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AN INVENTORY MODEL FOR DETERIORATING ITEMS IN FUZZY ENVIRONMENT FOR JOINT PRICING AND INVENTORY CONTROL WITH PARTIAL BACKLOGGING AND TIME AND PRICE DEPENDENT DEMAND

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ABSTRACT

This study proposes an advanced inventory model that addresses the challenges associated with deteriorating items in a fuzzy environment. The model integrates joint pricing and inventory control while considering partial backlogging and time- and price-dependent demand. The deteriorating items are characterized by their decreasing value over time, and the fuzzy environment accounts for the inherent uncertainty in the real business world. The model recognizes the interdependence of pricing and inventory decisions, acknowledging that pricing strategies can influence inventory levels and vice versa. The model incorporates a partial backlog to accommodate customers who are prepared to wait for out-of-stock items, thereby minimizing lost sales and enhancing customer satisfaction. The model's application has the potential to enhance operational efficiency, improve customer satisfaction, and maximize profitability for businesses operating in diverse industries. A trapezoidal fuzzy number is assigned to the cost parameter and defuzzyfy by applying the graded mean representation method. A numerical example has been considered to illustrate the model, and the significant features of the results are discussed. Finally, based on these examples, sensitivity analyses have been studied by taking one parameter at a time and keeping the other parameters the same.

Keywords: Price and Time Dependent Demand, Partial Backlogging, Trapezoidal Fuzzy Number, Graded Mean Representation Method, Deterioration

1. INTRODUCTION

Effective inventory management is crucial for businesses operating in dynamic and uncertain environments. Among the various factors that affect inventory control, demand patterns, item deterioration, and pricing dynamics play significant roles. Traditional inventory models often assume constant demand, neglecting the impact of price changes and item deterioration, leading to suboptimal decisions. To address these limitations, a fuzzy inventory model that incorporates price and time-dependent demand for deteriorating items has emerged as a powerful approach for better inventory control. The fuzzy inventory model embraces the inherent uncertainty associated with demand, recognizing that customer preferences and market conditions can vary over time. Moreover, the model accounts for the deterioration of items, which is particularly relevant for perishable or time-sensitive goods, such as food, pharmaceuticals, or electronic components. By considering both price and time-dependent demand, the fuzzy inventory model enables businesses to make more accurate and informed decisions regarding inventory replenishment and pricing strategies. In the fuzzy inventory model, demand uncertainty is represented using fuzzy logic, allowing for a more flexible and realistic characterization of customer preferences. Fuzzy logic enables the modeling of vague and imprecise information, capturing the fuzzy nature of demand patterns in a more nuanced way. By incorporating fuzzy logic, the model can handle fluctuating demand, seasonality, and market dynamics more effectively, leading to improved inventory management outcomes. Furthermore, the inclusion of price dynamics in the fuzzy inventory model recognizes the impact of pricing decisions on customer behavior. Price changes can significantly influence demand patterns, with lower prices potentially stimulating higher demand and vice versa. By considering price as a variable that affects demand, the fuzzy inventory model allows businesses to optimize both inventory levels and pricing strategies simultaneously, maximizing profitability and customer satisfaction. Md. Akhtar et. al. [1], introduced an inventory model for non-instantaneous deteriorating items with price and time dependent demand over finite time horizon. Sadaf FATIMA et. al. [2], gives an inventory model for instantaneous deteriorating items with time sensitive demand for post covid-19 recovery. Priti Chaudhary and Tanuj Kumar, [3], provided an Intuitionistic fuzzy inventory model with quadratic demand rate, time-dependent holding cost and shortages. An investigation done by Preety Poswal et. al. [4], on of fuzzy EOQ model for price sensitive and stock dependent demand under shortages. An study on lock fuzzy EPQ model with deterioration and stock and unit selling price-dependent demand using preservation technology, done by Mustafijur Rahman et. al. [5]. Sujata Saha [6], introduced a fuzzy inventory model for deteriorating items in a supply chain system with price dependent demand and without backorder. An inventory model developed by Md. Anwar Hossen, et. al. [7], with price and time dependent demand with fuzzy valued inventory costs under inflation. Saha, S., & Chakrabarti, T. (2017). Fuzzy inventory model for deteriorating items in a supply chain system with price dependent demand and without backorder Maragatham, M., & Lakshmidevi, P. K. [9], A fuzzy inventory model for deteriorating items with price dependent demand. Maihami, R., & Kamalabadi, I. N. [10]. Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand.

Overall, the fuzzy inventory model for deteriorating items with price and time-dependent demand represents an advancement in inventory management theory and practice. By integrating fuzzy logic, it captures the uncertainty in demand patterns, while accounting for item deterioration and pricing dynamics. As businesses face increasingly complex and competitive environments, the adoption of this model can enhance their ability to make informed decisions, optimize inventory control, and ultimately achieve higher profitability and customer satisfaction.

2. PRELIMINARIES

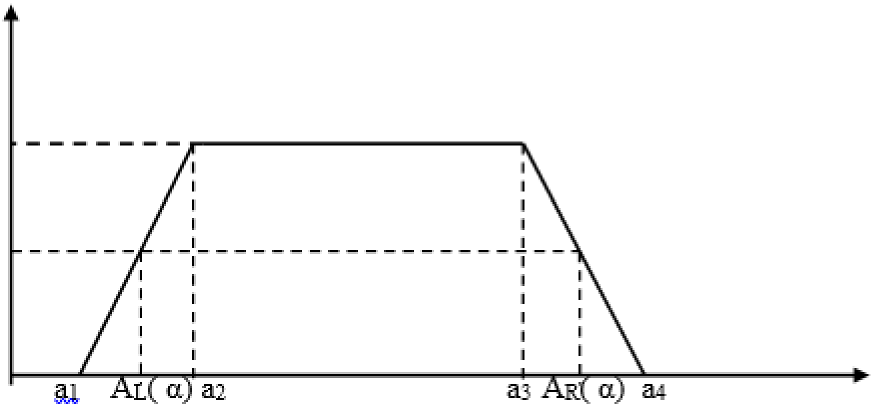
a. **DEFINITION:** A fuzzy set is a mathematical concept that extends the notion of a crisp set by allowing elements to have varying degrees of membership. In a crisp set, an element either belongs to the set or does not. However, in a fuzzy set, an element can have a degree of membership ranging between 0 and 1, representing the extent to which it belongs to the set.

