Original Article
ISSN (Online): 2582-7472

ShodhKosh: Journal of Visual and Performing Arts
June 2024 5(6), 1768–1774

INTRODUCE SOME CONCEPTS OF THE FRAMES IN HILBERT AND BANACH SPACES IN RANDOM VARIABLES

Kamal Kumar[1], Virender[2], Nikita Dalal[3]

[1]Department of Mathematics, Baba Mastnath University Rohtak, India
[2]Department of Mathematics, Shyam Lal College, University of Delhi, India
[3]Department of Mathematics, Baba Mastnath University Rohtak, India

Corresponding Author
Nikita Dalal,
nikitadalal54321@gmail.com

DOI
10.29121/shodhkosh.v5.i6.2024.2557

Funding: This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

ABSTRACT

In this work, we establish various functional-analytical features of these decompositions and demonstrate their applicability to wavelet and gabor systems. First, we demonstrate the stability of atomic decompositions and frames under minor perturbations. This is motivated by analogous classical perturbation outcomes for bases, such as the Kato perturbation theorem and the Paley-Wiener basis stability requirements. The methodological contributions are concentrated on creating confidence bands, change-point tests, and two-sample tests because these procedures seem appropriate for the suggested situation. The reason for its selection and analysis in the thesis is its connection to operators. Frame sequences are created, and an investigation is conducted into a class of operators connected to a specific Bessel sequence, which turns it into a frame for every operator in the class.

1. INTRODUCTION

Over a century since the introduction, the Fourier transform or Fourier series has been used as an important tool in analysis for approximating a function in the space of square integrable functions over the reals, which is denoted by $L2(R)$. Though it has been an important tool, it has a significant lacking for signal analysis. A signal has two main components i.e. the intensity of the signal and the duration of the particular intensity. To know of a signal fully, one needs to get the information of the intensity and the duration of the signal.

An application to annual temperature profiles and a simulation study are used to investigate the properties of the finite sample. The creation of statistical methodology for the analysis of functional data collected over time and/or space has been an active subject of research because to the recent significant evolution in improved data collection technologies. The majority of the research has focused on creating technique based on Hilbert space, for which a complete theory currently exists. Nonetheless, Ramsay and Silverman (2005) have extensively explored the crucial importance of

smoothness, and practically all functions that are fit in practice are at least continuous. In these situations, fully functional solutions may be more advantageous than dimension reduction strategies, which may result in a loss of information. The required two-sample and change-point tests in Sections will be developed using the theoretical contributions. Here, the suggested method's utility is more obvious because it can be challenging to distinguish variations between two smooth curves in real-world scenarios. Furthermore, in many practical scenarios, minor differences might not even be significant.
Consequently, the "relevant" setting is chosen, which permits predetermined departures from an assumed null function rather than attempting to test for precise equality under the null hypothesis.

Example 1: If μ_1 and $\mu_2$are the mean functions corresponding to two samples, and C([0,1]), the space of continuous functions on the compact interval [0,1], is endowed with the sup-norm $\|f\| = \sup_{t\in[0,1]}|f(t)|$, then hypotheses of the type

$$H_0: \|\mu_1 - \mu_2\| \leq \Delta \text{ and } H_1: \|\mu_1 - \mu_2\| > \Delta \ldots \ldots (1)$$

where $\Delta \geq 0$ signifies a prespecified constant.
Therefore, a particular case of (1) is the classical instance of verifying perfect equality, which is obtained by the choice Δ = 0. In applications, nevertheless, it could make sense to thoroughly consider your options and pinpoint the exact change size that you are truly interested in. Testing pertinent hypotheses specifically avoids the consistency issue that was noted by Berkson (1938), according to which, if the sample size is large enough, any consistent test will discover any arbitrary tiny change in the mean functions. This viewpoint can also be seen as a specific kind of bias-variance trade-off. A Monte Carlo simulation study is used in Section to assess the finite-sample qualities of the pertinent two-sample and change-point tests and, in particular, the bootstrap techniques' performance. Several scenarios are examined, and the results indicate that the suggested methodology functions rather well. Additionally, a two-sample and change-point test application for an archetypal data example—annual temperature profiles obtained at Australian measurement stations—is provided in Section.

