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AN APPLICATION OF GRAPH COLORING: NURSE SCHEDULING PROBLEM

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ABSTRACT

The Nurse Rostering Problem (NRP), also known as the Nurse Scheduling Problem (NSP), represents a challenging scheduling issue that significantly impacts hospital staff worldwide on a daily basis. Nursing staff scheduling is one of the most important but complex problem for the hospital management. NSP is a critical subclass of scheduling problems known for their complexity. This paper introduces an efficient approach to solving the Nurse Scheduling Problem (NSP), aiming to meet the needs of nurses, patients, and hospital management. It requires critical thinking and good decision-making skills to develop a roster that effectively balances patient care, employee and other needs in the organization. To reduce the problem's complexity, graph coloring techniques have been utilized.

Keywords: Graph Coloring, NSP- Nurse scheduling problem, NR- Nurse Roster, Scheduling techniques.

1. INTRODUCTION

Nursing is a noble and respectable profession. It is a profession that is acknowledged worldwide due to its significant contribution to the societal well-being. However, this profession brings considerable challenges that affect the nurses' performance. Hence, a scientific investigation to improve their performance is timely. Creating a schedule for nurses has always been a challenging task. Nurse scheduling is a significant issue faced by many reputable hospitals worldwide. Typically, nurses operate in three shifts, as patients require continuous care throughout the day. Hospital administrators struggle with the complex task of assigning nurses to different shifts based on the ward's needs. Crafting a nurse schedule involves the intricate process of organizing work shifts for hospital nurses. The Nurse Scheduling Problem (NSP) is a daily challenge encountered by hospitals everywhere. Briefly, Ernst et.al (2004) and Yi (2005) identifies 28 different categories of methods that have been used in personnel scheduling problems. These methods include optimization approaches (i.e. mathematical programming), constraint logic programming, constructive heuristic, expert systems, genetic algorithms, simple local search, simulated annealing, search, knowledge-based systems, artificial neural networks and hybrid systems. This wide range of approaches and techniques has been performed in nurse scheduling and classified into four categories which are optimization, search, constructive heuristics, and hybrid techniques. The process of scheduling nurses is a critical task in managing duty rosters within the hospital system, and it is a pivotal component of healthcare services. At present, the creation and updating of these schedules demand significant time and effort from highly skilled nurses and administrative

staff. Implementing graph coloring techniques can streamline this process, leading to more equitable and efficient scheduling solutions.

2. PROBLEM DEFINITION

Creating a nurse scheduling system is a complex task, particularly when done manually as in the case described in Sri Lanka. Here's a unique rephrasing of the given scenario. In Sri Lanka, nurse shift scheduling is traditionally handled manually by a head or chief nurse. This individual is responsible for drafting and publishing the monthly nurse roster before the start of each new month, typically beginning the process around the fifteenth of the current month. The head nurse utilizes a manual table, which features the days of the month as columns and the names of all nurses as rows.

Nurses work in shifts due to the nature of hospital operations. There are three primary shifts:

- Morning shift: 7:00 am to 1:00 pm
- Evening shift: 1:00 pm to 7:00 pm
- Night shift: 7:00 pm to 7:00 am

The head nurse's role includes filling out the table with the necessary night shifts for each nurse, usually by identifying patterns that meet both the required number of nurses and their availability. Following a night shift, nurses are granted a day off. Additionally, each nurse is entitled to Sundays off in a given month. The head nurse often assigns overtime (OT) to prevent conflicts within the roster, although OT assignments are subject to certain constraints and cannot exceed the basic allotment for each individual. Excessive OT can lead to overworked staff and increased error rates.

The scheduling process must ensure the right number of nurses with the appropriate skills for each shift, while also considering budget constraints. This balancing act is crucial in meeting the hospital's minimum staffing requirements. Furthermore, the process must accommodate staff preferences and provide the necessary flexibility to keep them satisfied. Patient needs are another critical factor. On wards where patients require complex treatments, the number of nurses needed can fluctuate based on patient acuity, complicating the scheduling process. The head nurse aims to meet as many staff preferences as possible without compromising hospital needs.