b. **DEFINITION***:* A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is represented with membership function $\mu_{\tilde{a}}$ as:

$$\mu_{\tilde{a}}(x) = \begin{cases} L(x) = \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ R(x) = \dfrac{a_4 - x}{a_2 - a_1}, & a_3 \le x \le a_4 \\ 0, & otherwise \end{cases}$$

c. **The α-cut** of $\tilde{A} = (a_1, a_2, a_3, a_4)$, $0 \le \alpha \le 1$ is A(α) = [$A_L(\alpha)$, $A_R(\alpha)$],

Where $A_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $A_R(\alpha) = a_4 - (a_4 - a_3)\alpha$ are the left and right end points of A(α).



α-Cut of a Trapezoidal Fuzzy Number

d. **DEFINITION:** If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number then the *graded mean representation* of $\tilde{a}$ is defined as

$$P(\tilde{A}) = \frac{\int_0^w \frac{L^{-1}(h) + R^{-1}(h)}{2} dh}{\int_0^w h dh} \quad \text{with } 0 < h \le w_A \text{ and } 0 < w_A \le 1$$

$$P(\tilde{A}) = \frac{\frac{1}{2}\int_0^1 h[a_1 + h(a_2 - a_1) + a_4 - h(a_4 - a_3)]dh}{\int_0^1 h dh} = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

3. NOTATIONS AND ASSUMPTIONS

c purchase cost per unit
C_h holding cost per unit per unit time
Cs the backordering cost per unit per unit time
Co the cost of lost sale per unit
C_d the cost of deterioration per unit
p p the selling price per unit, where p>c
θ the parameter of deterioration rate of the stock
t_1 the length of time in which there is no inventory shortage
T the length of replenishment cycle time
Q the order quantity
p^* the optimal selling price per unit
t^* the optimal length of time in which there is no inventory shortage
T^* the optimal length of the replenishment cycle time
Q^* the optimal order quantity
$I_1(t)$ the inventory level at time $t \in [0,t_1]$
$I_2(t)$ the inventory level at time $t \in [t_2,T]$
q_1 the maximum inventory level
q_2 the maximum amount of demand backlogged
$Z(p,t_1,T)$ the total profit per unit time of the inventory system
$Z^*(p^*,t^*_1,T^*)$ the optimal total profit per unit time of the inventory system

ASSUMPTIONS:

- A single instantaneous deteriorating item is assumed.
- The replenishment rate is infinite and lead time is zero.
- The time horizon is finite and length of one cycle is assumed to be unity.
- Shortages are allowed. We adopt the notation used in Abad (1996) where the unsatisfied demand is backlogged, and the fraction of shortage backordered is $S(x)=k_0 e^{\delta t},(0<k_0\le 1,\delta>0)$, where x is the waiting time up to the next replenishment and δ is a positive constant and $0\le S(x)\le 1,\ S(0)=1$. To guarantee the existence of an optimal solution, we assume that $S(x)+H(S^{'}(x))>0$, where $S^{'}(x)$ is the first derivative of $S(x)$. Note that if $S(x)=1(or\,0)$ for all *x*, then shortage is completely backlogged.
- The basic demand rate, $D(p,t)=(\alpha-\beta p)e^{\lambda t}$ (where *α>0, β>0*) is a linearly decreasing function of the price and decreases (increases) exponentially with time when λ<0 (λ >0). Thus, the demand rate is a function of price and time, which should reflect a real situation: i.e. the demand may increase when the price decreases, or it may vary through time. Here we adopt the form of the multiplicative exponential time effect for the basic demand rate, which is suitable for describing the time-varying demand. Given a different λ, which can be either positive or negative, this form can represent most cases where demand rate varies with time. The consideration of the time and price dependent demand is useful for the deteriorate items such as: fashion goods, high-tech product, fruits and vegetables (Tsao et al., 2008).

4. MATHEMATICAL FORMULATION

CRISP MODEL

We assume that the length of time in which there is no shortage is larger than or equal to the length of time of shortages. During the time interval [*0, t_1*], the inventory level decreases due to both demand and deterioration during the time interval [0, t_1]. Finally, a shortage occurs due to demand and partial backlogging during the time interval [t_1, *T*].

$$\frac{dI_1(t)}{dt}+\theta I(t)=-D(p,t),\qquad 0\le t\le t_1 \qquad \text{... (1)}$$

with initial condition $I_1(0)=q_1$ and boundary conditions: $I_1(t_1)=0$

solution of equation (1) is given by

$$I_1(t)e^{\theta t}=-\frac{(\alpha-\beta p)}{(\theta+\lambda)}e^{(\theta+\lambda)t_1}+K_1 \qquad \text{... (2)}$$

Using, $I(0)=q_1$, we obtain $K_1=q_1+\dfrac{(\alpha-\beta p)}{(\theta+\lambda)}$,

The equation (3), becomes

$$I_1(t)=q_1e^{-\theta t}+\frac{(\alpha-\beta p)}{(\theta+\lambda)}e^{-\theta t}+\frac{(\alpha-\beta p)}{(\theta+\lambda)}e^{\lambda t} \qquad \text{... (3)}$$