2. $C(T)$-VALUED RANDOM VARIABLES

The set of continuous functions from T into the real line R is denoted by C(T), and some fundamental information regarding central limit theorems and invariance principles for C(T)-valued random variables is given in this section. The sup norm $\|\cdot\|$, which is defined as$\|f\| = \sup_{t\in T}|f(t)|$, will be applied to C(T) in the following, unless otherwise specified, making $(C(T), \|\cdot\|)$ a Banach space.
The open sets with respect to the sup norm $\|\cdot\|$ then generate the natural Borel σ-field B(T) over C(T). It is understood that the measurability of random variables on $(\Omega, \mathcal{A}, P)$ having values in C(T) is related to B(T). It is assumed that the underlying probability space $(\Omega, \mathcal{A}, \mathbb{P})$ is complete. Furthermore, it is assumed that (T, ρ) is completely limited in terms of a metric ρ on T. T metrizability suggests that $C(T)$ is separable, avoiding problems with measurability. Moreover, $C(T)$ is tight for any random variable X.
Assume that X is a random variable with values in $C(T)$ and on $(\Omega, \mathcal{A}, P)$. Formally, expectations and higher-order moments of random variables valued in Banach space can be introduced in many ways. Every time $\mathbb{E}[\|X\|] < \infty$, the expectation $\mathbb{E}[X]$ of a random variable X in $C(T)$ exists as an element of $C(T)$. Every time $\mathbb{E}[\|X\|^k] = \mathbb{E}[\sup_{t\in T}|X(t)|^k] < \infty$ the k th moment is present. Point-wise evaluation can be used to calculate the kth order moments, which are expressed as $\mathbb{E}[X(t_1)\cdots X(t_k)]$. Because it enables the point wise computation of covariance kernels, the case $k = 2$ is significant. When a sequence of random variables $(X_n: n \in \mathbb{N})$ is asymptotically tight and its finite-dimensional distributions converge weakly to the finite-dimensional distributions of a random variable X in $C(T)$, that is,

$$\left(X_n(t_1), \ldots, X_n(t_k)\right) \Rightarrow \left(X(t_1), \ldots, X(t_k)\right) \ldots\ldots (1(a))$$

for every $t_1, \ldots, t_k \in T$ and whichever $k \in \mathbb{N}$, where the sign " ⇒ " designates convergence in distribution in $\mathbb{R}^k$. For every $t_1, \ldots, t_k$, $\left(X(t_1), \ldots, X(t_k)\right) \sim \mathcal{N}_k(0, \Sigma)$, where the (i, j) th entry of the covariance matrix Σ is given by $\mathbb{E}\left[X(t_i)X(t_j)\right], i, j = 1, \ldots, k$, then a centred random variable X in $C(T)$ is said to be Gaussian if its finite-dimensional distributions are multivariate normal. Thus, the covariance function $k(t, t') = \mathbb{E}[X(t)X(t')]$ fully characterises the distribution of X.; Deriving conditions under which the central limit theorem (CLT) applies in generic Banach spaces is a challenging issue, notably more intricate than its counterpart for real-valued random variables. The finiteness of the underlying random variables' second moments does not offer a necessary and sufficient condition in Banach spaces. In order to overcome the problem, complex theory was developed, leading to the concepts of type 2 and cotype 2 Banach spaces (for a summary, refer to Ledoux and Talagrand's 1991 book). To achieve the CLT, however, additional assumptions are required because the Banach space of continuous functions on a compact interval lacks the necessary type and cotype qualities. This is particularly true when integrating time series of continuous functions into the framework. The idea of φ-mixing sequences $(\eta_j : j \in \mathbb{N})$ of $C(T)$-valued random variables is proposed to explain the dependence of the observations. Firstly, for any two σ-fields $\mathcal{F}$ and $\mathcal{G}$, define

