\mu_{ij})^{\upsilon_k}\right], \\ \left[\prod_{k=1}^{e}(\lambda_{ij})^{\upsilon_k},\ \prod_{k=1}^{e}(\mu_{ij})^{\upsilon_k},\ \prod_{k=1}^{e}(\kappa_{ij})^{\upsilon_k},\ \prod_{k=1}^{e}(\iota_{ij})^{\upsilon_k}\right], \\ \left[\prod_{k=1}^{e}(\phi_{ij})^{\upsilon_k},\ \prod_{k=1}^{e}(\rho_{ij})^{\upsilon_k},\ \prod_{k=1}^{e}(\psi_{ij})^{\upsilon_k},\ \prod_{k=1}^{e}(\sigma_{ij})^{\upsilon_k}\right] \end{array} \right\rangle \quad (7)$$

$$a_{ij} = \left\langle \begin{array}{c} \left[\prod_{k=1}^{e}(\alpha_{ij})^{\upsilon_k},\ \prod_{k=1}^{e}(\beta_{ij})^{\upsilon_k},\ \prod_{k=1}^{e}(\gamma_{ij})^{\upsilon_k},\ \prod_{k=1}^{e}(\mu_{ij})^{\upsilon_k}\right], \\ \left[1 - \prod_{k=1}^{e}(1-\lambda_{ij})^{\upsilon_k},\ 1 - \prod_{k=1}^{e}(1-\mu_{ij})^{\upsilon_k},\ 1 - \prod_{k=1}^{e}(1-\kappa_{ij})^{\upsilon_k},\ 1 - \prod_{k=1}^{e}(1-\iota_{ij})^{\upsilon_k}\right], \\ \left[1 - \prod_{k=1}^{e}(1-\phi_{ij})^{\upsilon_k},\ 1 - \prod_{k=1}^{e}(1-\rho_{ij})^{\upsilon_k},\ 1 - \prod_{k=1}^{e}(1-\psi_{ij})^{\upsilon_k},\ 1 - \prod_{k=1}^{e}(1-\sigma_{ij})^{\upsilon_k}\right] \end{array} \right\rangle \quad (8)$$

Step D: Evaluate the average solution AV as follows: AV = $[\tilde{a}_{(\upsilon_j)}]_{1\times q}$, where $\tilde{a}_{(\upsilon_j)} = \frac{1}{p}\sum_{i=1}^{p} a_{ij}$ aij is defined as the average of the sum of aij for i ranging from 1 to p, in accordance with Definition 4.

$$\sum_{i=1}^{p} a_{ij} = \left\langle \begin{array}{c} \left[1 - \prod_{k=1}^{e}(1-\alpha_{ij})^{\frac{1}{p}},\ 1 - \prod_{k=1}^{e}(1-\beta_{ij})^{\frac{1}{p}},\ 1 - \prod_{k=1}^{e}(1-\gamma_{ij})^{\frac{1}{p}},\ 1 - \prod_{k=1}^{e}(1-\mu_{ij})^{\frac{1}{p}}\right], \\ \left[\prod_{k=1}^{e}(\lambda_{ij})^{\frac{1}{p}},\ \prod_{k=1}^{e}(\mu_{ij})^{\frac{1}{p}},\ \prod_{k=1}^{e}(\kappa_{ij})^{\frac{1}{p}},\ \prod_{k=1}^{e}(\iota_{ij})^{\frac{1}{p}}\right], \\ \left[\prod_{k=1}^{e}(\phi_{ij})^{\frac{1}{p}},\ \prod_{k=1}^{e}(\rho_{ij})^{\frac{1}{p}},\ \prod_{k=1}^{e}(\psi_{ij})^{\frac{1}{p}},\ \prod_{k=1}^{e}(\sigma_{ij})^{\frac{1}{p}}\right] \end{array} \right\rangle \quad (9)$$

Step E: Compute the positive distance from the average solution, denoted as PDA = [pij]p×q, and the negative distance, denoted as NDA = [nij]p×q, using equation (10) for each, respec- tively

$$p_{ij} = \frac{\max\{0, S(a_{ij}) - S(\tilde{a}_{(\upsilon_j)})\}}{S(\tilde{a}_{(\upsilon_j)})} \quad (10)$$

$$n_{ij} = \frac{\max\{0, S(\tilde{a}_{(\upsilon_j)}) - S(a_{ij})\}}{S(\tilde{a}_{(\upsilon_j)})}$$

Step F: Compute these weighted sums of the positive distance **PDA** along with the negative distance **NDA**, denoted as **WP** = [Spi]px1 and **WN** = [Sni]px1, respectively

$$Sp_i = \sum_{j=1}^{q} w_j p_{ij},$$

$$Sn_i = \sum_{j=1}^{q} w_j n_{ij}.$$

Step G: Standardize the values Spi and Sni to derive WPt = [Spt] px1 and WNt = [Snt] px1 utilizing the normalization technique.