By applying the boundary condition $I(t_1)=0$, we get

$$q_1=\frac{(\alpha-\beta p)}{(\theta+\lambda)}(e^{(\theta+\lambda)t_1}-1) \qquad \text{... (4)}$$

Equation (4), becomes

$$I_1(t)=\frac{(\alpha-\beta p)}{(\theta+\lambda)}(e^{\theta(t_1-t)}e^{\lambda t_1}-e^{\lambda t}) \qquad 0\le t\le t_1 \qquad \text{... (5)}$$

The unsatisfied demand is backlogged at the rate $e^{-\delta(T-t)},\delta>0$ and waiting time upto the next replenishment *(T-t)* and $t\in[t_1,T]$

$$\frac{dI_2(t)}{dt}=-(\alpha-\beta p)e^{\lambda t}e^{-\delta(T-t)},\qquad t_1\le t\le T \qquad \text{... (6)}$$

with initial condition s and boundary conditions: $-I_2(T) = q_2$

solution of (7), given by

$$I_2(t) = \frac{(\alpha - \beta p)}{(\lambda + \delta)} e^{-\delta T} [e^{(\lambda+\delta)t_1} - e^{(\lambda+\delta)t}], \qquad t_1 \le t \le T \qquad \text{... (7)}$$

Maximum backordered quantity

$$q_2 = -I_2(T),$$

$$q_2 = \frac{(\alpha - \beta p)}{(\lambda + \delta)} e^{-\delta T} [e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1}] \qquad \text{...(8)}$$

Total quantity ordered per cycle

$$Q = q_1 + q_2$$

$$Q = \frac{(\alpha - \beta p)}{(\theta + \lambda)} (e^{(\theta+\lambda)t_1} - 1) + \frac{(\alpha - \beta p)}{(\lambda + \delta)} e^{-\delta T} [e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1}] \qquad \text{...(9)}$$

Now, we can calculate inventory costs and sales revenue per cycle that consists following cost components:

1. *A:* the ordering cost.
2. *HC:* the inventory holding cost

$$HC = C_h \int_0^{t_1} I_1(t) dt$$

$$= \frac{C_h(\alpha - \beta p)}{\theta\lambda(\theta + \lambda)} \left\{ \lambda e^{(\theta+\lambda)t_1} - (\theta + \lambda) e^{\lambda t_1} + \theta \right\} \qquad \text{... (10)}$$

3. *DC:* the Deterioration cost

$$DC = C_d \left\{ Q - \int_0^{t_1} D(p,t) dt - \int_{t_1}^{T} e^{-\delta(T-t)} D(p,t) dt \right\}$$

$$= \frac{C_d(\alpha - \beta p)}{\lambda(\theta + \lambda)} \left\{ \lambda e^{(\theta+\lambda)t_1} - \lambda - (\theta + \lambda)(e^{\lambda t_1} - 1) \right\} \qquad \text{... (11)}$$

4. *SC:* the shortage cost due to backlog

$$SC = \int_{t_1}^{T} -I_2(t) dt$$

$$SC = \frac{C_s(\alpha - \beta p)}{(\lambda + \delta)^2} e^{-\delta T} \left\{ e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1} - (\lambda + \delta)(T - t_1) e^{(\lambda+\delta)t_1} \right\} \qquad \text{... (12)}$$

5. The Opportunity cost due lost sale

$$OC = C_o \int_{t_1}^{T} D(p,t)(1 - e^{-\delta(T-t)}) dt$$

$$OC = \frac{C_o(\alpha - \beta p)}{\lambda(\lambda + \delta)} \left\{ (\lambda + \delta)(e^{\lambda T} - e^{\lambda t_1}) - \lambda e^{-\delta T} (e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1}) \right\} \qquad \text{...(13)}$$

6. *PC:* the purchase cost

$$PC = cQ = c(q_1 + q_2)$$

$$PC = c \left\{ \frac{(\alpha - \beta p)}{(\theta + \lambda)} (e^{(\theta+\lambda)t_1} - 1) + \frac{(\alpha - \beta p)}{(\lambda + \delta)} e^{-\delta T} (e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1}) \right\} \qquad \text{... (14)}$$

7. *SR:* the sales revenue

$$SR = p\left\{\int_0^{t_1} D(p,t)dt + q_2\right\}$$

$$= p\left\{\frac{(\alpha-\beta p)}{\lambda}(e^{\lambda t_1}-1) + \frac{(\alpha-\beta p)}{(\lambda+\delta)}e^{-\delta T}(e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1})\right\} \quad \text{... (15)}$$

Therefoe, the total profit per unit time is given by

$$Z(p,t_1,T) = \frac{SR - A - HC - DC - SC - OC - PC}{T}$$

$$= -\frac{1}{T}\left[\begin{array}{l} A + \frac{C_h(\alpha-\beta p)}{\theta\lambda(\theta+\lambda)}\left\{\lambda e^{(\theta+\lambda)t_1} - (\theta+\lambda)e^{\lambda t_1} + \theta\right\} + \\ \frac{C_d(\alpha-\beta p)}{\lambda(\theta+\lambda)}\left\{\lambda e^{(\theta+\lambda)t_1} - \lambda - (\theta+\lambda)(e^{\lambda t_1}-1)\right\} \\ + \frac{C_s(\alpha-\beta p)}{(\lambda+\delta)^2}e^{-\delta T}\left\{e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1} - (\lambda+\delta)(T-t_1)e^{(\lambda+\delta)t_1}\right\} \\ + \frac{C_o(\alpha-\beta p)}{\lambda(\lambda+\delta)}\left\{(\lambda+\delta)(e^{\lambda T} - e^{\lambda t_1}) - \lambda e^{-\delta T}(e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1})\right\} \\ + c\left\{\frac{(\alpha-\beta p)}{(\theta+\lambda)}(e^{(\theta+\lambda)t_1}-1) + \frac{(\alpha-\beta p)}{(\lambda+\delta)}e^{-\delta T}(e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1})\right\} \\ - p\left\{\frac{(\alpha-\beta p)}{\lambda}(e^{\lambda t_1}-1) + \frac{(\alpha-\beta p)}{(\lambda+\delta)}e^{-\delta T}(e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1})\right\} \end{array}\right] \quad \text{...(16)}$$

The objective of this paper is to maximizing the total profit per unit time. To determine the optimal an ordering policies to maximizing the total profit per unit time. For this purpose, first we find optimal solution of *(t_1, T)* not only exists but also is unique for any given p. Then, for any given value of t_1 and T, there exists a unique p that maximize the total profit per unit time.