$$\phi(\mathcal{F},\mathcal{G}) = \sup\{|\mathbb{P}(G \mid F) - \mathbb{P}(G)| : F \in \mathcal{F}, G \in \mathcal{G}, \mathbb{P}(F) > 0\} \ldots\ldots.(2)$$

where $\mathbb{P}(G \mid F)$ indicates the conditional likelihood of G specified F. Subsequent, signify by $\mathcal{F}_k^{k'}$ the σ-field created by $(\eta_j : k \leq j \leq k')$. Then describe the φ-mixing measurement as

$$\varphi(k) = \sup_{k' \in \mathbb{N}} \phi\left(\mathcal{F}_1^{k'}, \mathcal{F}_{k'+k}^{\infty}\right) \ldots..(3)$$

and call the sequence $(\eta_j : j \in \mathbb{N})\varphi$-mixing every time $\lim_{k\to\infty} \varphi(k) = 0$.
The following requirements are enforced to get a CLT and an invariance principle for sequences of φ-mixing random elements in $C(T)$.

ASSUMPTION 1: $(X_{n,j} : n \in \mathbb{N}, j = 1, \ldots, n)$ is an selection of $C(T)$-valued arbitrary variables where, for some $j = 1, \ldots, n$ and $n \in \mathbb{N}$,

$$X_{n,j} = \eta_j + \mu_{n,j} \ldots \ldots . . (4)$$

with outlooks $\mathbb{E}[X_{n,j}] = \mu_{n,j}$ and error procedure $(\eta_j : j \in \mathbb{N}) \subset C(T)$. Additionally, the subsequent conditions are expected to hold: (A1) There is a continuous K s.t., for all $j \in \mathbb{N}$,

$$\mathbb{E}\left[\|\eta_j\|^{2+v}\right] \leq K,\ \mathbb{E}\left[\|\eta_j\|^{J}\right] < \infty \ldots\ldots(5)$$

for certain $v > 0$ and some straight integer $J \geq 2$. (A2) The miscalculation procedure $(\eta_j : j \in \mathbb{N})$ is stationary. There occurs a real-valued nonnegative arbitrary variable M with $\mathbb{E}[M^J] < \infty$, s.t., for some $n \in \mathbb{N}$ and $j = 1, \ldots, n$, the disparity

$$\left|X_{n,j}(t) - X_{n,j}(t')\right| \leq M\rho(t,t') \ldots..(6)$$

holds almost confidently for all $t, t' \in T$. The continuous J is the identical as in (A1). (A4) $(\eta_j : j \in \mathbb{N})$ is φ-mixing through mixing coefficients sustaining for particular $\bar{\tau} \in (1/(2+2v), 1/2)$ the circumstance

$$\sum_{k=1}^{\infty} k^{1/(1/2-\bar{\tau})} \varphi(k)^{1/2} < \infty,\ \sum_{k=1}^{\infty} (k+1)^{J/2-1} \varphi(k)^{1/J} < \infty \ldots\ldots..(7)$$

where the coefficients v and J are the similar as in (A1). It should be noted that similar presumptions can be made for random variable sequences $(X_n : n \in \mathbb{N})$ in $C(T)$. The covariance structure is the same in every row for triangular arrays that satisfy Assumption 1 since they only differ in their means from row to row.

$$\mathrm{Cov}\left(X_{n,j}(t), X_{n,j'}(t')\right) = \mathrm{Cov}\left(\eta_j(t), \eta_{j'}(t')\right) = \gamma(j-j', t, t') \ldots..(8)$$

for all $n \in \mathbb{N}$ and $j, j' = 1, \ldots, n$ (note that $\gamma(-j, t, t') = \gamma(j, t', t)$). The CLT that follows is implied by Assumption 1 and is demonstrated in Section. In this chapter, $D(\omega, \rho)$ is the packing number with respect to the metric ρ, which is the maximal number of ω-separated points in T. The symbol $\rightsquigarrow$ indicates weak convergence in $(C(T))^k$ or $(C([0,1] \times T))^k$ for some $k \in \mathbb{N}$