$$Sp_i^t = \frac{Sp_i}{\max_{1\le i\le p}\{Sp_i\}}$$

$$Sn_i^t = \frac{Sn_i}{\max_{1\le i\le p}\{Sn_i\}}$$

Step H: Determine these appraisal score AS = [asi]px1 for every alternative utilizing the formula:

$$as_i = \mu\,(Sp_i^t) + (1-\mu)\,(Sn_i^t) \qquad (13)$$

where decision-makers is able to adjust the measurements of μ based on their evaluation of this positive and negative distances. Specifically, if the decision-makers judgment is balanced, setting μ = 0.5 simplifies equation (13) to:

$$as_i = \frac{1}{2}\left(Sp_i^t + (1 - Sn_i^t)\right) \qquad (14)$$

Step I: Generate a ranked collection of alternatives based on the arranged values of ASi. These alternative with the highest AS_i value is considered the optimal choice.

4. NUMERIC ILLUSTRATION

4.1. CASE STUDY FOR TNNS MCGDM PROBLEM

In this part, the author presents a case analysis related to the allocation of resources to sup- port underprivileged business groups through the implementation of an Productive Economic Endeavours (PEE) Program. The Ministry of Social Affairs of the Republic of Indonesia has initiated this program for provide assistance to micro, small, and medium enterprises (MSMEs) with these dual objectives of elevating the income levels of these business groups and fostering social harmony among the local residents.

The Ministry of Social Affairs seeks to identify and support small and medium joint busi- ness groups that require venture capital assistance. To accomplish this, the Minister of Social Affairs has convened an committee responsible for overseeing these PEE program. The com- mittee comprises three decision makers, denoted as DM1,DM2, and DM3 each possessing dis- tinct assessment skills. The minister has assigned assessment weights to these decision makers, with values of 0.25, 0.25 and 0.5, respectively.

Subsequently, the committee members, in agreement, have identified four key criteria for the selection process: eligibility of the business (C1), quality of the management team (C2), household income levels (C3), along with the quality of the business plan (C4). Additionally, they have assigned weights to these criteria, with values of 0.25, 0.2, 0.4, and 0.15, respectively. Furthermore, they have categorized C1, C2, and C4 as benefit-type criteria, while considering C3 as a cost-type criterion.

They employed a set of linguistic variables represented by S, which includes EL, VL, ML, L, NE, E, MH, VH, and EH, to assess all the alternatives across various criteria using their Trapezoidal Neutrosophic Number (TNN) values for the linguistic categories, as detailed in Table 1.

Table 1. The variable of linguistic under trapezoidal neutrosophic numbers

Linguistic Variable	**TNNs**
Excessively high (EH)	((0.4,0.5,0.6,0.7),(0.0,0.1,0.2,0.3),(0.1,0.1,0.1,0.1))
Very high (VH)	((0.3,0.4,0.5,0.5),(0.1,0.2,0.3,0.4),(0.0,0.1,0.1,0.1))
Midst high (MH)	((0.1,0.1,0.1,0.1),(0.1,0.1,0.1,0.1),(0.6,0.7,0.8,0.9))
Enough (E)	((0.7,0.7,0.7,0.7),(0.0,0.1,0.2,0.3),(0.1,0.1,0.1,0.1))
Not enough (NE)	((0.3,0.4,0.5,0.6),(0.1,0.1,0.1,0.1),(0.1,0.2,0.3,0.4))
Low (L)	((0.3,0.4,0.5,0.5),(0.0,0.1,0.2,0.3),(0.0,0.1,0.1,0.2))
Midst low (ML)	((0.1,0.2,0.3,0.4),(0.1,0.1,0.1,0.1),(0.3,0.4,0.5,0.6))
Very low (VL)	((0.1,0.2,0.3,0.4),(0.1,0.1,0.1,0.1),(0.4,0.5,0.6,0.6))
Excessively low (EL)	((0.1,0.2,0.3,0.3),(0.1,0.2,0.3,0.4),(0.2,0.3,0.4,0.5))

Table 2. The linguistic preferences of Decision Maker DM1 were evaluated.

	C_1	C_2	C_3	C_4
U_1	ML	E	EH	EL
U_2	VH	EL	E	L
U_3	NE	EH	ML	VH
U_4	L	NE	MH	VL
U_5	E	EH	EH	E

Table 3. The linguistic preferences of Decision Maker DM2 were evaluated.

	C_1	C_2	C_3	C_4
U_1	VH	ML	VL	E
U_2	E	NE	ML	EH
U_3	EH	MH	NE	MH
U_4	VL	EL	EL	EH
U_5	EH	ML	VH	ML

Table 4. The linguistic preferences of Decision Maker DM3 were evaluated.