Some of the constraints that must be considered in the scheduling process include:

- Nurses cannot work all three shifts (morning, evening, and night) on the same day.
- Nurses on holiday are exempt from shifts during that time.
- Nurses have the day off following a night shift.
- Limits on the minimum and maximum number of successive workdays.
- Limits on the minimum and maximum number of hours worked.
- Limits on the minimum and maximum number of successive free days.
- Maximum number of assignments per day of the week.
- Maximum number of assignments for each shift type.
- Maximum number of a particular shift type per week.
- Assigning identical shift types during weekends.
- Balancing the workload among all personnel.

The head nurse's goal is to create a schedule that satisfies as many of these constraints and preferences as possible while ensuring the hospital's operational needs are met.

3. GRAPH COLORING

Graph coloring is a way of assigning labels, often called "colors," to elements of a graph subject to certain constraints. In the most common case, it involves coloring the vertices of a graph such that no two adjacent vertices share the same color. This problem is fundamental in graph theory and has applications in scheduling, map coloring, register allocation in compilers, and more.

3.1 DEFINITIONS AND CONCEPTS:

1. **Graph**: A set of vertices V and edges E connecting pairs of vertices.
2. **Vertex Coloring**: Assigning colors to vertices such that no two adjacent vertices have the same color.
3. **Chromatic Number**: The smallest number of colors needed to color a graph G.

3.2 GRAPH COLORING PROBLEM:

Given a graph G, assign a color to each vertex such that no two adjacent vertices have the same color and minimize the number of colors used.

3.3 APPLYING GRAPH THEORY FOR THE NSP:

To address the complexities of scheduling in a hospital setting, Graph Theory has proven to be a valuable tool. In particular, the use of graph coloring techniques can help avoid conflicts and allocate resources efficiently. In this specific nurse scheduling problem, several constraints must be considered:

1. Nurses with varying skill levels can be assigned to the same shift, but shifts cannot consist solely of junior or senior nurses.
2. Certain nurses have personal preferences or conflicts that prevent them from working together, necessitating their placement in different groups.
3. Hospital management may have specific requirements to ensure a high-quality roster, which may include separating certain nurses across different shifts.
4. Patient preferences also play a role, as they might request that specific nurses be assigned to different shifts.

For this Nurse Scheduling Problem (NSP), a ward twelve nurses have been considered, designated as A1, A2, A3, ..., A12. Based on seniority, the most experienced three nurses are grouped together as Group 1, while the least experienced three are placed in Group 2. Considering the various requirements and constraints from nurses, hospital management, and patients, the nurses are grouped as shown in the following table 1:

Groups	Skills	Nurses
GROUP 1	Most experienced (senior)	A_1, A_2, A_3
GROUP 2	Junior	A_4, A_5, A_6
GROUP 3	Hospital Management	A_1, A_{10}, A_{11}
GROUP 4	Patients Requirement	A_1, A_5, A_7

Table 1

Using the data described, a matrix was constructed for the nurses, resulting in a 12 x 12 grid. In this matrix, the rows and columns represent the nurses, denoted as i and j respectively. The entry in the ij^{th} position is determined by their group assignments as outlined in the previous table.

If two nurses are in the same group, the ij^{th} entry is set to '1'; if they are in different groups, the entry is '0'. The following figure illustrates the resulting matrix:

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
A_1	0	1	1	0	1	0	1	0	0	1	1	0
A_2	1	0	1	0	0	0	0	0	0	0	0	0
A_3	1	1	0	0	0	0	0	0	0	0	0	0
A_4	0	0	0	0	1	1	0	0	0	0	0	0
A_5	0	0	0	1	0	1	0	0	0	0	0	0
A_6	0	0	0	1	1	0	0	0	0	0	0	0
A_7	1	0	0	0	1	0	0	0	0	0	0	0
A_8	0	0	0	0	0	0	0	0	0	0	0	0
A_9	0	0	0	0	0	0	0	0	0	0	0	0
A_{10}	1	0	0	0	0	0	0	0	0	0	1	0
A_{11}	1	0	0	0	0	0	0	0	0	1	0	0
A_{12}	0	0	0	0	0	0	0	0	0	0	0	0
12x12 adjacency matrix												

Figure 1. Resulting matrix

Using the matrix described earlier, a graph was constructed with nurses represented as vertices. If two nurses belong to the same group according to the given table, their corresponding vertices are connected by edges. In graph theory, this matrix is referred to as an adjacency matrix. It is n×n matrix where the ij^{th} entry indicates the presence of an edge between vertex i and vertex j.

