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$\mathcal{G}\hat{G}$-CLOSED SETS IN GRILL GENERALIZED TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce $\mathcal{G}\hat{g}$-closed sets in grill generalized topological spaces. Also, we investigate the properties of the μ-$\mathcal{G}\hat{g}$-closed sets and discuss some characterization of μ-$\mathcal{G}$-regular and μ-$\mathcal{G}$-normal spaces.

Keywords: Grill, topology Φμ, μ-gs-closed, μ-$\mathcal{G}\hat{g}$-closed.

Mathematical Subject Classification: 54C05, 54C08, 54D10.

1. INTRODUCTION

In 1947, Choquet ([5]) introduced the concept of grill on a topological space. In ([5], [23], [32]) the concept ofgrill is a powerful tool in nets and filters, deeply study of a topological notion such as proximity spaces, closurespaces, theory of compactifications and extension problems of different kinds. In 2007, Roy and Mukherjee ([22])was defined and studied a typical topology induced by grill. In 1970, Levine ([13]) introduced generalized closedsets in topological space. In 2012, Shyamapada Modak and Sukalyan Mistry ([30]) introduced the generalized closed sets in grill generalized topological space. In 2003, M. K. R. S. Veerakumar ([33]) introduced ĝ-closed sets in topological spaces. In 1998, Arockiarani ([1], [2]) studies on generalizations of generalized closed sets and maps in topology and explore the concept of semi closed sets to define a new class of generalized semi closed sets via Grill. In this paper we define μ-$\mathcal{G}\hat{g}$-closed in grill topological spaces and discuss the characterization ofμ-$\mathcal{G}$-regular and μ-$\mathcal{G}$-normal spaces.

Throughout this paper, X or (X, μ, $\mathcal{G}$) represent a grill topological space with no separation axioms assumed unless explicitly stated. For a subset A of a space X, the closure of A and the interior of A are denoted by $C_\mu(A)$ and $i_\mu(A)$, respectively.

2. PRELIMINARIES

Definition 2.1[5, 22]. A nonempty collection $\mathcal{G}$ of nonempty subsets of a topological space (X, τ) is called grill if (i) $A \in \mathcal{G}$ and $A \subseteq B \subseteq X \Longrightarrow B \in X$, and (ii) $A, B \subseteq X$ and $A \cup B \in \mathcal{G} \Longrightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$. If $\mathcal{G}$ is grill on X, then $(X, \tau, \mathcal{G})$ is called a grill topological space.

Definition 2.2 [22]. Let $(X, \tau, \mathcal{G})$ be a grill topological space. An operator Φ: $\exp(X) \rightarrow \exp(X)$ is called a local function with respect to μ and $\mathcal{G}$ is defined as follows: for $A \subseteq X$, $\Phi(A)\ (\mathcal{G}, \tau) = \Phi(A) = \{x \in X: U \cap A \in \mathcal{G}$ for every $U \in \tau\ (x)\}$ where $\tau(x) = \{U \in \tau: x \in U\}$. It is well known from [22], $A \cap \Phi(A) = \psi(A)$ is a Kuratowsi closure operator.

Definition 2.3[22]. Corresponding to a grill on a topological space (X, τ), there exists a unique topology $\tau_{\mathcal{G}}$ on X given by $\tau_{\mathcal{G}} = \{U \subseteq X: \psi(X - A) = (X - A)\}$, where for any $A \subseteq X$, $\psi(A) = A \cup \Phi(A) = \tau_{\mathcal{G}}\text{-}cl(A)$.

Definition 2.4. [9] Let $(X, \tau, \mathcal{G})$ be a grill topological space. A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is $\tau_{\mathcal{G}}$-closed (resp. $\tau_{\mathcal{G}}$-dense in itself $\tau_{\mathcal{G}}$-perfect), if (A) = A or equivalently if $\Phi(A) \subseteq A$ (resp. $A \subseteq \Phi(A)$, $A = \Phi(A)$)

Definition 2.5. [9] Let $(X, \tau, \mathcal{G})$ be a grill topological space. A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is g-closed with respect to the grill $\mathcal{G}$ (briefly, $\mathcal{G}$-g-closed) if $\Phi(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
A subset A of X is said to be $\mathcal{G}$-g-open if $X - A$ is $\mathcal{G}$-g-closed.