THEOREM 1: Let $(X_{n,j} : n \in \mathbb{N}, j = 1, \ldots, n)$ denote a triangular selection of arbitrary variables in $C(T)$ with outlooks $E[X_{n,j}] = \mu_{n,j}$ s.t. Assumption 4.1 is fulfilled and $\int_0^{\tau} D(\omega,\rho)^{1/J} d\omega < \infty$ for some $\tau > 0$. Before

$$G_n = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \left(X_{n,j} - \mu_{n,j}\right) \rightsquigarrow Z \ldots..(9)$$

in $C(T)$, where Z exists a centered Gaussian arbitrary variable with covariance function

$$C(s,t) = \mathrm{Cov}\left(Z(s), Z(t)\right) = \sum_{i=-\infty}^{\infty} \gamma(i, s, t) \ldots \ldots (10)$$

REMARK 1:

(a) For φ-mixing processes with exponentially diminishing mixing coefficients, that is, $\varphi(k) \leq ca^k (k \in \mathbb{N})$ for some $a \in (0,1)$, Assumption 4.1 condition (A4) is satisfied.

(b) The interval $T = [0,1]$ equipped with the metric $\rho(s,t) = |s-t|^{\theta}$ for a positive constant $\theta \in (0,1]$ is the subject of the sections that follow. The packing number in this instance satisfies $D(\omega,\rho) \lesssim \lceil \tau^{-1/\theta} \rceil$, suggesting

$$\int_0^{\tau} D(\omega,\rho)^{1/J} d\omega \lesssim \int_0^{\tau} \lceil \omega^{-1/\theta} \rceil^{1/J} d\omega \lesssim \frac{\tau^{1-1/(J\theta)}}{1-1/(J\theta)} < \infty \ldots..(11)$$

whenever the even integer J fulfills $J > 1/\theta$. Thus, under this assumption, Theorem 1 is applicable to Hölder continuous processes. For every $\theta \in (0,1/2)$, for instance, the trajectories of the Brownian motion on the interval [0,1] are Hölder continuous of order θ; in this scenario, we must suppose $J \geq 4$ in Assumption 1. Generally speaking, a larger summability condition on the mixing coefficients is needed for less smoothness. $J = 2$ is enough to obtain the CLT in Theorem 1 for processes with Hölder continuous routes with $\theta > 1/2$, which includes Lipschitz continuity. The process $\left(\hat{\mathbb{V}}_n: n \in \mathbb{N}\right)$ will then have a weak invariance principle verified by

$$\hat{\mathbb{V}}_n(s) = \frac{1}{\sqrt{n}} \sum_{j=1}^{\lfloor sn \rfloor} \left(X_{n,j} - \mu_{n,j}\right) \dots \dots . (12)$$

$$+\sqrt{n}\left(s - \frac{\lfloor sn \rfloor}{n}\right)\left(X_{n,\lfloor sn \rfloor+1} - \mu_{n,\lfloor sn \rfloor+1}\right) \dots \dots (13)$$

useful for the change-point analysis suggested. Letter that the procedure $\left(\hat{\mathbb{V}}_n(s): s \in [0,1]\right)$ is an division of the Banach space $C([0,1], C(T)) = \{\phi: [0,1] \to C(T) \mid \phi \text{ is continuous}\}$, where the norm on this space is specified by

$$\sup_{s\in[0,1]} \sup_{t\in T} |\phi(s,t)| = \|\phi\|_{C([0,1]\times T)} \dots \dots . (14)$$

Observe moreover that the quantity $\phi(s)$ for each s in the interval $s \in [0,1]$ is an element of $C(T)$, a real-valued continuous function whose domain is T. Express the value of ϕ(s) at the point t∈T as $\phi(s,t)$. Additionally, every element in $C([0,1], C(T))$ can also be thought of as an element in $C([0,1] \times T)$.. The notation $\|\cdot\|$ is used here and throughout the chapter to indicate any of the arising standards because the context will indicate which space corresponds to it. Additionally, we utilise the notation $s \wedge s' = \min\{s, s'\}$ quite a bit.