$Z(p,t_1,T)$ is function of t_1, T and p. So, any value of p, the necessary conditions for the total profit per unit time (16) to be maximize are $\frac{\partial Z(p,t_1,T)}{\partial t_1}$=0, and $\frac{\partial Z(p,t_1,T)}{\partial T} = 0$, simultaneously. *i.e.*

$$\frac{\partial Z(p,t_1,T)}{\partial t_1} = -\frac{1}{T}\left[\begin{array}{l} c\mathrm{e}^{t_1(\theta+\lambda)}(\alpha-p\beta) - c\mathrm{e}^{-T\delta+t_1(\delta+\lambda)}(\alpha-p\beta) - \mathrm{e}^{t_1\lambda}p(\alpha-p\beta) \\ +\mathrm{e}^{-T\delta+t_1(\delta+\lambda)}p(\alpha-p\beta) \\ +\frac{(\alpha-p\beta)\left(-\mathrm{e}^{t_1\lambda}\lambda(\theta+\lambda)+\mathrm{e}^{t_1(\theta+\lambda)}\lambda(\theta+\lambda)\right)C_d}{\lambda(\theta+\lambda)} \\ +\frac{(\alpha-p\beta)\left(-\mathrm{e}^{t_1\lambda}\lambda(\theta+\lambda)+\mathrm{e}^{t_1(\theta+\lambda)}\lambda(\theta+\lambda)\right)C_h}{\theta\lambda(\theta+\lambda)} \\ +\frac{(\alpha-p\beta)\left(-\mathrm{e}^{t_1\lambda}\lambda(\delta+\lambda)+\mathrm{e}^{-T\delta+t_1(\delta+\lambda)}\lambda(\delta+\lambda)\right)C_o}{\lambda(\delta+\lambda)} \\ -\mathrm{e}^{-T\delta+t_1(\delta+\lambda)}(-t_1+T)(\alpha-p\beta)C_s \end{array}\right] = 0 \quad \text{... (17)}$$

and

$$\frac{\partial Z(p,t_1,T)}{\partial T}=\left[\begin{array}{l}\frac{1}{T^2}\left(\begin{array}{l}A-\frac{\left(-1+e^{t_1\lambda}\right)p(\alpha-p\beta)}{\lambda}+\frac{ce^{-T\delta}\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)(\alpha-p\beta)}{\delta+\lambda}\\ -\frac{e^{-T\delta}\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)p(\alpha-p\beta)}{\delta+\lambda}+\frac{c\left(-1+e^{t_1(\theta+\lambda)}\right)(\alpha-p\beta)}{\theta+\lambda}\\ +\frac{(\alpha-p\beta)\left(-\lambda+e^{t_1(\theta+\lambda)}\lambda-\left(-1+e^{t_1\lambda}\right)(\theta+\lambda)\right)C_d}{\lambda(\theta+\lambda)}\\ +\frac{(\alpha-p\beta)\left(\theta+e^{t_1(\theta+\lambda)}\lambda-e^{t_1\lambda}(\theta+\lambda)\right)C_h}{\theta\lambda(\theta+\lambda)}\\ +\frac{(\alpha-p\beta)\left(-e^{-T\delta}\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)\lambda+\left(-e^{t_1\lambda}+e^{T\lambda}\right)(\delta+\lambda)\right)C_o}{\lambda(\delta+\lambda)}\\ +\frac{e^{-T\delta}(\alpha-p\beta)\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}-e^{t_1(\delta+\lambda)}(-t_1+T)(\delta+\lambda)\right)C_s}{(\delta+\lambda)^2}\end{array}\right)\\ -\frac{1}{T}\left(\begin{array}{l}ce^{-T\delta+T(\delta+\lambda)}(\alpha-p\beta)-e^{-T\delta+T(\delta+\lambda)}p(\alpha-p\beta)\\ -\frac{ce^{-T\delta}\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)(\alpha-p\beta)\delta}{\delta+\lambda}\\ +\frac{e^{-T\delta}\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)p(\alpha-p\beta)\delta}{\delta+\lambda}\\ +\frac{(\alpha-p\beta)\left(\begin{array}{l}e^{-T\delta}\left(-e^{t(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)\delta\lambda\\ +e^{T\lambda}\lambda(\delta+\lambda)-e^{-T\delta+T(\delta+\lambda)}\lambda(\delta+\lambda)\end{array}\right)C_o}{\lambda(\delta+\lambda)}\\ +\frac{e^{-T\delta}(\alpha-p\beta)\left(-e^{t_1(\delta+\lambda)}(\delta+\lambda)+e^{T(\delta+\lambda)}(\delta+\lambda)\right)C_s}{(\delta+\lambda)^2}\\ -\frac{e^{-T\delta}(\alpha-p\beta)\delta\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}-e^{t_1(\delta+\lambda)}(-t_1+T)(\delta+\lambda)\right)C_s}{(\delta+\lambda)^2}\end{array}\right)\end{array}\right]=0 \quad ...(18)$$