Definition 2.6. [13] Let (X, τ) be a topological space. A subset A of a space (X, τ) is said to be g-closed set if $(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Remark 2.1[9]. Every g-closed set is a $\mathcal{G}$-g-closed but not vice versa.

Remark 2.2[31]. Every closed set is g - closed.

Remark 2.3[33]. Every closed set is $\hat{g}$ - closed.

Very interesting notion in literature has been introduced by Csaszar [6] on 1997. Using this notion topology has been reconstructed. The concept is: A map τ: $\exp(X) \rightarrow \exp(X)$ is possessing the property monotony (i.e., such that $A \subseteq B$ implies $\tau(A) \subseteq \tau(B)$). We denote by $\Gamma(X)$ the collections of all mapping having this property.
One of the consequences of the above notion is generalized topological space (GTS) [7,8], its formal definition is:

Definition 2.7. [7, 8, 19, 20] Let X be a non-empty set, and $\mu \subseteq \exp(X)$, μ is called a generalized topological space (GTS) on X if $\phi \in \mu$ and the union of elements of μ belongs to μ. The member of μ is called μ -open set and the complement of μ -open set is called μ -closed set. Again, cμ is notation of μ-closure.

Definition 2.8[21]. Let (X, μ) be a generalized topological space. Then the generalized kernel of $A \subseteq X$ is denoted by g-ker(A) and defined as $g\text{-ker}(A) = \cap\{\mathcal{G} \in \mu: A \subseteq \mathcal{G}\}$.

Lemma 2.1 [21]. Let (X, μ) be a generalized topological space and $A \subseteq X$. Then $g\text{-ker}(A) = \{x \in X: c_{\mu}(\{x\}) \cap A \neq \phi\}$. If $\mathcal{G}$ is a grill on X, then $(X, \mu, \mathcal{G})$ is called a grill generalized topological space.

3. GRILL GENERALIZED TOPOLOGICAL SPACES (GGTS)

Definition 3.1. [7] Let $(X, \mu, \mathcal{G})$ be a grill generalized topological space. A mapping $()^{\Phi\mu}$: $\exp(X) \rightarrow \exp(X)$ is defined as follows: $(A)^{\Phi_\mu} = (A)^{\Phi_\mu}(\mathcal{G}, \mu) = \{x \in X: A \cap U \in \mathcal{G}\}$, where $U \in \psi(x)$.
The mapping is called the local function associated with the grill $\mathcal{G}$ and generalized topology μ.

Theorem 3.1. [7]. Let $(X, \mu, \mathcal{G})$ be a Grill Generalized Topological Spaces. Then

(i) $(\phi)^{\Phi_\mu} = \phi$.

(ii) for $A, B \subseteq X$ and $A \subseteq B$, $(A)^{\Phi_\mu} \subseteq (B)^{\Phi_\mu}$.

(iii) $(A)^{\Phi_\mu} \subseteq c_\mu(A)$.

(iv) $((A)^{\Phi_\mu})^{\Phi_\mu}) \subseteq c_\mu(A)$.

(v) $(A)^{\Phi_\mu}$ is a μ-closed set.

(vi) $((A)^{\Phi_\mu})^{\Phi_\mu}) \subseteq (A)^{\Phi_\mu}$.

(vii) for $\mathcal{G} \subseteq \mathcal{G}1$ implies $(A)^{\Phi_\mu}(\mathcal{G}1) \supseteq (A)^{\Phi_\mu}(\mathcal{G})$.
(viii) for $U \in \mu$, $U \cap (U \cap A)^{\Phi_\mu} \subseteq U \cap (A)^{\Phi_\mu}$.
(ix) for $G \notin \mathcal{G}$, $(A - G)^{\Phi_\mu} = (A)^{\Phi_\mu} = (A \cup G)^{\Phi_\mu}$.

Proof: Obvious

Definition 3.2. Let (X, μ) be a GTS with a grill $\mathcal{G}$ on X. The set operator c^{Φ_μ} is called a generalized ϕ_μ-closure and is defined as $c^{\Phi_\mu}(A) = A \cup A^{\Phi_\mu}$, for $A \subseteq X$. We will denote by $\mu^\Phi(\mu; \mathcal{G})$ the generalized structure, generatedby c^{Φ_μ}, that is, $\mu^\Phi(\mu; \mathcal{G}) = \{U \subseteq X: c^{\Phi_\mu}(X - U) = (X - U)\}$. $\mu^\Phi(\mu; \mathcal{G})$ is called Φ_μ-generalized structure with respect to μ and $\mathcal{G}$ (in short ϕ_μ-generalized structure) which is finer than μ. The element of $\mu^\Phi(\mu; \mathcal{G})$ are called μ^Φ-open and the complement of μ^Φ-open is called μ^Φ-closed.