THEOREM 2: Assume that Theorem 1 premises are met. The weak invariance principle is then valid, which means

$$\hat{\mathbb{V}}_n \rightsquigarrow \mathbb{V} \dots \dots (4.15)$$

in $C([0,1] \times T)$, where $\mathbb{V}$ stays a centered Gaussian portion on $C([0,1] \times T)$ categorized by

$$\mathrm{Cov}\left(\mathbb{V}(s,t), \mathbb{V}(s',t')\right) = (s \wedge s')C(t,t') \dots \dots (4.16)$$

and long-run covariance function C is specified in (11).

REMARK 2: L^p-m-approximability is another concept of dependency that is commonly applied to Hilbert space valued time series. A similar notion may be defined for the case of $C(T)$-valued time series that we study here, where we essentially need that an m-dependent process be able to approximate the error process in model (1). More specifically, this means that it admits a representation of the form $\eta_j = f\left(\varepsilon_j, \varepsilon_{j-1}, \varepsilon_{j-2}, \dots\right)$ with a sequence $(\varepsilon_n: n \in \mathbb{N})$ of random variables and that there exists, for each j, an independent copy of $\left(\varepsilon_n^{(j)}: n \in \mathbb{N}\right)$ of $(\varepsilon_n: n \in \mathbb{N})$ s.t. the random variables $\eta_{j,m} = f\left(\varepsilon_j, \dots, \varepsilon_{j-m+1}, \varepsilon_{j-m}^{(j)}, \varepsilon_{j-m-1}^{(j)}, \dots\right)$ satisfy

$$\sum_{m=1}^{\infty} \mathbb{E}\left[\left\|\eta_m - \eta_{m,m}\right\|^2\right]^{1/2} < \infty \text{.....}(17)$$

(note that $\|\cdot\|$ is the sup-norm on $C(T)$). Substituting in Assumption 1 the state (A4) by

$$\sum_{m=1}^{\infty} m^{1/(1/2-\bar{\tau})} \mathbb{E}\left[\left\|\eta_m - \eta_{m,m}\right\|^2\right]^{1/2} < \infty$$

$$\sum_{m=1}^{\infty} (m+1)^{J/2-1} \mathbb{E}\left[\left\|\eta_m - \eta_{m,m}\right\|^J\right]^{1/J} < \infty \text{.....}(18)$$

a CLT can be demonstrated for Banach space-valued phase series of the procedure (1) through an m approximable error method in $C(T)$. These findings can then be applied to m-approximable $C([0,1])$-valued time series to construct a statistical approach similar to that described in Section. Furthermore, it is demonstrated using comparable arguments to those presented in Hörmann and Kokoszka (2010)'s Example 4.1 that this dependency model encompasses fAR(1) processes.

3. THE TWO-SAMPLE PROBLEM

Moving forward, the situation $T = [0,1]$ will be considered, as this is the standard option for functional data analysis. For every ∈(0,1]), the equivalent metric is $\rho(s,t) = |s-t|^{\theta}$. Statistics has a long history with two-sample problems, and the corresponding tests are among the most widely used statistical techniques. Numerous contributions have also been made for the functional setting. In the current context, two are noteworthy. The impact of smoothing while transforming discrete observations into functional data was examined by Hall and van Keilegom (2007). Based on Hilbert-space theory, Horváth et al. (2013) presented two-sample tests for $L^p - m$ approximable functional time series. In the Banach-space framework of Section, a two-sample test is proposed here. In order to achieve this, take two independent samples of $C([0,1])$- valued random variables, $X_1, \dots, X_m$ and $Y_1, \dots, Y_n$. Assumption 1 (A2) states that expectation functions and covariance kernels exist. They are represented by the notations $\mu_1 = \mathbb{E}[X_1]$ and $\mu_2 = \mathbb{E}[Y_1]$,, and the formulas $k_1(t,t') = \mathrm{Cov}\big(X_1(t), X_1(t')\big)$ and $k_2(t,t') = \mathrm{Cov}\big(Y_1(t), Y_1(t')\big)$, respectively. The magnitude of the highest deviation is thus of interest.