	C_1	C_2	C_3	C_4
U_1	EH	L	E	EH
U_2	EH	MH	ML	L
U_3	EL	VH	E	L
U_4	MH	NE	VH	MH
U_5	NE	ML	MH	EH

Tables 2–4 display evaluations submitted by the three decision makers (DMs) employing Trapezoidal Neutrosophic Numbers (TNNs) for the five alternatives across the four criteria. In order to determine the recipients of venture capital assistance, the authors employ the enhanced EDAS approach evaluates these five collaborative business groups. The technical assessment procedures for these refined EDAS method are described above:

Step A: Following these construction of linguistic value matrices by decision makers, the au- thors convert these matrices into Trapezoidal Neutrosophic Number (TNN) decision-making matrices.

Step B: Normalize these TNN decision matrices according to benefit along with the cost crite- ria, resulting in the normalized TNN matrices as presented in Table 5.

Step C: Combine these normalized TNN matrices. Utilizing the provided decision makers weight matrix, denoted as v (with values {0.25, 0.25, 0.5}), the authors will obtain the aggre- gated TNN matrix, which can be present in Table 6 (utilizing the TNNWAA operator) and

Table 7 (employing the TNNWGA operator).

Table 5. The Decision Maker DMK normalized values for option alternative Ui with that match criterion Cj.

DM	U_i	C_1	C_2	C_3	C_4
DM_1	U_1	((0.1, 0.2, 0.3, 0.4)	((0.7, 0.7, 0.7, 0.7)	((0.6, 0.5, 0.4, 0.3)	((0.1, 0.2, 0.3, 0.3)
		(0.1, 0.1, 0.1, 0.1)	(0.0, 0.1, 0.2, 0.3)	(1, 0.9, 0.8, 0.7)	(0.1, 0.2, 0.3, 0.4)
		(0.3, 0.4, 0.5, 0.6))	(0.1, 0.1, 0.1, 0.1))	(0.9, 0.9, 0.9, 0.9))	(0.2, 0.3, 0.4, 0.5))
	U_2	((0.3, 0.4, 0.5, 0.5)	((0.1, 0.2, 0.3, 0.3)	((0.3, 0.3, 0.3, 0.3)	((0.3, 0.4, 0.5, 0.5)
		(0.1, 0.2, 0.3, 0.4)	(0.1, 0.2, 0.3, 0.4)	(1, 0.9, 0.8, 0.7)	(0.0, 0.1, 0.2, 0.3)
		(0.0, 0.1, 0.1, 0.1))	(0.2, 0.3, 0.4, 0.5))	(0.9, 0.9, 0.9, 0.9))	(0.0, 0.1, 0.1, 0.2))
	U_3	((0.3, 0.4, 0.5, 0.6)	((0.4, 0.5, 0.6, 0.7)	((0.9, 0.8, 0.7, 0.6)	((0.3, 0.4, 0.5, 0.5)
		(0.1, 0.1, 0.1, 0.1)	(0.0, 0.1, 0.2, 0.3)	(0.9, 0.9, 0.9, 0.9)	(0.1, 0.2, 0.3, 0.4)
		(0.1, 0.2, 0.3, 0.4))	(0.1, 0.1, 0.1, 0.1))	(0.7, 0.6, 0.5, 0.4))	(0.0, 0.1, 0.1, 0.1))
	U_4	((0.3, 0.4, 0.5, 0.5)	((0.3, 0.4, 0.5, 0.6)	((0.9, 0.9, 0.9, 0.9)	((0.1, 0.2, 0.3, 0.4)
		(0.0, 0.1, 0.2, 0.3)	(0.1, 0.1, 0.1, 0.1)	(0.9, 0.9, 0.9, 0.9)	(0.1, 0.1, 0.1, 0.1)
		(0.0, 0.1, 0.1, 0.2))	(0.1, 0.2, 0.3, 0.4))	(0.4, 0.3, 0.2, 0.1))	(0.4, 0.5, 0.6, 0.6))
	U_5	((0.7, 0.7, 0.7, 0.7)	((0.4, 0.5, 0.6, 0.7)	((0.6, 0.5, 0.4, 0.3)	((0.7, 0.7, 0.7, 0.7)
		(0.0, 0.1, 0.2, 0.3)	(0.0, 0.1, 0.2, 0.3)	(1, 0.9, 0.8, 0.7)	(0.0, 0.1, 0.2, 0.3)
		(0.1, 0.1, 0.1, 0.1))	(0.1, 0.1, 0.1, 0.1))	(0.9, 0.9, 0.9, 0.9))	(0.1, 0.1, 0.1, 0.1))
DM_2	U_1	((0.3, 0.4, 0.5, 0.5)	((0.1, 0.2, 0.3, 0.4)	((0.9, 0.8, 0.7, 0.6)	((0.7, 0.7, 0.7, 0.7)
		(0.1, 0.2, 0.3, 0.4)	(0.1, 0.1, 0.1, 0.1)	(0.9, 0.9, 0.9, 0.9)	(0.0, 0.1, 0.2, 0.3)
		(0.0, 0.1, 0.1, 0.1))	(0.3, 0.4, 0.5, 0.6))	(0.6, 0.5, 0.4, 0.4))	(0.1, 0.1, 0.1, 0.1))
	U_2	((0.7, 0.7, 0.7, 0.7)	((0.3, 0.4, 0.5, 0.6)	((0.9, 0.8, 0.7, 0.6)	((0.4, 0.5, 0.6, 0.7)
		(0.0, 0.1, 0.2, 0.3)	(0.1, 0.1, 0.1, 0.1)	(0.9, 0.9, 0.9, 0.9)	(0.0, 0.1, 0.2, 0.3)
		(0.1, 0.1, 0.1, 0.1))	(0.1, 0.2, 0.3, 0.4))	(0.7, 0.6, 0.5, 0.4))	(0.1, 0.1, 0.1, 0.1))
	U_3	((0.3, 0.4, 0.5, 0.6)	((0.4, 0.5, 0.6, 0.7)	((0.9, 0.8, 0.7, 0.6)	((0.3, 0.4, 0.5, 0.5)
		(0.1, 0.1, 0.1, 0.1)	(0.0, 0.1, 0.2, 0.3)	(0.9, 0.9, 0.9, 0.9)	(0.1, 0.2, 0.3, 0.4)
		(0.1, 0.2, 0.3, 0.4))	(0.1, 0.1, 0.1, 0.1))	(0.7, 0.6, 0.5, 0.4))	(0.0, 0.1, 0.1, 0.1))
	U_4	((0.3, 0.4, 0.5, 0.5)	((0.3, 0.4, 0.5, 0.6)	((0.9, 0.9, 0.9, 0.9)	((0.1, 0.2, 0.3, 0.4)
		(0.0, 0.1, 0.2, 0.3)	(0.1, 0.1, 0.1, 0.1)	(0.9, 0.9, 0.9, 0.9)	(0.1, 0.1, 0.1, 0.1)
		(0.0, 0.1, 0.1, 0.2))	(0.1, 0.2, 0.3, 0.4))	(0.4, 0.3, 0.2, 0.1))	(0.4, 0.5, 0.6, 0.6))
	U_5	((0.7, 0.7, 0.7, 0.7)	((0.4, 0.5, 0.6, 0.7)	((0.6, 0.5, 0.4, 0.3)	((0.7, 0.7, 0.7, 0.7)
		(0.0, 0.1, 0.2, 0.3)	(0.0, 0.1, 0.2, 0.3)	(1, 0.9, 0.8, 0.7)	(0.0, 0.1, 0.2, 0.3)
		(0.1, 0.1, 0.1, 0.1))	(0.1, 0.1, 0.1, 0.1))	(0.9, 0.9, 0.9, 0.9))	(0.1, 0.1, 0.1, 0.1))
DM_3	U_1	((0.4, 0.5, 0.6, 0.7)	((0.1, 0.2, 0.3, 0.4)	((0.9, 0.8, 0.7, 0.6)	((0.7, 0.7, 0.7, 0.7)