Theorem 3.2. [7]. The set operator c^{Φ_μ} satisfy following conditions:
(i) $A \subseteq c^{\Phi_\mu}(A)$, for $A \subseteq X$.

(ii) $c^{\Phi_\mu}(\phi) = \phi$ and $c^{\Phi_\mu}(X) = X$.
(iii) $c^{\Phi_\mu}(A) \subseteq c^{\Phi_\mu}(B)$ if $A \subseteq B \subseteq X$.
(iv) $c^{\Phi_\mu}(A) \cup c^{\Phi_\mu}(B) \subseteq c^{\Phi_\mu}(A \cup B)$.
(v) $c^{\Phi_\mu} \in \Gamma(X)$.

Proof: Proof is obvious from Theorem 3.1.

Definition 3.3. Let (X, μ) be a Generalized Topological Space. A subset A of X is said to be $g\mu$-closed set [14] if $c_\mu(A) \subseteq M$ whenever $A \subseteq M$ and $M \in \mu$.

Definition 3.4. A subset A of a GGTS $(X, \mu, \mathcal{G})$ is μ^Φ-dense in itself (resp. μ^Φ-perfect) if $A \subseteq (A)^{\Phi_\mu}$ (resp. $(A)^{\Phi_\mu} = A$).

Definition 3.5. [30] A subset A of a GGTS $(X, \mu, \mathcal{G})$ is called μ-$\mathcal{G}$-generalized closed (briefly, μ-$\mathcal{G}g$-closed) if $(A)^{\Phi_\mu} \subseteq U$ whenever U is μ-open and $A \subseteq U$. A subset A of a GGTS $(X, \mu, \mathcal{G})$ is called μ-$\mathcal{G}$-generalized open (briefly, μ- $\mathcal{G}g$-open) if X- A is μ- $\mathcal{G}g$-closed.

Definition 3.6 [13]. Let $(X, \mu, \mathcal{G})$ be a grill topological space on X. Then for an y A, B ⊆ X the following statements hold:
(i) $(A \cup B)^{\Phi_\mu} = A^{\Phi_\mu} \cup B^{\Phi_\mu}$
(ii) $((A)^{\Phi_\mu})^{\Phi_\mu} \subseteq A^{\Phi_\mu} = c^{\Phi_\mu}(A) \subseteq c_\mu(A)$, and hence A^{Φ_μ} is closed in (X, μ), for all $A \subseteq X$.
(iii) $A \subseteq B \Rightarrow A^{\Phi_\mu} \subseteq B^{\Phi_\mu}$.

Definition 3.7[10] A subset A of a topological space X is said to be μ-θ-closed if $A = \mu\text{-}\theta C_\mu(A)$where $\mu\text{-}\theta C_\mu(A)$ is defined as $\mu\text{-}\theta C\mu(A) = \{x \in X - c_\mu(U) \cap A \neq \emptyset\}$for every $U \in \mu$ and $x \in U$.

Definition 3.8 [10] A subset A of X is said to be μ-θ-open if X - A is μ-θ-closed.

Definition 3.9. [10] A subset A of a topological space X is said to be μ-δ-closed if $A = \mu\text{-}\delta C_\mu(A)$ where $\mu\text{-}\delta C_\mu(A)$ is defined as $\mu\text{-}\delta C\mu(A) = \{x \in X / i_\mu C_\mu(U) \cap A \neq \emptyset\}$for every $U \in \mu$ and $x \in U$.

Definition 3.10[10]A subset A of X is said to be μ-δ-open if X - A is μ-δ-closed.

Definition 3.11. [10]A subset A of a topological space X is said to be μ-θg-closed if $\mu\text{-}\theta C_\mu(A) \subseteq U$ wheneverA ⊆ U and

U is μ-open.

Definition 3.12. [10]A subset A of a topological space X is said to be μ-gθ-closed if $C_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-θ-open.