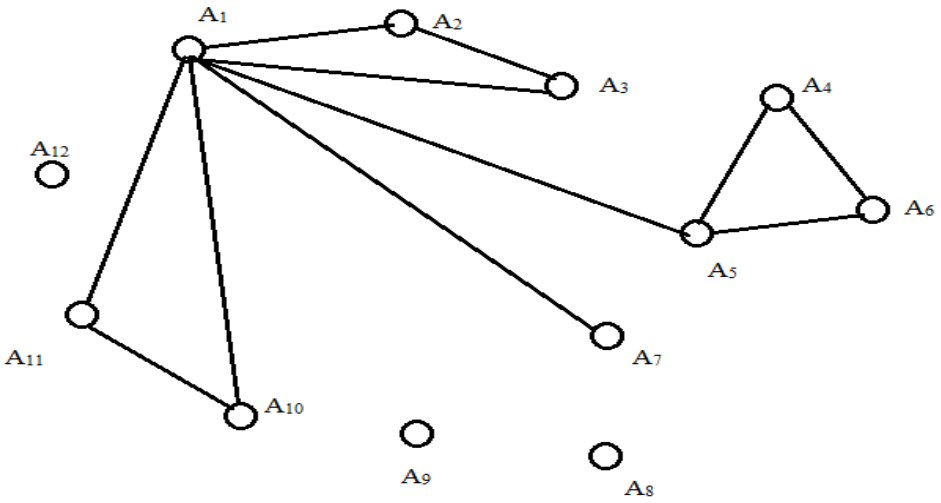

Figure 2. Graph with adjacency matrix

4. WELCH POWELL ALGORITHM

The Welch-Powell algorithm is a graph coloring algorithm that assigns colors to the vertices of a graph such that no two adjacent vertices share the same color. This method is particularly useful for scheduling problems, such as assigning shifts to nurses, where no two adjacent tasks can occur simultaneously. Here's a step-by-step explanation of the Welch-Powell algorithm:

4.1 WELCH-POWELL ALGORITHM STEPS:

1. **INPUT GRAPH AND VERTICES**:
   - Given a graph G with vertices V and edges E.
2. **DEGREE CALCULATION**:
   - Calculate the degree (the number of edges connected) of each vertex.
3. **SORT VERTICES**:
   - Sort the vertices in descending order based on their degree. This ensures that the vertex with the highest degree (most connections) is considered first.
4. **INITIALIZE COLORS**:
   - Start with the first color.
5. **COLOR ASSIGNMENT**:
   - Assign the first color to the highest-degree vertex and mark it as colored.
   - Move to the next highest-degree vertex:
   - Assign it the lowest-numbered color that hasn't been used by its adjacent vertices.
   - Continue this process for all vertices, ensuring that no two adjacent vertices share the same color.
6. **REPEAT FOR ALL VERTICES**:
   - If a vertex cannot be colored with the current set of colors (because all adjacent vertices have different colors), introduce a new color.
   - Repeat the process until all vertices are colored.

4.2 SOLUTION OF PROBLEM USING THE WELCH-POWELL ALGORITHM:

Consider a graph G with vertices V= {$A_1, A_2, \ldots\ldots A_{12}$,} and edges E= {($A_1, A_2$,), ($A_1, A_3$), ($A_2, A_1$), ($A_2, A_3$), ($A_1, A_5$), ($A_1, A_7$), ($A_1, A_{10}$), ($A_1, A_{11}$), ($A_4, A_5$), (A_5, A_6), ($A_4 A_6$),}.

1. **CALCULATED DEGREES**:
   - A_1: 6
   - A_2: 2
   - A_3: 2
   - A_4: 2
   - A_5: 3
   - A_6: 2
   - A_7: 1
   - A_8: 0
   - A_9: 0
   - A_{10}: 2

- A_{11}: 2
- A_{12}: 0

2. SORT VERTICES BY DEGREE: $A_1, A_5, A_2, A_3, A_4, A_6, A_{10}, A_{11}, A_7, A_8, A_9, A_{12},$

3. INITIALIZE COLORS:

- Start with color 1.