		(0.1, 0.2, 0.3, 0.4)	(0.1, 0.1, 0.1, 0.1)	(0.9, 0.9, 0.9, 0.9)	(0.0, 0.1, 0.2, 0.3)
		(0.0, 0.1, 0.1, 0.1))	(0.3, 0.4, 0.5, 0.6))	(0.6, 0.5, 0.4, 0.4))	(0.1, 0.1, 0.1, 0.1))
	U_2	((0.7, 0.7, 0.7, 0.7)	((0.3, 0.4, 0.5, 0.6)	((0.9, 0.8, 0.7, 0.6)	((0.4, 0.5, 0.6, 0.7)
		(0.0, 0.1, 0.2, 0.3)	(0.1, 0.1, 0.1, 0.1)	(0.9, 0.9, 0.9, 0.9)	(0.0, 0.1, 0.2, 0.3)
		(0.1, 0.1, 0.1, 0.1))	(0.1, 0.2, 0.3, 0.4))	(0.7, 0.6, 0.5, 0.4))	(0.1, 0.1, 0.1, 0.1))
	U_3	((0.3, 0.4, 0.5, 0.6)	((0.4, 0.5, 0.6, 0.7)	((0.9, 0.8, 0.7, 0.6)	((0.3, 0.4, 0.5, 0.5)
		(0.1, 0.1, 0.1, 0.1)	(0.0, 0.1, 0.2, 0.3)	(0.9, 0.9, 0.9, 0.9)	(0.1, 0.2, 0.3, 0.4)
		(0.1, 0.2, 0.3, 0.4))	(0.1, 0.1, 0.1, 0.1))	(0.7, 0.6, 0.5, 0.4))	(0.0, 0.1, 0.1, 0.1))
	U_4	((0.3, 0.4, 0.5, 0.5)	((0.3, 0.4, 0.5, 0.6)	((0.9, 0.9, 0.9, 0.9)	((0.1, 0.2, 0.3, 0.4)
		(0.0, 0.1, 0.2, 0.3)	(0.1, 0.1, 0.1, 0.1)	(0.9, 0.9, 0.9, 0.9)	(0.1, 0.1, 0.1, 0.1)
		(0.0, 0.1, 0.1, 0.2))	(0.1, 0.2, 0.3, 0.4))	(0.4, 0.3, 0.2, 0.1))	(0.4, 0.5, 0.6, 0.6))
	U_5	((0.7, 0.7, 0.7, 0.7)	((0.4, 0.5, 0.6, 0.7)	((0.6, 0.5, 0.4, 0.3)	((0.7, 0.7, 0.7, 0.7)
		(0.0, 0.1, 0.2, 0.3)	(0.0, 0.1, 0.2, 0.3)	(1, 0.9, 0.8, 0.7)	(0.0, 0.1, 0.2, 0.3)
		(0.1, 0.1, 0.1, 0.1))	(0.1, 0.1, 0.1, 0.1))	(0.9, 0.9, 0.9, 0.9))	(0.1, 0.1, 0.1, 0.1))