Definition 3.13. [10]A subset A of a topological space X is said to be μ-gθ-open(μ-θg-open) if X – A is μ-gθ-closed(μ-θg-closed)

4. GENERALIZED M-SEMICLOSED SETS WITH RESPECT TO A GRILL

Definition 4.1. A subset A of a topological space X is said to be μ-gs-closed if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-semi open.

Definition 4.2. Let $(X, \mu, \mathcal{G})$ be a grill topological space. Then a subset A of X is said to be μ-$\mathcal{G}\hat{g}$-closed if $A^{\Phi_\mu} \subseteq U$ whenever $A \subseteq U$ and U is μ-semi open in X.

Definition 4.3. A subset A of X is said to be μ-$\mathcal{G}\hat{g}$-open if X- A is μ-$\mathcal{G}\hat{g}$-closed.

Proposition 4.1. For a grill topological space $(X, \mu, \mathcal{G})$ on X

(i) Every μ-closed set in X is μ-$\mathcal{G}\hat{g}$-closed.
(ii) For any subset A in X, A^{Φ_μ} is μ-G$\hat{g}$-closed.
(iii) Every Φμ-closed set is μ-$\mathcal{G}\hat{g}$-closed.
(iv) Any non-member of $\mathcal{G}$ is μ-$\mathcal{G}\hat{g}$-closed.
(v) Every μ-$\mathcal{G}\hat{g}$-closed set is μ-$\mathcal{G}$g-closed.
(vi) Every μ-gs-closed set is μ-$\mathcal{G}\hat{g}$-closed.
(vii) Every μ-θ-closed set in X is μ-$\mathcal{G}\hat{g}$-closed.
(viii) Every μ-δ-closed set in X is μ-$\mathcal{G}\hat{g}$-closed.

Proof.

(i) Let A be a μ-closed set then Cμ(A) = A. Let U be a μ-semi open set in X such that $A \subseteq U$. Then, $A^{\Phi_\mu} \subseteq C\mu(A))$ $= A \subseteq U \Rightarrow A^{\Phi_\mu} \subseteq U \Rightarrow$ A is μ-$\mathcal{G}\hat{g}$-closed.

(ii) Let A be a subset in X. Then $A^{\Phi_\mu}(A^{\Phi_\mu}) \subseteq A^{\Phi_\mu} \subseteq U \Rightarrow A^{\Phi_\mu}$ is μ-$\mathcal{G}\hat{g}$-closed.

(iii) Let A be a Φ μ- closed set. Then $C^{\Phi_\mu}(A)) = A \Rightarrow A \cup A^{\Phi_\mu} = A \Rightarrow A^{\Phi_\mu} \subseteq A$. Therefore, $A^{\Phi_\mu} \subseteq U$ whenever $A \subseteq U$ and U is μ-semi open in X. This implies A is μ-$\mathcal{G}\hat{g}$-closed.

(iv) Let $A \notin \mathcal{G}$ then $A^{\Phi_\mu} = \emptyset \Rightarrow$ A is μ-$\mathcal{G}\hat{g}$-closed.

(v) Let A be a μ-$\mathcal{G}\hat{g}$-closed and $A \subseteq U$ and U is μ-open in X, we get $A^{\Phi_\mu} \subseteq U \Rightarrow$ A is μ- $\mathcal{G}g$-closed.Therefore, every μ-$\mathcal{G}\hat{g}$-closed set is μ- $\mathcal{G}g$ -closed.

(vi) Let A be a μ-gs-closed set and U be a μ-semi open set in X, such that $A \subseteq U$, then $c_\mu(A) \subseteq U$, consider $A^{\Phi_\mu} \subseteq$ $c\mu(A) \subseteq U \Rightarrow$ A is μ-$\mathcal{G}\hat{g}$-closed. Thus, every μ-gs-closed set is μ-$\mathcal{G}\hat{g}$-closed.

(vii) Let A be μ-θ closed. Then A = θCμ(A). Let U be a μ-semi open set in X such that $A \subseteq U$, then $A^{\Phi_\mu} \subseteq C\mu(A)$ $\subseteq \theta C\mu(A) = A \subseteq U$. Thus, A is μ- $\mathcal{G}\hat{g}$-closed.