Further, for the total profit function $Z(p,t_1,T)$ to be convex, the following condition must be satisfied

$$\frac{\partial Z^2(p,t_1,T)}{\partial t_1^2}>0,\frac{\partial Z^2(p,t_1,T)}{\partial T^2}>0$$

and

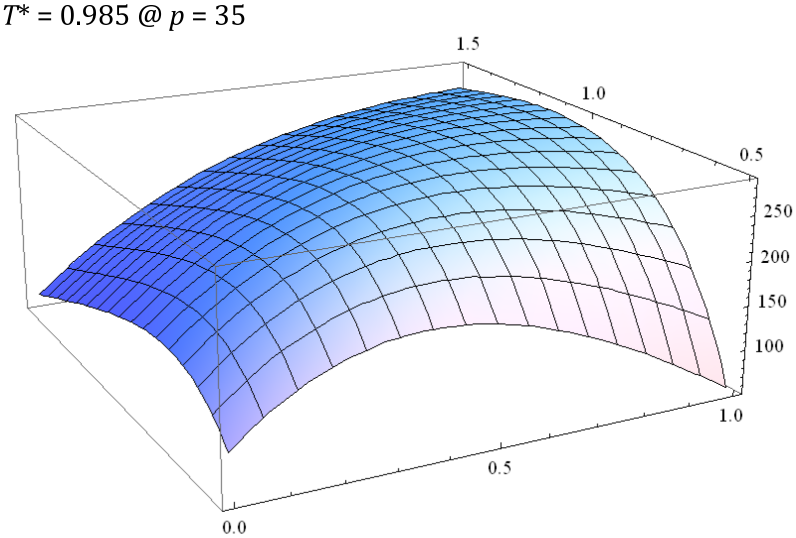
$$\left(\frac{\partial Z^2(p,t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial Z^2(p,t_1,T)}{\partial T^2}\right)-\left(\frac{\partial Z^2(p,t_1,T)}{\partial t_1\partial T}\right)>0$$

Now, we study the condition under which the optimal selling price exists and is unique. For any Z*, and t_1, the first order necessary condition for $Z(p,t_1^*,T^*)$ to be maximize is $\frac{\partial Z(p,t_1^*,T^*)}{\partial p}=0$ *i.e.*

$$\frac{\partial Z(p,t_1^*,T^*)}{\partial p}=-\frac{1}{T}\left[\begin{array}{l}\frac{\left(-1+e^{t_1\lambda}\right)p\beta}{\lambda}-\frac{\left(-1+e^{t_1\lambda}\right)(\alpha-p\beta)}{\lambda}-\frac{ce^{-T\delta}\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)\beta}{\delta+\lambda}\\ +\frac{e^{-T\delta}\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)p\beta}{\delta+\lambda}-\frac{e^{-T\delta}\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)(\alpha-p\beta)}{\delta+\lambda}\\ -\frac{c\left(-1+e^{t_1(\theta+\lambda)}\right)\beta}{\theta+\lambda}-\frac{\beta\left(-\lambda+e^{t_1(\theta+\lambda)}\lambda-\left(-1+e^{t_1\lambda}\right)(\theta+\lambda)\right)C_d}{\lambda(\theta+\lambda)}\\ -\frac{\beta\left(\theta+e^{t_1(\theta+\lambda)}\lambda-e^{t_1\lambda}(\theta+\lambda)\right)C_h}{\theta\lambda(\theta+\lambda)}\\ -\frac{\beta\left(-e^{-T\delta}\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}\right)\lambda+\left(-e^{t_1\lambda}+e^{T\lambda}\right)(\delta+\lambda)\right)C_o}{\lambda(\delta+\lambda)}\\ --\frac{e^{-T\delta}\beta\left(-e^{t_1(\delta+\lambda)}+e^{T(\delta+\lambda)}-e^{t_1(\delta+\lambda)}(-t_1+T)(\delta+\lambda)\right)C_s}{(\delta+\lambda)^2}\end{array}\right]=0 \quad ...(19)$$

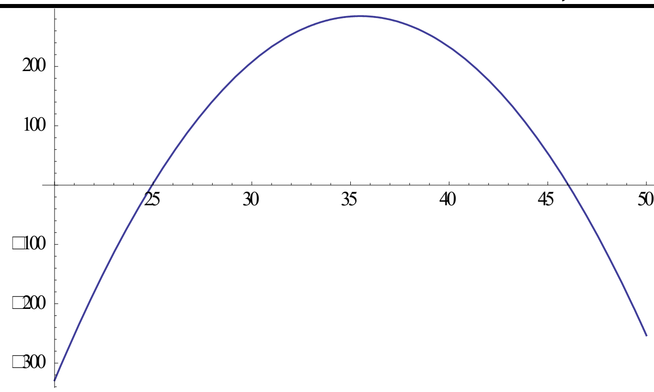
$$\frac{\partial^2 Z(p,t_1^*,T^*)}{\partial p^2}=-\frac{1}{T^*}\left[-\frac{2\left(1-e^{t_1^*\lambda}\right)\beta}{\lambda}+\frac{2e^{-T^*\delta}\left(-e^{t_1^*(\delta+\lambda)}+e^{T^*(\delta+\lambda)}\right)\beta}{\delta+\lambda}\right]<0$$

Example: 1. *(crisp model)*
This example is based on the following parameters
A=\$250/per order, c=\$20/per unit, Ch=\$1/per unit time, Cs=\$5/per unit per unit time, Co=\$25/per unit, θ = 0.08, α = 200, β = 4, λ = -0.98, δ = 0.1.
First we assume the value of p = 35 and optimized value of t_1 and T
Z* = 284.9, t^* = 0.7296 and T^* = 0.985 @ p = 35



Graphical representation of $Z(p,t_1^*,T^*)$

Now we used t^* = 0.732 and T* = 0.987 for optimize the value of p and we get
P^* = 35.4927



