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EXPLORING THE INTERPLAY BETWEEN COMMUNICATION COMPLEXITY AND NETWORK DENSITY IN PDNS

Pinki Sharma[1], Reshma Begum[2], Rakesh Kumar Katare[3], Dr. Akhilesh A. Waoo[2]

[1] 1Department of Computer Science and Engineering, AKS, University, Satna [M.P.], India
[2] Department of Mathematics, Govt. P.G. College, Seoni [M.P.], India
[3] Department of Computer Science APS University, Rewa [M.P.], India
[4] *Department of Computer Science and Engineering, AKS, University, Satna [M.P.], India

Corresponding Author
Akhilesh A. Waoo,
akhileshwaoo@gmail.com

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ABSTRACT

This research investigates the features of a PDN interconnection network for Parallel and Distributed architecture. PDN is a novel and evolving architecture for parallel and distributed systems. It is an extremely effective method of joining a group of nodes into a network with a diameter equal to 2. At a distance of one or two nodes are joined together. A Perfect Difference Set is a mathematical technique for estimating the improved number of nodes in an extremal manner for a perfect difference network. We represented the Adjacency Matrix of PDN in geometric form and separated it into four pieces i.e., represented the geometric relationship among PDN processors. We next estimated the Network Density and Communication Complexity of PDN and its subparts and presented a quick comparison of them.

Keywords: Interconnection Network, Adjacency Matrix, Communication Complexity, Network Density, Parallel and Distributed Architecture, PDN.

1. INTRODUCTION

In this research work, the PDN interconnection network for Parallel and Distributed architecture is chosen to study its properties. To do this a new approach has been discovered in which we have taken an adjacency matrix and does its division graphically.

2. PERFECT DIFFERENCE NETWORK [PDN]

PDN is a new and emerging architecture in Parallel and Distributed Systems. it is an extremely adequate way of connecting a set of nodes into a network with a diameter equal to 2, and one node is connected to another node in 1 or 2 hops. A Perfect Difference set is a mathematical tool for calculating and optimizing number nodes asymptotically for a perfect difference Network [7]. Perfect difference sets were first discussed by J. Singer in 1938 in terms of points and lines in a finite projective plane [9][15].

A graphical representation of PDN for n =7, based on the PDS {0,1,3}, is depicted in the following Fig. 1.

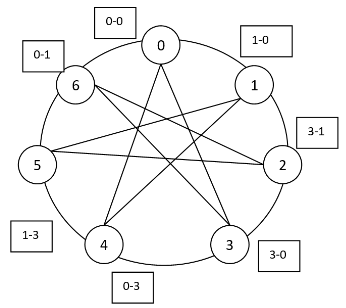


Fig 1. Graphical Representation of PDN

3. ADJACENCY MATRIX OF PDN

Suppose G is a simple directed graph with m nodes, and suppose the nodes of G have been ordered and are called $v_1, v_1, v_1, \dots\dots, v_1$. Then the adjacency matrix [33] A = (a_{ij}) of the graph G is the $m \times m$ matrix defined as follows:

$$a_{ij} = \begin{cases} 1 \ \ if\ v_i\ is\ adjacent\ to\ v_j, \\ \quad 0\ \ otherwise \end{cases}$$

$$L = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Fig 2. Adjacency Matrix of PDN

Following is the Tabular representation of Matrix L, here i represents the number of rows and j represents the number of columns. Here we have colored the blocks containing value 1 to distinguish them from the blocks containing 0 value. We have done so just to make the pattern of PDN clear.:

Table 1. Tabular Representation of Adjacency Matrix L of PDN

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

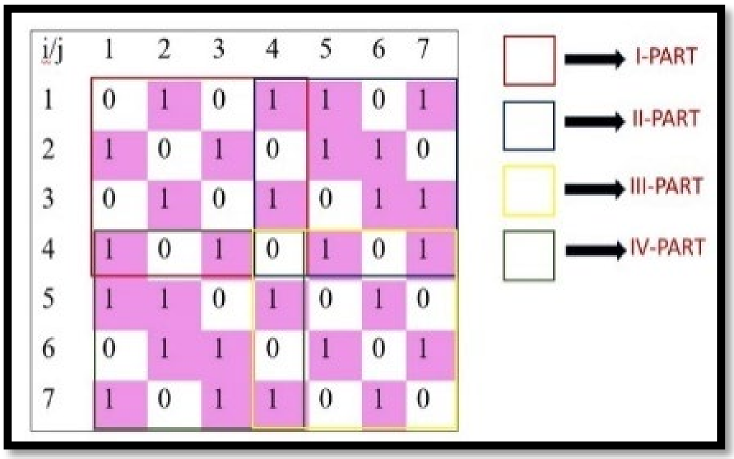
4. MATRIX DIVISION: GRAPHICALLY

Now, we will assume the division of the above matrix shown in Table 1 in two ways first by dividing it horizontally and vertically from the center of Matrix L and then by dividing it diagonally. This division is shown in Fig. 3 and Fig. 11 respectively. Then, we have represented each part separately and derived its architecture for each part. To make them clear we have used four colors red blue yellow and green. These colors represent I-Part, II-Part, III-part, and IV-Part respectively.

Let's see the division of Matrix in section 3.1 and section 3.2. After that derivation of architecture from subparts of matrix for both cases. These architectures and subparts are discussed in sub-sections of section 3.1 and section 3.2.

3.1. VERTICAL AND HORIZONTAL DIVISION

In the following Figure 3, we have taken a tabular representation of Matrix L and divided it horizontally and vertically into four parts (say Matrix A). The division is distinguished with the help of different colors as shown below in Fig. 3. The first part is separated using red color. The second part is separated using blue color. The third part is separated using yellow color and the fourth part is separated using green color.



i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 3. Matrix A Division Horizontally and Vertically (say Matrix A)

3.1.1 I-PART

Here, the first part of matrix A which is shown in Fig.1 is shown in Fig. 4. Let's name it L_A. This part represents connections among the first, second, third, and fourth nodes with themselves of PDN architecture shown in Fig. 1. Since, it shows a connection of four nodes, this part or L_A will be a matrix with 4 rows and 4 columns. Now, we have derived architecture using only this part which is shown in Fig.5. On comparing Fig 1 of PDN and the first part of architecture in Fig 5, we conclude that Fig 5 is a sub-part of Fig 1. That means a part of PDN is represented in the Fig. 5.

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 4. I-Part L_A of Matrix A

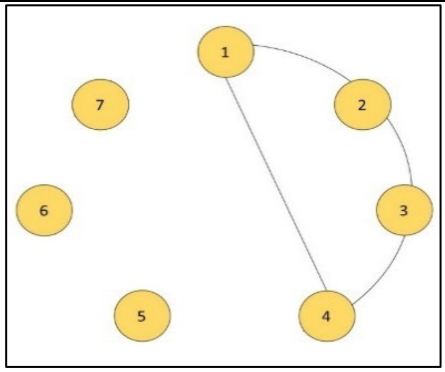


Fig 5. Architecture of I-Part L_A of Matrix A

3.1.2 II-PART

Here, the second part of matrix L which is shown in Fig. 1 is given in Fig. 6. Let's name it L_B. This part represents connections among nodes first, second, third, and fourth nodes with the fourth, fifth, sixth, and seventh nodes respectively of PDN architecture shown in Fig. 1. This part L_B forms a matrix with 4 rows and 4 columns. Now, we have derived architecture using this part which is shown in Fig.7. On comparing Fig 1 of PDN and the second part of architecture in Fig 7, we conclude that Fig 7 is a sub-part of Fig 1. That means a more part of PDN is represented in the Fig. 7.

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 6. II-Part L_B of Matrix A

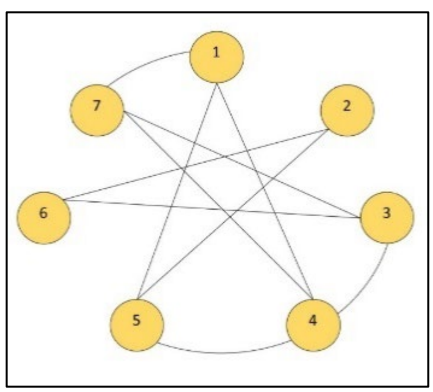


Fig 7. The architecture of II-Part L_B of Matrix A

3.1.3 III-PART

Here, the third part of matrix L which is shown in Fig. 1 is given in Fig. 8. Let's name it L_C. This part represents connections among the fourth, fifth, sixth, and seventh nodes with the first, second, third, and fourth nodes respectively of PDN architecture shown in Fig. 1. This part L_C forms a matrix with 4 rows and 4 columns. Now, we have derived architecture using this part which is shown in Fig.9. On comparing Fig 1 of PDN and the third part architecture in Fig 9, we conclude that Fig 9 is a sub-part of Fig 1. That means a more part of PDN is represented in the Fig. 9.