(viii) Let A be μ-δ closed. Then A = μ-δCμ(A). Let U be a μ -semi open set in X such that $A \subseteq U$, then $A^{\Phi_\mu} \subseteq C\mu(A)$ $\subseteq \delta C\mu(A) = A \subseteq U$. Thus, A is μ-$\mathcal{G}\hat{g}$-closed.

Remark 4.1. The sets μ-gθ-closed and μ-$\mathcal{G}\hat{g}$-closed are independent from each other. Similarly, μ-θg-closed and μ-$\mathcal{G}\hat{g}$-closed are independent from each other.

Remark 4.2. Every μ-gs-closed set is μ-$\mathcal{G}\hat{g}$-closed but the converse is not true as by the following example:

Example 4.1. Let X = {a, b, c, d}, μ = {X, {a, b}, {b, d}, {a, b, d}}, $\mathcal{G}$ = {X, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Then (X, μ) is a topological space and $\mathcal{G}$ is a grill on X.Let A = {a} then $A^{\Phi_\mu} = \emptyset$. Therefore, A is μ-$\mathcal{G}\hat{g}$-closed. But $A \subseteq$ {a, b} and $c_\mu(A)$ = {a, c} does not a subset of {a, b}. Therefore, A is not μ-gs-closed.

Definition 4.4. Let X be a space and (Ø ≠) A ⊆ X. Then [A] = {B ⊆ X: A ∩B ≠ Ø}is a grillon X, called the principal grill generated by A.

Proposition 4.2. In the case of [X] principal grill generated by X, it is known that τ = τ[X] so that any [X]- μ-gs-closed set becomes simply a μ-gs-closed set and vice-versa.

Theorem 4.1. Let (X, μ, $\mathcal{G}$) be a grill topological space. If a subset A of X is μ- $\mathcal{G}$ĝ-closed then c^{Φ_μ} (A) ⊆ U whenever A ⊆ U and U is μ-semi open.

Proof. Let A be a μ- $\mathcal{G}$ĝ-closed set and U be a μ-semi open in X such that A ⊆ U then A^{Φ_μ} ⊆ U ⇒ A ∪ A^{Φ_μ} ⊆ U ⇒ c^{Φ_μ} (A) ⊆ U. Thus, c^{Φ_μ} (A) ⊆ U whenever A ⊆ U and U is μ-semi open.

Theorem 4.2. Let (X, μ, $\mathcal{G}$) be a grill topological space. If a subset A of X is μ- $\mathcal{G}$ĝ-closed then for all$x$ ∈ c^{Φ_μ} (A), $c_\mu(\{x\})$ ∩ A ≠ Ø.

Proof. Let x ∈ c^{Φ}(A). If c_μ ({x}) ∩ A = Ø ⇒ A ⊆ X – $c_\mu(\{x\})$ then by Theorem 4.1, c^{Φ_μ} (A) ⊆ X – c_μ({x}) which is a contradiction to our assumption that x ∈ c^{Φ_μ}(A), Therefore, $c_\mu\{x\}$) ∩ A ≠ Ø.

Theorem 4.3. Let (X, μ, $\mathcal{G}$) be a grill topological space. If a subset A of X is μ- $\mathcal{G}$ĝ-closed then c^{Φ}(A) –A contains no non-empty closed set of (X, μ, $\mathcal{G}$). Also, A^{Φ_μ} – A contains no nonempty closed set of (X, μ, $\mathcal{G}$).

Proof. Let F be a closed set contained in c^{Φ_μ} (A) – A and let x ∈ F, since F ∩ A = Ø. we get $c_\mu(\{x\})$ ∩ A = Ø Which is a contradiction to the fact that cμ({x}) ∩ A ≠ Ø. c^{Φ_μ} (A) – A contains no non-emptyclosed set of (X, μ, $\mathcal{G}$). Since A^{Φ_μ} – A = c^{Φ}(A) – A, A^{Φ_μ} – A contains no non-empty closed set of (X, μ, $\mathcal{G}$).

Corollary 4.1. Let (X, μ, $\mathcal{G}$) be a T1-space in grill on X. Then every μ- $\mathcal{G}$ĝ-closed set is Φμ-closed.

Proof. Let A be a μ-$\mathcal{G}$ĝ-closed set and x ∈ A^{Φ_μ} then x ∈ c^{Φ_μ} (A). By Theorem 4.2, $c_\mu(\{x\})$ ∩ A ≠ Ø, {x}∩ A ≠ Ø, x ∈A. Therefore, $A^{\Phi\mu}$ ⊆ AThus, A is Φμ-closed.