Table 6. The TNNWAA operator generated the aggregated TNN matrix.

	TNN Matrix
U_1	
C_1	((0.3101, 0.5948, 0.5137, 0.5948), (0, 0.2448, 0.1860, 0.2440), (0, 0.0999, 0.1494, 0.1564))
C_2	((0.3969, 0.4579, 0.5214, 0.5395), (0, 0.1, 0.1189, 0.2279), (0, 0.1414, 0.1495, 0.2213))
C_3	((0.6259, 0.5296, 0.4551, 0.3914), (0.9740, 0.9, 0.8239, 0.7453), (0.8132, 0.7770, 0.7348, 0.7348))
C_4	((0.5441, 0.5051, 0.5719, 0.6293), (0, 0.1189, 0.2213, 0.3223), (0.1189, 0.1316, 0.1414, 0.1495))
U_2	
C_1	((0.4757, 0.5395, 0.6065, 0.6592), (0, 0.1189, 0.2213, 0.3223), (0, 0.1, 0.1, 0.1))
C_2	((0.1549, 0.2104, 0.2704, 0.31), (0.1, 0.1189, 0.1316, 0.1414), (0.2912, 0.4140, 0.5264, 0.64))
C_3	((0.8374, 0.7265, 0.8985, 0.54), (0.9240, 0.9, 0.8738, 0.8451), (0.7453, 0.6640, 0.5791, 0.4898))
C_4	((0.3265, 0.4268, 0.5272, 0.56), (0, 0.1, 0.2, 0.3), (0, 0.1, 0.1, 0.1681))
U_3	
C_1	((0.2363, 0.3381, 0.4405, 0.5076), (0, 0.1414, 0.2059, 0.2632), (0.1414, 0.2059, 0.2632, 0.3162))
C_2	((0.2828, 0.3656, 0.4523, 0.4903), (0, 0.1414, 0.2059, 0.2632), (0, 0.1626, 0.5318, 0.1732))
C_3	((0.6518, 0.5551, 0.4794, 0.4144), (0.9486, 0.9, 0.8485, 0.7937), (0.8451, 0.7896, 0.7296, 0.6640))
C_4	((0.2547, 0.336, 0.4209, 0.4209), (0, 0.1189, 0.1861, 0.2449), (0, 0.1626, 0.1681, 0.2449))
U_4	
C_1	((0.1549, 0.2104, 0.2704, 0.2979), (0, 0.1, 0.1189, 0.1316), (0, 0.3956, 0.4426, 0.5583))
C_2	((0.2547, 0.3553, 0.4562, 0.54), (0.1, 0.1189, 0.1316, 0.1414), (0.1189, 0.2213, 0.3223, 0.4229))
C_3	((0.8268, 0.7622, 0.7058, 0.7058), (0.9, 0.8239, 0.7453, 0.6640), (0.7521, 0.6422, 0.5583, 0.4486))
C_4	((0.729, 0.2456, 0.31, 0.3821), (0, 0.1, 0.1189, 0.1316), (0.3464, 0.3956, 0.4426, 0.4695))
U_5	
C_1	((0.4551, 0.518, 0.5839, 0.6536), (0, 0.1, 0.1414, 0.1732), (0.1, 0.1414, 0.1732, 0.2))
C_2	((0.1868, 0.2887, 0.3914, 0.5947), (0, 0.1, 0.1189, 0.1316), (0.2279, 0.2828, 0.3343, 0.33))
C_3	((0.8139, 0.7886, 0.766, 0.7568), (0.9240, 0.8738, 0.8206, 0.7637), (0.6160, 0.5196, 0.4142, 0.9486))
C_4	((0.4417, 0.5051, 0.5719, 0.4551), (0, 0.1, 0.1681, 0.2279), (0.1316, 0.1414, 0.1495, 0.1565))

Table 7. The TNNWGA operator generated the aggregated TNN matrix.