4. ASSIGN COLORS:

- Assign color 1 to vertex A_1,.
- Next, assign color 1 to the highest-degree vertex A_4, that is not adjacent to A_1: A_5 cannot have color 1 (it is adjacent to A_1,), so assign color 2.
- A_6 cannot have color 2 (it's adjacent to A_5,), so assign color 3.
- A_2 cannot have color 1 (it's adjacent to A_1,), so assign color 2.
- A_3 cannot have color 1 0r 2 (it is adjacent to A_1 and A_2), so assign color 3.
- A_7 cannot have color 1 (it's adjacent to A_1,), so assign color 2
- A_{10} cannot have color 1 (it's adjacent to A_1,), so assign color 2.
- A_{11} cannot have color 1 0r 2 (it is adjacent to A_1 and A_{10}), so assign color 3.
- Now Assign color 4-to-zero-degree vertex i.e., A_8, A_9, A_{12}.

4.3. FINAL COLORS:

- A_1, A_4 : 1 (Red)
- A_2, A_5, A_7, A_{10} : 2 (Blue)
- A_3, A_6, A_{11} : 3 (Green)
- A_8, A_9, A_{12} : 4 (Yellow)

The vertices in the graph were colored. Adjacent vertices were assigned different colors to ensure that no two connected vertices shared the same color. This process helps in visualizing and solving the scheduling problem more effectively. The following figure illustrates the resulting graph with the vertices and edges appropriately colored.

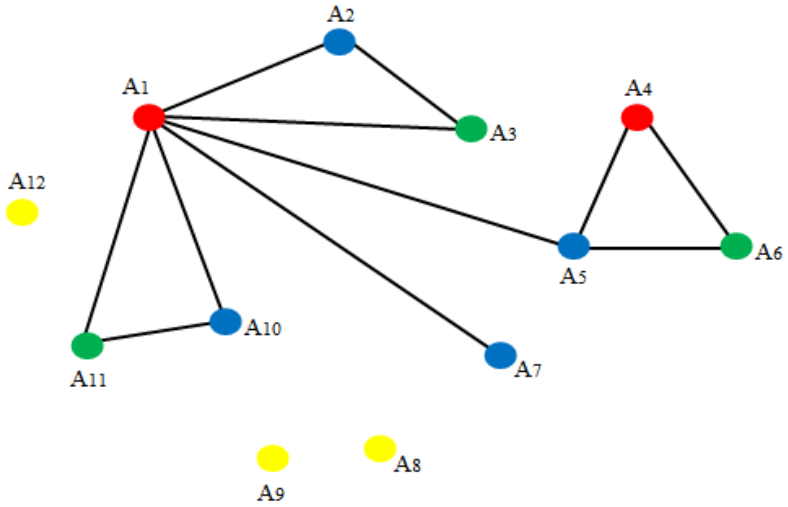

Figure 3. Resulting colored graph with the vertices and edges

After coloring the vertices, the nurses were regrouped based on their assigned colors. Nurses sharing the same color were placed into the same group. Typically, in graph coloring, if a vertex is not adjacent to any other vertex (for example, A_8, A_9, A_{12}), it is assigned an existing color already used in the graph. However, in this study, such vertices were assigned a new color and placed into a separate group called the "bench group" (D). Table 2 below details the group assignments following the application of graph coloring techniques:

Groups	Nurses
A	A_1, A_4
B	A_2, A_5, A_7, A_{11}
C	A_3, A_6, A_{11}
D	A_8, A_9, A_{12}

Table 2

With the help of this table we can schedule nurse roster and divide it into three shifts morning, evening and night.

5. CONCLUSION

The Nurse Scheduling Problem (NSP) is a real-world challenge characterized by its complexity and numerous constraints. In this study, while not all soft constraints were addressed, all hard constraints were rigorously considered. Consequently, the results present a feasible solution to the problem, demonstrating the practical applicability of this approach. By employing graph coloring techniques, and using Welch Powell algorithm for graph coloring the complexity of the NSP was significantly reduced. The Welch-Powell algorithm provides an efficient way to handle graph coloring, especially in scheduling and resource allocation problems, ensuring that adjacent tasks do not overlap. This method proves effective in generating an improved solution for such a complex scheduling issue. Therefore, applying graph coloring to problems like the NSP can yield better, more manageable outcomes.

6. FUTURE WORK

Many hospitals take a long time to prepare nurses roster, which is fair to everybody. Instead of consuming time in generating it, we hope to generalize in future work a software, where minimum data is required to have a roster for any month.

CONFLICT OF INTERESTS

None.

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