Corollary 4.2. Let (X, μ, $\mathcal{G}$) be a T1-space in grill on X. Then A (⊆ X) is μ-$\mathcal{G}$ĝ-closed set iff A is Φμ-closed.

Proposition 4.3. Let (X, μ, $\mathcal{G}$) be a grill topological space and A be a μ-$\mathcal{G}$ĝ-closed. Then thefollowing statements are equivalent:

(i) A is Φμ-closed

(ii) c^{Φ}(A)–A is closed in (X, μ, $\mathcal{G}$).

(iii) A^{Φ_μ} – A is closed in (X, μ, $\mathcal{G}$).

Proof.: (i) ⇒ (ii) Let A be Φμ-closed. Then c^{Φ_μ} (A) – A = Ø. so c^{Φ_μ} (A) – A is a closed set.

(ii) ⇒ (iii) Since c^{Φ_μ} (A) –A = A^{Φ_μ} – A.

(iii)⇒ (i) Let A^{Φ_μ} – A be closed in (X, μ, $\mathcal{G}$). Since A is μ-$\mathcal{G}$ĝ-closed by Theorem 4.3, A^{Φ_μ}–A=Ø.So, A is Φμ-closed.

Lemma 4.1. Let (X, μ, $\mathcal{G}$) be a grill topological space. If A (⊆ X) is Φμ-dense in itself, then A^{Φ_μ} = $c_\mu(A^{\Phi\mu})$ = $c^{\Phi_\mu}(A)$ = $c_\mu(A)$.

Proof. A is Φμ-dense in itself ⇒A ⊆ A^{Φ_μ} ⇒ $c_\mu(A)$ ⊆ $c_\mu(A^{\Phi_\mu})$ = A^{Φ_μ} ⊆ $c_\mu(A)$ ⇒ $c_\mu(A)$ = A^{Φ_μ} = $c_\mu(A^{\Phi_\mu})$ now by definition $c^{\Phi_\mu}(A)$ = A ∪ A^{Φ_μ} = A ∪ $c_\mu(A)$ =c(A).Therefore, A^{Φ_μ} = c_μ (A^{Φ_μ}) = c^{Φ_μ} (A) = $c_\mu(A)$.

Theorem 4.4. Let (X, μ, $\mathcal{G}$) be a grill topological space. If A (⊆ X) is Φμ-dense in itself and μ-$\mathcal{G}$ĝ-closed,then A is μ-gs-closed.

Proof. Follows from Lemma 4.1.

Corollary 4.3. For a grill $\mathcal{G}$ on a space (X, μ). Let A (⊆X) be Φμ-dense in itself. Then A is μ-$\mathcal{G}$ĝ-closed if and only if it is A is μ-gs closed.

Proof. Follows from Proposition 4.1(vi) and Theorem 4.4.

Theorem 4.5. For any grill $\mathcal{G}$ on a space (X, μ) the following statements are equivalent:

(i) Every subset of X is μ-$\mathcal{G}$ĝ-closed;

(ii) Every semi open subset of (X, μ) is Φμ-closed.

Proof. (i) ⇒ (ii) Let A be μ -semi open in (X, μ) then by (i), A is μ-$\mathcal{G}$ĝ-closed so that A^{Φ_μ} ⊆ A ⇒ A is Φμ-closed.

(ii) ⇒ (i) Let A ⊆X and U be μ -semi open in (X, μ) such that A ⊆U. Since U is μ -semi open by (ii), $U^{\Phi\mu}$ ⊆ U. Now A ⊆ U ⇒ A^{Φ_μ} ⊆ $U^{\Phi\mu}$ ⊆ U ⇒ A is μ-$\mathcal{G}$ĝ-closed.

Theorem 4.6. For any subset A of a space (X, μ) and a grill G on X. If A is μ-$\mathcal{G}\hat{g}$-closed then $A \cup (X - A^{\Phi\mu})$ is μ-$\mathcal{G}\hat{g}$-closed.