	TNN Matrix
U_1	
C_1	((0.2632, 0.3760, 0.4820, 0.5595), (0.0514, 0.1262, 0.2032, 0.2828), (0.1323, 0.1, 0.223, 0.2652))
C_2	((0.2817, 0.3868, 0.4786, 0.5143), (0.026, 0.1, 0.1761, 0.2547), (0.1091, 0.1868, 0.223, 0.3072))
C_3	((0.4695, 0.4355, 0.3984, 0.3567), (1, 0.9, 0.8319, 0.7721), (0.8586, 0.8505, 0.8435, 0.8435))
C_4	((0.3253, 0.4325, 0.5243, 0.5663), (0.026, 0.1262, 0.2263, 0.3265), (0.1262, 0.1549, 0.1868, 0.2223))
U_2	
C_1	((0.4281, 0.5143, 0.5957, 0.6435), (0.026, 0.1262, 0.2263, 0.3265), (0.076, 0.1, 0.1, 0.1))
C_2	((0.1316, 0.1681, 0.1967, 0.2059), (0.1, 0.1262, 0.1549, 0.6143), (0.4175, 0.5262, 0.64, 0.766))
C_3	((0.6838, 0.6260, 0.5663, 0.5045), (1, 0.9, 0.8811, 0.8684), (0.7721, 0.7172, 0.6657, 0.6167))
C_4	((0.3223, 0.4229, 0.5233, 0.5438), (0, 0.1, 0.2, 0.3), (0.026, 0.1, 0.1, 0.1761))
U_3	
C_1	((0.1861, 0.2990, 0.4053, 0.4409), (0.076, 0.1515, 0.2294, 0.31), (0.1515, 0.2294, 0.31, 0.3939))
C_2	((0.0670, 0.0945, 0.1237, 0.1285), (0.769, 0.1515, 0.2063, 0.2652), (0.2255, 0.3162, 0.3821, 0.4804))
C_3	((0.4879, 0.4551, 0.4212, 0.3833), (1, 0.9, 0.8586, 0.8268), (0.8684, 0.8319, 0.8033, 0.7787))
C_4	((0.2279, 0.1414, 0.3343, 0.3343), (0,.0514, 0.1262, 0.2032, 0.2828), (0.2048, 0.3162, 0.3821, 0.5102))
U_4	
C_1	((0.4161, 0.5318, 0.6223, 0.6687), (0.076, 0.1, 0.1262, 0.1549), (0.4434, 0.5514, 0.6536, 0.7622))
C_2	((0.2279, 0.3363, 0.4400, 0.5045), (0.1, 0.1262, 0.1549, 0.1868), (0.1262, 0.2263, 0.3265, 0.4268))
C_3	((0.7937, 0.7135, 0.6299, 0.6299), (0.9, 0.8319, 0.7721, 0.7172), (1, 0.786, 0.7622, 0.741))
C_4	((0.14140.1778, 0.2059, 0.2300), (0.076, 0.1, 0.1262, 0.1549), (0.4579, 0.5514, 0.6536, 0.7551))
U_5	
C_1	((0.3984, 0.4864, 0.5692, 0.6480), (0.0514, 0.1, 0.1515, 0.2063), (0.1, 0.1515, 0.2063, 0.2652))
C_2	((0.1414, 0.2514, 0.3567, 0.4600), (0.076, 0.1, 0.1262, 0.1549), (0.2547, 0.336, 0.4209, 0.5102))
C_3	((0.7637, 0.7021, 0.6344, 0.5903), (1, 0.8811, 0.8435, 0.8139), (1, 0.7355, 0.7172, 0.7))
C_4	((0.3253, 0.4325, 0.5243, 0.6086), (0.026, 0.1, 0.1761, 0.2547), (0.1549, 0.1868, 0.223, 0.2652))

Step D: Compute these average solution values utilizing equation (9) along with the acquire the average solution matrix as indicated in Table 8.