$$d_\infty = \|\mu_1 - \mu_2\| = \sup_{t\in[0,1]} |\mu_1(t) - \mu_2(t)| \ldots..(19)$$

between the two mean curves, or in evaluating the theories regarding a meaningful difference

$$H_0: d_\infty \le \Delta \text{ versus } H_1: d_\infty > \Delta \ldots \ldots (20)$$

where $\Delta \ge 0$ is a predetermined constant that the test's user determines. Again, take note that this setup contains the exceptional case Δ=0 for the "classical" two-sample problem $H_0: \mu_1 = \mu_2$ vs $H_0: \mu_1 \ne \mu_2$--which, to the best of our knowledge, has not yet been studied for $C([0,1])$--valued data. Note that studies looking for significant variations between two finite-dimensional characteristics that relate to various populations have primarily been examined in the literature on biostatistics, as demonstrated by Wellek (2010). The samples are considered to be balanced in this section in the sense that

$$\frac{m}{n+m} \longrightarrow \lambda \in (0,1) \ldots..(21)$$

as $m,n \to \infty$. Furthermore, let $X_1, \dots, X_m$ and $Y_1, \dots, Y_n$ be sampled as of independent period series $\big(X_j : j \in \mathbb{N}\big)$ and $\big(Y_j : j \in \mathbb{N}\big)$ that content Assumption 1 with $J > 1/\theta$. Under given these prerequisites, both functional time series meet the CLT, and Theorem 1 therefore dictates that

$$\sqrt{n+m}\left(\frac{1}{m}\sum_{j=1}^{m}(X_j - \mu_1), \frac{1}{n}\sum_{j=1}^{n}(Y_j - \mu_2)\right)$$

$$\rightsquigarrow \left(\frac{1}{\sqrt{\lambda}}Z_1, \frac{1}{\sqrt{1-\lambda}}Z_2\right), \ldots..(22)$$

where Z_1 and Z_2 are independent, scattered Gaussian processes retaining covariance functions

$$C_1(t,t') = \sum_{j=-\infty}^{\infty} \gamma_1(j,t,t') \text{ and } C_2(t,t') = \sum_{j=-\infty}^{\infty} \gamma_2(j,t,t') \ldots \ldots (23)$$

here, γ_1 and γ_2 are specified in the discussion that follows Assumption 4.1 and correspond to the corresponding sequences $\big(X_j : j \in \mathbb{N}\big)$ and $\big(Y_j : j \in \mathbb{N}\big)$. Currently, the samples' independence and the modest convergence in (4.23) suggest right away that

$$Z_{m,n} = \sqrt{n+m}\left(\frac{1}{m}\sum_{j=1}^{m} X_j - \frac{1}{n}\sum_{j=1}^{n} Y_j - (\mu_1 - \mu_2)\right) \rightsquigarrow Z \ldots \ldots (24)$$

in $C([0,1])$ as $m,n \to \infty$, anywhere $Z = Z_1/\sqrt{\lambda} + Z_2/\sqrt{1-\lambda}$ is a centered Gaussian procedure thru covariance function

$$C(t,t') = \mathrm{Cov}\big(Z(t), Z(t')\big) = \frac{1}{\lambda}C_1(t,t') + \frac{1}{1-\lambda}C_2(t,t') \ldots \ldots (25)$$