Proof. Let $A \cup (X - A^{\Phi\mu}) \subseteq U$, where U is μ-semi open in X. Then $X - U \subseteq X - (A \cup (X - A^{\Phi\mu})) = A^{\Phi\mu} - A$. Since A is μ-$\mathcal{G}\hat{g}$-closed, by Theorem 3.2, we have $X - U = \emptyset$, i.e., X = U. Since X is the only semiopen set containing $A \cup (X - A^{\Phi\mu})$, $A \cup (X - A^{\Phi\mu})$ is μ-$\mathcal{G}\hat{g}$-closed.

Proposition 4.4. For any subset A of a space (X, μ, $\mathcal{G}$) be a grill $\mathcal{G}$ on X, the following statements are equivalent:

(i) $A \cup (X - A^{\Phi\mu})$ is μ-$\mathcal{G}\hat{g}$-closed.

(ii) $A^{\Phi\mu} - A$ is μ-$\mathcal{G}\hat{g}$-open.

Proof. Follows from the fact that $X - (A^{\Phi\mu} - A) = A \cup (X - A^{\Phi\mu})$.

Theorem 4.7. Let (X, μ, $\mathcal{G}$) be a grill topological space and A, B be subsets of X such that $A \subseteq B \subseteq c^{\Phi}(A)$. If A is μ-$\mathcal{G}\hat{g}$-closed, then B is μ-$\mathcal{G}\hat{g}$-closed.

Proof. Let $B \subseteq U$, where U is μ-semi open in X. Since A is μ-$\mathcal{G}\hat{g}$-closed, $A^{\Phi\mu} \subseteq U \Rightarrow c^{\Phi\mu}(A) \subseteq U$. Now, $A \subseteq B \subseteq c^{\Phi\mu}(A) \Rightarrow c^{\Phi\mu}(A) \subseteq c^{\Phi\mu}(B) \subseteq c^{\Phi\mu}(A)$. Thus, $c^{\Phi\mu}(B) \subseteq U$ and hence B is μ-$\mathcal{G}\hat{g}$- closed.

Corollary 4.4. Φμ-closure of every μ- $\mathcal{G}\hat{g}$-closed set is μ- $\mathcal{G}\hat{g}$-closed**.**

Theorem 4.8. Let (X, μ, $\mathcal{G}$) be a grill topological space and A, B be subsets of X such that $A \subseteq B \subseteq A^{\Phi\mu}$. If A is μ-$\mathcal{G}\hat{g}$-closed. Then A and B are μ-gs-closed.

Proof. $A \subseteq B \subseteq A^{\Phi\mu} \Rightarrow A \subseteq B \subseteq c^{\Phi\mu}(A)$ and hence by Theorem 4.7, B is μ-$\mathcal{G}\hat{g}$-closed. Again, $A \subseteq B \subseteq A^{\Phi\mu} \Rightarrow A^{\Phi\mu} \subseteq B^{\Phi\mu} \subseteq ((A)^{\Phi\mu})^{\Phi\mu} \subseteq A^{\Phi\mu} \Rightarrow A^{\Phi\mu} = B^{\Phi\mu}$. Thus, A and B are Φμ-dense in itself and hence by Theorem 4.4, A and B are μ-gs-closed.

Theorem 4.9. Let (X, μ, $\mathcal{G}$) be a grill topological space. Then a subset A of X is μ-$\mathcal{G}\hat{g}$-open if and only if $F \subseteq i^{\Phi\mu}(A)$ whenever $F \subseteq A$ and F is μ-closed.

Proof. Let A be μ-$\mathcal{G}\hat{g}$-open and $F \subseteq A$, where F is μ-closed in (X, μ, $\mathcal{G}$). Then $X - A \subseteq X - F \Rightarrow \Phi(X - A) \subseteq X - F \Rightarrow c^{\Phi\mu}(X - A) \subseteq X - F \Rightarrow F \subseteq i^{\Phi\mu} A$. Conversely, $X - A \subseteq U$ where U is open in (X, τ) $\Rightarrow X - U \subseteq i^{\Phi\mu}(A) \Rightarrow c^{\Phi\mu}(X - A) \subseteq U$. Thus (X − A) is μ-$\mathcal{G}\hat{g}$-closed and hence A is μ-$\mathcal{G}\hat{g}$-open.

5. SOME CHARACTERIZATIONS OF M-REGULAR AND M-NORMAL SPACES

Theorem 5.1. Let X be a μ- normal space and $\mathcal{G}$ be a grill on X then for each pair of disjoint closed sets F and K, there exist disjoint μ-$\mathcal{G}\hat{g}$-open sets U and V such that $F \subseteq U$ and $K \subseteq V$.