Graphical representation of $Z(p^*, t_1^*, T^*)$

FUZZY MODEL

Cost can not defined precisely in real world *i.e.* in some situations vaguely defined. So, to handle this vagueness cost components viz. c, C_h, C_d, C_s, C_o, and p are assumed as fuzzy number. Trapezoidal fuzzy number introduced for the parameters as below

$$\tilde{c} = (c_1, c_2, c_3, c_4),\ \tilde{C}_h = (C_{h1}, C_{h2}, C_{h3}, C_{h4}),\ \tilde{C}_d = (C_{d1}, C_{d2}, C_{d3}, C_{d4})$$

$$\tilde{C}_s = (C_{s1}, C_{s2}, C_{s3}, C_{s4}),\ \tilde{C}_o = (C_{o1}, C_{o2}, C_{o3}, C_{o4}),\ and\ \tilde{p} = (p_1, p_2, p_3, p_4)$$

$$\tilde{A} = (a_1, a_2, a_3, a_4)$$

Now, total profit obtained in equation (16) in fuzzy sense expressed as

$$\tilde{Z}(p, t_1, T) = -\frac{1}{T}\left[\begin{array}{l} \tilde{A} + \dfrac{\tilde{C}_h(\alpha - \beta\tilde{p})}{\theta\lambda(\theta+\lambda)}\left\{\lambda e^{(\theta+\lambda)t_1} - (\theta+\lambda)e^{\lambda t_1} + \theta\right\} + \\ \dfrac{\tilde{C}_d(\alpha - \beta\tilde{p})}{\lambda(\theta+\lambda)}\left\{\lambda e^{(\theta+\lambda)t_1} - \lambda - (\theta+\lambda)(e^{\lambda t_1} - 1)\right\} \\ + \dfrac{\tilde{C}_s(\alpha - \beta\tilde{p})}{(\lambda+\delta)^2} e^{-\delta T}\left\{e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1} - (\lambda+\delta)(T - t_1)e^{(\lambda+\delta)t_1}\right\} \\ + \dfrac{\tilde{C}_o(\alpha - \beta\tilde{p})}{\lambda(\lambda+\delta)}\left\{(\lambda+\delta)(e^{\lambda T} - e^{\lambda t_1}) - \lambda e^{-\delta T}(e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1})\right\} \\ + \tilde{c}\left\{\dfrac{(\alpha - \beta\tilde{p})}{(\theta+\lambda)}(e^{(\theta+\lambda)t_1} - 1) + \dfrac{(\alpha - \beta\tilde{p})}{(\lambda+\delta)} e^{-\delta T}(e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1})\right\} \\ - \tilde{p}\left\{\dfrac{(\alpha - \beta\tilde{p})}{\lambda}(e^{\lambda t_1} - 1) + \dfrac{(\alpha - \beta\tilde{p})}{(\lambda+\delta)} e^{-\delta T}(e^{(\lambda+\delta)T} - e^{(\lambda+\delta)t_1})\right\} \end{array}\right] \quad \ldots (20)$$

By applying the technique of graded mean integration representation method for defuzzification, we have

$$d(\tilde{Z}(p,t_1,T),0)=-\frac{1}{T}\begin{bmatrix} d(\tilde{A},0)+\dfrac{d(\tilde{C}_h,0)(\alpha-\beta d(\tilde{p},0))}{\theta\lambda(\theta+\lambda)}\left\{\lambda e^{(\theta+\lambda)t_1}-(\theta+\lambda)e^{\lambda t_1}+\theta\right\}+ \\ \dfrac{d(\tilde{C}_d,0)(\alpha-\beta d(\tilde{p},0))}{\lambda(\theta+\lambda)}\left\{\lambda e^{(\theta+\lambda)t_1}-\lambda-(\theta+\lambda)(e^{\lambda t_1}-1)\right\} \\ +\dfrac{d(\tilde{C}_s,0)(\alpha-\beta d(\tilde{p},0))}{(\lambda+\delta)^2}e^{-\delta T}\left\{e^{(\lambda+\delta)T}-e^{(\lambda+\delta)t_1}-(\lambda+\delta)(T-t_1)e^{(\lambda+\delta)t_1}\right\} \\ +\dfrac{d(\tilde{C}_o,0)(\alpha-\beta d(\tilde{p},0))}{\lambda(\lambda+\delta)}\left\{(\lambda+\delta)(e^{\lambda T}-e^{\lambda t_1})-\lambda e^{-\delta T}(e^{(\lambda+\delta)T}-e^{(\lambda+\delta)t_1})\right\} \\ +d(\tilde{c},0)\left\{\dfrac{(\alpha-\beta d(\tilde{p},0))}{(\theta+\lambda)}(e^{(\theta+\lambda)t_1}-1)+\dfrac{(\alpha-\beta d(\tilde{p},0))}{(\lambda+\delta)}e^{-\delta T}(e^{(\lambda+\delta)T}-e^{(\lambda+\delta)t_1})\right\} \\ -d(\tilde{p},0)\left\{\dfrac{(\alpha-\beta d(\tilde{p},0))}{\lambda}(e^{\lambda t_1}-1)+\dfrac{(\alpha-\beta d(\tilde{p},0))}{(\lambda+\delta)}e^{-\delta T}(e^{(\lambda+\delta)T}-e^{(\lambda+\delta)t_1})\right\} \end{bmatrix} \quad ...(21)$$