Beneath the merging in (24), the measurement

$$\hat{d}_\infty = \left\|\frac{1}{m}\sum_{j=1}^{m} X_j - \frac{1}{n}\sum_{j=1}^{n} Y_j\right\| \ldots \ldots.(26)$$

is a plausible estimator of the maximal deviation $d_\infty = \|\mu_1 - \mu_2\|$, and for large values of $\hat{d}_\infty$, the null hypothesis in (4.3) is rejected. To create a test with a predefined asymptotic level, the following steps are taken to establish the limit distribution of $\hat{d}_\infty$. In order to achieve this, let

$$\mathcal{E}^{\pm} = \{t \in [0,1]: \mu_1(t) - \mu_2(t) = \pm d_\infty\} \ldots \ldots (27)$$

if $d_\infty > 0$, and outline $\mathcal{E}^+ = \mathcal{E}^- = [0,1]$ if $d_\infty = 0$. In conclusion, signify by $\mathcal{E} = \mathcal{E}^+ \cup \mathcal{E}^-$the set of extremal points of the variance $\mu_1 - \mu_2$ of the dual mean functions. The principal main outcome establishes the asymptotic circulation of the statistic $\hat{d}_\infty$.

THEOREM 4.3: If $X_1, \dots, X_m$ and $Y_1, \dots, Y_n$ are sampled as of independent time series $(X_j: j \in \mathbb{N})$ and $(Y_j: j \in \mathbb{N})$ in $C([0,1])$, both sustaining Assumption 4.1 thru metric $\rho(s,t) = |s-t|^\theta, \theta \in (0,1], J\theta > 1$, then

$$T_{m,n} = \sqrt{n+m}(\hat{d}_\infty - d_\infty) \xrightarrow{\mathcal{D}} T(\mathcal{E}) \dots \dots (28)$$

Anywhere

$$T(\mathcal{E}) = \max\left\{\sup_{t\in\mathcal{E}^+} Z(t), \sup_{t\in\mathcal{E}^-} - Z(t)\right\} \dots \dots (29)$$

and the centered Gaussian procedure Z is specified by (27) and sets $\mathcal{E}^+$and $\mathcal{E}^-$are distinct in (29). It is important to note that even with i.i.d. data, the limit distribution is not distribution-free because it depends intricately on the set E of extremal points of the difference $\mu_1 - \mu_2$. Specifically, two sets of processes with matching mean functions μ_1, μ_2 and $\tilde{\mu}_1, \tilde{\mu}_2$ are possible, such that $\|\mu_1 - \mu_2\| = \|\tilde{\mu}_1 - \tilde{\mu}_2\|$. However, if the corresponding sets of extremal points E and E ˜ do not coincide, then the respective limit distributions in Theorem 3 will disagree completely. Theorem 3 proof is provided in Section, For the random variable ([0,1]) , it follows in Theorem 4.3 that in the case $d_\infty = 0, \mathcal{E}^+ = \mathcal{E}^- = [0,1]$.

$$T = \max_{t\in[0,1]} |Z(t)| \dots \dots (30)$$

This is a straightforward outcome of the continuous mapping theorem and the weak convergence (4.24) of the process $Z_{m,n}$.

Asymptotic inference: Testing the classical hypothesis $H_0: \mu_1 \equiv \mu_2$.

Theorem 3 also affords the asymptotic circulations of the test measurement $\hat{d}_\infty$ in the case of dual identical mean functions, that is, if $\mu_1 \equiv \mu_2$. At this time, it agrees to the special case $\Delta = 0$, and thus $d_\infty = 0, \mathcal{E}^\pm = [0,1]$. Thus,

$$T_{m,n} \xrightarrow{\mathcal{D}} T \ (m,n \to \infty) \dots\dots (31)$$

where the arbitrary variable T is definite in (4.30). An asymptotic level α test aimed at the standard hypotheses

$$H_0: \mu_1 = \mu_2 \ \text{ versus } \ H_1: \mu_1 \neq \mu_2 \dots \dots (32)$$

may hence be attained by rejecting H_0 when

$$\hat{d}_\infty > \frac{u_{1-\alpha}}{\sqrt{n+m}} \dots \dots (33)$$

where $u_{1-\alpha}$ is the distribution's $(1-\alpha)$-quantile for the random variable T, which is defined in (30). Keep in mind that the long-run covariance operator, which must be determined in applications, is the only dependency this quantile possesses. It is simple to see that the test defined by (33) has asymptotic level α and is consistent with Theorem 3.