Proof. It is obvious, since every open set is μ-$\mathcal{G}\hat{g}$-open.

Theorem 5.2. Let X be a μ- normal space and $\mathcal{G}$ be a grill on X then for each closed set F and any open set V containing F, there exist a μ-$\mathcal{G}\hat{g}$-open set U such that $F \subseteq U \subseteq c^{\Phi\mu}(U) \subseteq V$.

Proof. Let F be a closed set and V be an open set in (X, μ) such that $F \subseteq V$. Then F and X − V are disjoint closed sets. By Theorem 5.1, there exist disjoint μ-$\mathcal{G}\hat{g}$-open sets U and W such that $F \subseteq U$ and $X - V \subseteq W$. Since W is μ-$\mathcal{G}\hat{g}$-open and $X - V \subseteq W$ where X − V is closed, $X - V \subseteq i^{\Phi}(W)$. So, $X - i^{\Phi\mu}(W) \subseteq V$. Again, $U \cap W = \emptyset \Rightarrow U \cap i^{\Phi\mu}(W) = \emptyset$. Hence $c^{\Phi\mu}(U) \subseteq X - i^{\Phi\mu}(W) \subseteq V$. Thus $F \subseteq U \subseteq c^{\Phi\mu}(V) \subseteq V$, where U is a μ-$\mathcal{G}\hat{g}$-open set.

The following theorems gives characterizations of a μ- normal space in terms of μ-$\mathcal{G}\hat{g}$-open sets. Which are the consequence of Theorems 5.1, 5.2 and proposition 4.2 if one takes $\mathcal{G}$ = [X]

Theorem 5.3. Let X be a normal space and $\mathcal{G}$ be a grill on X then for each pair of disjointclosed sets F and K, there exist disjoint μ-$\mathcal{G}\hat{g}$-open sets U and V such that $F \subseteq U$ and $K \subseteq V$.

Theorem 5.4. Let X be a normal space and $\mathcal{G}$ be a grill on X then for each closed set F and any open set V containing F, there exist a μ-$\mathcal{G}\hat{g}$-open set U such that $F \subseteq U \subseteq C^{\Phi\mu}(U) \subseteq V$.

Theorem 5.5. Let X is regular and $\mathcal{G}$ be a grill on a space (X, μ). Then for each closed set F and each $x \in X - F$, there exist disjoint μ-$\mathcal{G}\hat{g}$-open sets U and V such that $x \in U$ and $F \subseteq V$.

Proof. The proof is obvious.

Theorem 5.6. Let X be a regular space and $\mathcal{G}$ be a grill on a space (X, μ). Then for each open setV of (X, μ) and each point $x \in V$ there exist a μ-$\mathcal{G}_{\hat{g}}$-open set U such that $x \in U \subseteq c^{\Phi\mu}(U) \subseteq V$.

Proof. Let V be any semi-open set in (X, μ) containing a point x of X. Then by Theorem 5.5, there exist disjoint μ-$\mathcal{G}$ĝ-open sets U and W such that $x \in$ U and X − V ⊆ W. Now, U ∩ W = ∅ implies c^{Φ_μ} (U) ⊆ X−W ⊆ V. Thus $x \in$ U ⊆ c^{Φ_μ} (U) ⊆ V.

The following theorems gives characterizations of a regular space in terms of μ-$\mathcal{G}$ĝ-open sets which are the consequence of Theorems 5.5,5.6 and Proposition 4.2 if one takes $\mathcal{G}$ = [X].

Theorem 5.7. Let X be a regular and $\mathcal{G}$ be a grill on a space (X, μ). Then for each closed set Fand each $x \in$ X− F, there exist disjoint μ-$\mathcal{G}_{ĝ}$-open sets U and V such that $x \in$ U and F ⊆ V.

Theorem 5.8. Let X be a regular space and $\mathcal{G}$ be a grill on a space (X, μ). Then for each open set V of (X, μ) and each point $x \in$ V there exist a μ-$\mathcal{G}_{ĝ}$-open set U such that $x \in$ U ⊆ c^{Φ}(U) ⊆ V.

CONFLICT OF INTERESTS

None.

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