In equation (21), the values of expressions $d(\tilde{A},0)$, $d(\tilde{C}_h,0)$, $d(\tilde{p},0)$, $d(\tilde{C}_d,0)$, $d(\tilde{C}_s,0)$, $d(\tilde{C}_o,0)$, and $d(\tilde{c},0)$ are obtained by using the arithmetic operations on the trapezoidal fuzzy number and graded mean integration representation function as

$$d(\tilde{A},0)=\frac{a_1+2a_2+2a_3+a_4}{6},\qquad d(\tilde{C}_h,0)=\frac{C_{h1}+2C_{h2}+2C_{h3}+C_{h4}}{6},$$

$$d(\tilde{C}_s,0)=\frac{C_{s1}+2C_{s2}+2C_{s3}+C_{s4}}{6},\quad d(\tilde{C}_d,0)=\frac{C_{d1}+2C_{d2}+2C_{d3}+C_{d4}}{6},\ d(\tilde{c}_o,0)=\frac{C_{o1}+2C_{o2}+2C_{o3}+C_{o4}}{6},\quad d(\tilde{c},0)=\frac{c_1+2c_2+2c_3+c_4}{6},$$

Using all these values in equation (21), we get

$$
\tilde{Z}(p,t_1,T) = -\frac{1}{T}\left[\begin{array}{l}
\left(\frac{a_1+2a_2+2a_3+a_4}{6}\right)+\left(\frac{C_{h1}+2C_{h2}+2C_{h3}+C_{h4}}{6}\right) \\
\times\frac{\left(\alpha-\beta\left(\frac{p_1+2p_2+2p_3+p_4}{6}\right)\right)}{\theta\lambda(\theta+\lambda)}\left\{\lambda e^{(\theta+\lambda)t_1}-(\theta+\lambda)e^{\lambda t_1}+\theta\right\} \\
+\frac{\left(\frac{C_{d1}+2C_{d2}+2C_{d3}+C_{d4}}{6}\right)\left(\alpha-\beta\left(\frac{p_1+2p_2+2p_3+p_4}{6}\right)\right)}{\lambda(\theta+\lambda)} \\
\times\left\{\lambda e^{(\theta+\lambda)t_1}-\lambda-(\theta+\lambda)(e^{\lambda t_1}-1)\right\} \\
+\frac{\left(\frac{C_{s1}+2C_{s2}+2C_{s3}+C_{s4}}{6}\right)\left(\alpha-\beta\left(\frac{p_1+2p_2+2p_3+p_4}{6}\right)\right)}{(\lambda+\delta)^2} \\
\times e^{-\delta T}\left\{e^{(\lambda+\delta)T}-e^{(\lambda+\delta)t_1}-(\lambda+\delta)(T-t_1)e^{(\lambda+\delta)t_1}\right\} \\
+\frac{\left(\frac{C_{o1}+2C_{o2}+2C_{o3}+C_{o4}}{6}\right)\left(\alpha-\beta\left(\frac{p_1+2p_2+2p_3+p_4}{6}\right)\right)}{\lambda(\lambda+\delta)} \\
\times\left\{(\lambda+\delta)(e^{\lambda T}-e^{\lambda t_1})-\lambda e^{-\delta T}(e^{(\lambda+\delta)T}-e^{(\lambda+\delta)t_1})\right\} \\
+\left(\frac{c_1+2c_2+2c_3+c_4}{6}\right)\left\{\begin{array}{l}
\frac{\left(\alpha-\beta\left(\frac{p_1+2p_2+2p_3+p_4}{6}\right)\right)}{(\theta+\lambda)} \\
(e^{(\theta+\lambda)t_1}-1) \\
+\frac{\left(\alpha-\beta\left(\frac{p_1+2p_2+2p_3+p_4}{6}\right)\right)}{(\lambda+\delta)} \\
e^{-\delta T}(e^{(\lambda+\delta)T}-e^{(\lambda+\delta)t_1})
\end{array}\right\} \\
-\left(\frac{p_1+2p_2+2p_3+p_4}{6}\right)\left\{\begin{array}{l}
\frac{\left(\alpha-\beta\left(\frac{p_1+2p_2+2p_3+p_4}{6}\right)\right)}{\lambda} \\
(e^{\lambda t_1}-1) \\
+\frac{\left(\alpha-\beta\left(\frac{p_1+2p_2+2p_3+p_4}{6}\right)\right)}{(\lambda+\delta)} \\
e^{-\delta T}(e^{(\lambda+\delta)T}-e^{(\lambda+\delta)t_1})
\end{array}\right\}
\end{array}\right] \quad \ldots(22)
$$

Example 2*. (*Fuzzy model)

Here we assigned trapezoidal fuzzy number to the cost parameter as below

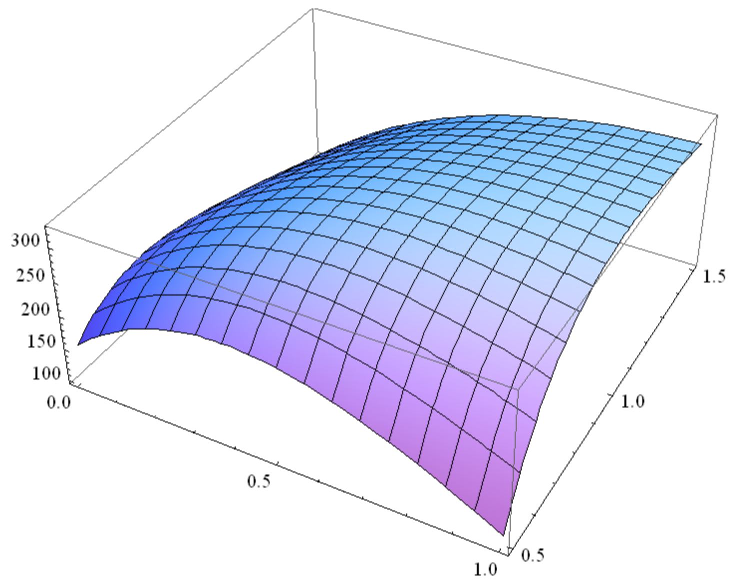
□=0.08; □=200;□=4;□=0.98;□=0.1;p=35.4

$\tilde{A} = (a_1, a_2, a_3, a_4) = (230, 240, 255, 260)$, $\tilde{c} = (c_1, c_2, c_3, c_4) = (17, 19, 21, 22)$