THEOREM 4: Assume that Theorem 3 conditions are met. Then, for $\alpha \in (0,1)$, identify the $(1-\alpha)$-quantile of the random variable T as defined in (30) by $u_{1-\alpha}$, and construct the functions

$$\mu^\pm_{m,n}(t) = \tfrac{1}{m}\textstyle\sum_{j=1}^m X_j - \tfrac{1}{n}\sum_{j=1}^n Y_j \pm \tfrac{u_{1-\alpha}}{\sqrt{n+m}} \dots (34)$$

Then the established

$$C_{\alpha,m,n} = \{\mu \in C([0,1]): \mu^-_{m,n}(t) \le \mu(t) \le \mu^+_{m,n}(t) \text{ for all } t \in [0,1]\}$$

defines a instantaneous asymptotic $(1-\alpha)$ self-assurance band aimed at $\mu_1 - \mu_2$, that is,

$$\lim_{m,n\to\infty} \mathbb{P}(\mu_1 - \mu_2 \in C_{\alpha,m,n}) = 1 - \alpha \dots (35)$$

It should be noted that the simultaneous confidence bands presented in Theorem 4 apply for all $t \in [0,1]$ and not only practically everywhere, in contrast to their counterparts in Hilbert space. This property makes the suggested bands easier to understand and potentially more effective for applications.

4. CONCLUSION

For a series of continuous linear functionals constructed on a Banach space, the concept of frames for operators is introduced. It has been demonstrated that the new idea is a logical development of the Banach frames that Casazza et al. defined in 2005. Results on generating frames for operators are provided, along with the necessary and sufficient conditions that must be met. Furthermore, unless the corresponding sequence spaces fail to meet certain additional constraints, it is demonstrated that the concepts of "atomic systems" and "frame for operators" as stated in the thesis are not generally identical. Frame sequences are created, and an investigation is conducted into a class of operators connected to a specific Bessel sequence, which turns it into a frame for every operator in the class.

CONFLICT OF INTERESTS

None.

ACKNOWLEDGMENTS

None.

REFERENCES

Alizadeh, E., Faroughi, M.H. and Rahmani, M. (2017). Continuous K-g-frames in Hilbert Spaces. arXiv, 1-17.

Rahimi, A. (2011). Multipliers of generalized frames in Hilbert spaces. Bull. Iranian Math. Soc., 37, 63-80.

Grochenig, K. and Ron, A. (1998). Tight compactly supported wavelets frame of arbitrarily high smoothness. Proc. Amer. Math. Soc., 126(4), 1101-1107.

Han, D. and Larson, D. (2000). Frames, bases and group representations. Mem. Amer. Math. Soc., 147, 2000.

Alizadeh, E., Rahimi, A., Osgooei, E. and Rahmani, M. (2019). Some properties of continuous K-g-frames in Hilbert Spaces. U.P.B. Sci. Bull., Series A, 81(3).

Deepshikha and Vashisht, L.K. (2015). On perturbation of local atoms for subspaces. Poincare J.Anal. Appl., 2, 129-137.

Deepshikha, Vashisht, L.K., Verma, G. (2017). Generalized weaving frames for operators in Hilbert spaces. Results Math. 72(3), 1369–1391.

Poumai, K.T. and Kaushik, S.K. (2017). Some results concerning Riesz bases and frames in Banach spaces. Jordan Journal of Mathematics and Statistics, 10(1), 11-32.

Poumai, K.T., Rathore, G.S. and Bhati, S. (2017). Some results on block frames. Global Journal of Pure and Applied Mathematics, 13(7), 2983- 2996.