Table 8. The Average Solution Matrix AV

	$(\tilde{a}_{v_j})$
C_1	((0.338, 0.4575, 0.4961, 0.5598),
	(0, 0.1327, 0.1701, 0.2161),
	(0, 0.1627, 0.1976, 0.2230))
C_2	((0.2603, 0.3408, 0.4242, 0.503),
	(0, 0.1148, 0.1381, 0.1736),
	(0, 0.2264, 0.3394, 0.3304))
C_3	((0.7674, 0.6897, 0.712, 0.589),
	(0.9337, 0.8790, 0.8212, 0.7599),
	(0.7499, 0.6621, 0.5903, 0.6330))
C_4	((0.4897, 0.412, 0.4897, 0.4983),
	(0, 0.1011, 0.1752, 0.2345),
	(0, 0.1642, 0.1735, 0.2745))

Step E: Compute every elements of the PDA and NDA matrices by using the average solution matrix, the aggregated TNN matrix obtained from the TNNWAA operator, and these score function values of the option presented in in Table 9. The results of these calculations are displayed in Table 10.

Table 9. The values of the score function for aij and a˜(vj).

	C_1	C_2	C_3	C_4
U_1	0.71	0.75	0.29	0.75
U_2	0.76	0.55	0.42	0.74
U_3	0.67	0.68	0.30	0.69
U_4	0.60	0.67	0.46	0.64
U_5	0.77	0.66	0.44	0.74
$\tilde{S}(a_{(v_j)})$	0.73	0.69	0.39	0.75

Table 10. The PDA and NDA matrix.

	C_1	C_2	C_3	C_4
U_1	0	0.0869	0	0
U_2	0.0410	0	0.0769	0
U_3	0	0	0	0
U_4	0	0	0.1794	0
U_5	0.0547	0	0.1282	0
	C_1	C_2	C_3	C_4
U_1	0.0273	0	0.2564	0
U_2	0	0.2028	0	0.0133
U_3	0.0821	0.0144	0.2307	0.08
U_4	0.1780	0.0289	0	0.1466
U_5	0	0.0434	0	0.0133

Step F: Calculate there weighted matrix for PDA by multiplying the PDA matrix with the criteria weight matrix W. Likewise, perform the same operation for the weighted matrix of NDA. Using the criteria weight vector W ={0.25, 0.2, 0.4, 0.15} and equation (11), the authors derive the results.

$$WP = \begin{bmatrix} sp_1 = 0.0174 \\ sp_2 = 0.0410 \\ sp_3 = 0 \\ sp_4 = 0.0718 \\ sp_5 = 0.06409 \end{bmatrix}$$

$$WN = \begin{bmatrix} sn_1 = 0.1094 \\ sn_2 = 0.0426 \\ sn_3 = 0.1277 \\ sn_4 = 0.0723 \end{bmatrix}$$

Step G: After applying equation (12) to normalize the values of $sp^t{}_i$ and $sn^t{}_i$, the authors subsequently acquire these values of $sp^t{}_i$ and $sn^t{}_i$.

$$WP^t = \begin{Bmatrix} sp^t_1 = 0.2424 \\ sp^t_2 = 0.5710 \\ sp^t_3 = 0 \\ sp^t_4 = 1 \\ sp^t_5 = 0.8914 \end{Bmatrix}$$

$$WN^t = \begin{Bmatrix} sn^t_1 = 0.8567 \\ sn^t_2 = 0.3336 \\ sn^t_3 = 1 \\ sn^t_4 = 0.5662 \\ sn^t_5 = 0.0838 \end{Bmatrix}$$

Step H: When the value of μ is set to 0.5 as per equation (13), you can determine the appraisal score (ASi) for i = 1, 2, 3, 4, 5, as outlined below. This ASi value represents the ultimate evaluation for the alternative U_A:

$$AS = \begin{Bmatrix} as_1 = 0.1929 \\ as_2 = 0.6187 \\ as_3 = 0 \\ as_4 = 0.7169 \\ as_5 = 0.9038 \end{Bmatrix}$$

Step I: Based on the AS matrix values, these alternatives are ranked in the following order: U5 > U2 > U4 > U1 > U3. It is evident that U5 is the most favourable alternative, while U3 is the least preferable choice.

During Step H of the enhanced EDAS algorithm, decision-makers have the flexibility to tailor the value of μ based on their distinct preferences regarding both positive and negative distances. To assess these impact of varying preference values based on the ranking outcomes, the authors assign different values of μ used to evaluate the results regarding the alternatives rankings, as detailed in Table 11.

Table 11. The ranking outcomes obtained using various parameter values for μ in the enhanced EDAS approach.