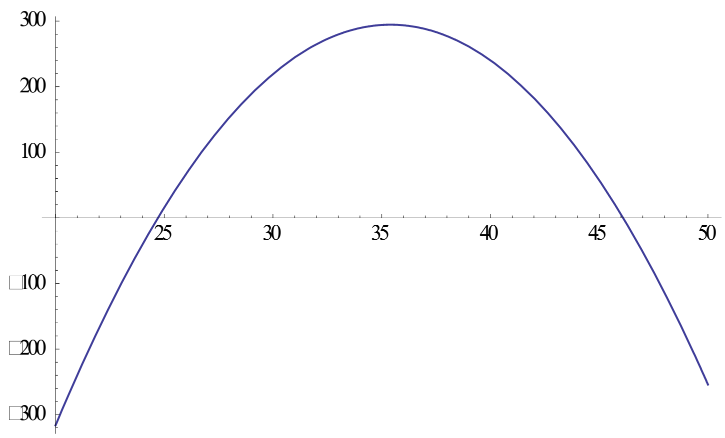
$\tilde{C}_h = (C_{h1}, C_{h2}, C_{h3}, C_{h4}) = (0.7, 0.9, 1.1, 1.3)$, $\tilde{C}_d = (C_{d1}, C_{d2}, C_{d3}, C_{d4}) = (3, 4, 6, 7)$

$\tilde{C}_s = (C_{s1}, C_{s2}, C_{s3}, C_{s4}) = (3, 4, 6, 7)$, $\tilde{C}_o = (C_{o1}, C_{o2}, C_{o3}, C_{o4}) = (21, 23, 27, 29)$,

$\tilde{Z}(p, t_1^*, T^*) = 326.493$, $t^* \to 0.701689, T^* \to 0.938103$



Now we used t^* = 0.719 and T* = 0.97 for optimize the value of p and we get
p^* = 35.4031



5. SENSITIVITY ANALYSIS

For the given examples sensitivity analysis has been performed to study the effect of changes of different parameters like deterioration, price and demand related parameters and backlogging parameter by changing (increasing and decreasing) the parameters at a time in both crisp and fuzzy model. Results of this analysis are shown in the following tables

Table1. Crisp Model

Parameter	*Variation*	*t**	*T**	*Z*(p*,t*,T*)*
α	180	1.02608	1.39075	113.307
	200	0.732521	0.987613	284.903
	220	0.594834	0.800036	479.859
β	3	0.527434	0.708553	641.027
	4	0.732521	0.987613	284.903
	5	1.87601	2.58481	12.1342
γ	-0.90	0.745374	1.00517	302.913
	-0.98	0.732521	0.987613	284.903
	-1.06	0.721602	0.972702	267.827
δ	0.08	0.715336	0.988121	285.623
	0.10	0.732521	0.987613	284.908
	0.12	0.747627	0.987181	284.277
θ	0.06	0.773803	0.996424	290.272
	0.08	0.732521	0.987613	284.903
	0.10	0.0694736	0.979193	279.886

Observation of above table is that the profit is directly proportional to the value of α *i.e.* profit increased as α increased and profit decreased as the value of α decreased. On the other side total profit is inversely proportional the value of $\beta, \gamma, \delta, and\, \theta$ *i.e.* the total profit increase as $\beta, \gamma, \delta, and\, \theta$ decreased andvice-versa.

Table 2 Fuzzy Model

Parameter	*Variation*	*t**	*T**	*Z*(p*,t*,T*)*
α	180	0.972192	1.30587	137.336
	200	0.701689	0.938103	326.501
	220	0.572182	0.76328	539.311
β	3	0.508292	0.677324	714.468
	4	0.701689	0.938103	326.501
	5	1.67796	2.28332	22.4278
γ	-0.90	0.715271	0.956483	345.457
	-0.98	0.701689	0.938103	326.501
	-1.06	0.69001	0.922305	308.513
δ	0.08	0.686122	0.938577	327.162
	0.10	0.701689	0.938103	326.501
	0.12	0.715409	0.937697	325.925
θ	0.06	0.739142	0.945755	331.563
	0.08	0.701689	0.938103	326.501
	0.10	0.667269	0.930792	321.761

6. CONCLUSION

The development of an inventory model for deteriorating items with partial backlogging and time and price-dependent demand represents a significant advancement in business world. This model tackles the complex dynamics of inventory management, considering multiple factors that impact decision-making. The inclusion of fuzziness in the environment acknowledges the inherent uncertainty and vagueness that exist in real-world businesses. This allows decision-makers to account for imprecise information and make more robust and adaptive inventory control decisions. It acknowledges that pricing decisions can influence inventory levels and vice versa, enabling a more holistic approach to optimizing profitability and customer satisfaction. Partial backlogging is also incorporated, acknowledging the reality that some customers may be willing to wait for out-of-stock items, avoiding lost sales and potentially improving overall customer retention. The model accounts for time and price-dependent demand, recognizing that consumer behavior can vary over time and in response to changes in pricing. By factoring in these demand dynamics, the model provides a more accurate representation of customer preferences and allows for improved decision-making in terms of inventory replenishment and pricing strategies.

Trapezoidal fuzzy number assigned to the cost parameter to make inventory model more reasonable. We optimized the total profit function by applying graded mean integration method. Observation of results shows that under various uncertainties in cost, fuzzy model provides improved results. This model presents a valuable tool for businesses operating in dynamic and uncertain environments, offering a comprehensive framework for joint pricing and inventory control. Its application has the potential to enhance operational efficiency, improve customer satisfaction, and maximize profitability in a wide range of industries.

CONFLICT OF INTERESTS

None

ACKNOWLEDGMENTS

None

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