μ	The best alternative	The worst alternative	Ranking
0.1	U_5	U_3	U5 > U2 > U4 > U1 > U3
0.2	U_5	U_3	U5 > U2 > U4 > U1 > U3
0.3	U_5	U_3	U5 > U2 > U4 > U1 > U3
0.4	U_5	U_3	U5 > U2 > U4 > U1 > U3
0.5	U_5	U_3	U5 > U2 > U4 > U1 > U3
0.6	U_5	U_3	U5 > U2 > U4 > U1 > U3
0.7	U_5	U_3	U5 > U2 > U4 > U1 > U3
0.8	U_5	U_3	U4 > U5 > U2 > U1 > U3

0.9	U_5	U_3	$U4 > U5 > U2 > U1 > U3$

Table 11 demonstrates the consistency of alternative rankings for values of μ less than 0.8 and equal to or greater than 0.8. Conversely, there is a slight variance in rankings between μ values below 0.8 and those at or above 0.8. Specifically, when μ is less than 0.8, U5 is the top-rated alternative, and U3 is considered the least favourable. On the other hand, when μ is greater than or equal to 0.8, U4 is the preferred choice, while U3 remains the least desirable option. The selection of the best alternative, either U5 or U4, hinges on the μ value. In contrast, U3 consistently ranks as the least favourable choice across all μ values. As a result, the findings remain consistent despite variations in the μ value.

When the authors contrast the enhanced EDAS method with the original EDAS, which fixed the μ value at 0.5, the improved EDAS method empowers decision-makers to flexibly adapt the μ parameter within a range of 0 to 1. This flexibility allows for the incorporation of DMs preferences regarding trade-offs between losses and gains when determining the average ideal solution. Decision-makers have the liberty to fine-tune the μ value based on their judgment, enabling them to derive a satisfactory solution that aligns with the specifics of the actual decision-making problem.

4.2. COMPARATIVE ANALYSIS

Comparing Results with TNNWAA and TNNWGA Operators. In this section, our aim is to contrast the outcomes obtained through the use of the TNNWAA and TNNWGA operators, either in terms of ranking order or final score values. The authors will examine and analyse the performance of the enhanced EDAS method when employing the TNNWAA and TNNWGA operators. In the preceding section, the authors employed the TNNWAA operator to combine these assessed linguistic values of DMs. With this part, our objective is to recombine these values using a different operator, namely the TNNWGA operator. Since Steps A and B involve the transformation and normalization of linguistic information into TNNs, the authors will ignore these steps and proceed directly with the choice method from Steps 3 to 9. As in the prior part, the authors able to obtain this evaluation scores and a collection of alternative rankings utilizing the TNNWGA operator, as exemplified in Table 12.

Table 12 reveals that their appraisal scores obtained using the TNNWAA and TNNWGA op- erators exhibit only marginal differences, whereas there is a slight disparity in the list of ranked choices. When employing the TNNWAA operator, the alternatives U4 and U1 are positioned as the third and fourth, accordingly. In contrast, the EDAS employing the TNNWGA operator

Table 12. The Alternatives Ranking and those A_{Si} Values

Method	as_1	as_2	as_3	as_4	as_5	Ranking
EDAS (with TNNWA)	0.192 9	0.618 7	0	0.716 9	0.903 8	$U_5 > U_2 > U_4 > U_1 > U_3$
EDAS (with TNNWG)	0.549 7	0.514 7	0	0.472 7	0.774 5	$U_5 > U_1 > U_2 > U_4 > U_3$

results in a reversal of positions for these two alternatives. The findings indicate that the list of ranked choices generated by the TNNWAA and TNNWGA operators aligns with their results obtained through their enhanced EDAS when utilizing the TNNWGA operator. This suggests that the improved EDAS is suitable, efficient, along with viable to addressing these MCGDM problem. So, the enhanced EDAS method is reliable and effective for addressing the MCGDM issue.

5. CONCLUSION

The notion of TNNs proves to be a valuable tool for addressing situations characterized by vagueness, inconsistency, incompleteness, and uncertainty-common challenges on differ- ent decision-making scenarios. In the article, the authors extend the improved EDAS method to create an improved EDAS specifically designed for MCGDM problems involving TNNs. This approach involves computing the mean alternative by aggregating TNNs for every cri- terion, employing both these TNNWAA and TNNWGA operators. Subsequently, it assesses the positive and negative metrics between every judged option along with these norm solution. By altering the values of the parameter μ based on the decision makers preferences, the en- hanced EDAS method evaluates options by determining their appraisal scores and establishing their rankings. The authors use a case study for find the best MSME for distributing the PEE program. This case study serves as a practical demonstration that the proposed EDAS method consistently produces ranking results for alternatives across various parameter values. It further establishes the method as a rational and practical solution for addressing MCGDM problems with TNNs, as it is compared against other aggregation TNN methods.

In the future, we can expand the improved EDAS method using trapezoidal neutrosophic numbers by applying it to more complex group decision-making scenarios, like team projects and public policy discussions. We could integrate tools that allow participants to express their opinions more clearly, helping to address uncertainties and conflicting preferences. Addition- ally, using technology such as data analytics and machine learning could help streamline the decision-making process and improve accuracy. Exploring its applications in various fields, such as healthcare or finance, will also demonstrate its versatility. Overall, making this method more accessible and practical will empower groups to make better-informed decisions together.

CONFLICT OF INTERESTS

None.

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