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 8. III-Part of Matrix A

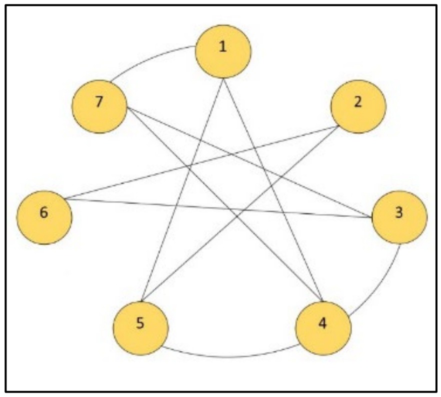


Fig 9. The architecture of III-Part of Matrix A

3.1.4 IV-PART

Here, the fourth part of matrix L which is shown in Fig. 1 is given in Fig. 10. This part represents connections among the fourth, fifth, sixth, and seventh nodes with themselves of PDN architecture shown in Fig. 1. Assume this part as a matrix with 4 rows and 4 columns. Now, we have derived architecture using this part which is shown in Fig. 11. On comparing Fig 1 of PDN and the fourth part of architecture in Fig 11, we conclude that Fig 11 is a sub-part of Fig 1. That means a more part of PDN is represented in the Fig. 11.

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 10. IV-Part of Matrix A

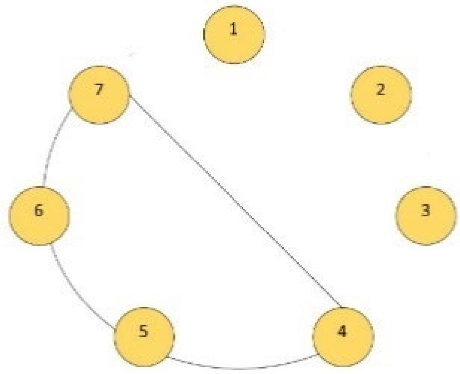
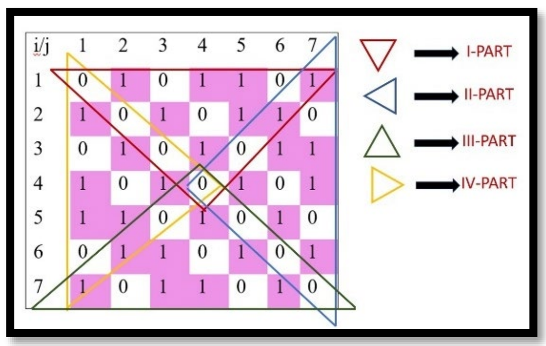


Fig 11. The architecture of IV-Part of Matrix *A*

3.2. DIAGONAL DIVISION

In the following Figure 12, we have taken a tabular representation of Matrix L and divided it diagonally into four parts (say Matrices $L_1, L_2, L_3,$ and L_4). Let's say this matrix B. The division is distinguished with the help of different colors as shown below in Fig. 12. The first part is separated using red color. The second part is separated using blue color. The third part is separated using green color and the fourth part is separated using yellow color.



i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 12. Matrix L Division Diagonally (say Matrix B)

3.2.1. I-PART

Here, the first part of matrix L which is shown in Fig.1 is shown in Fig. 13. Let's name it L_1. This part represents partial connections of all seven nodes with the first second, third, and fourth nodes of PDN architecture shown in Fig. 1. Assume this part as a matrix with 4 rows and 7 columns. Now, we have derived architecture using this part which is shown in Fig.14. On comparing Fig 1 of PDN and the first part of architecture in Fig 14, we conclude that Fig. 14 is a sub-part of Fig 1. That means a part of PDN is represented in the Fig. 14.

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 13. I-Part of Matrix B

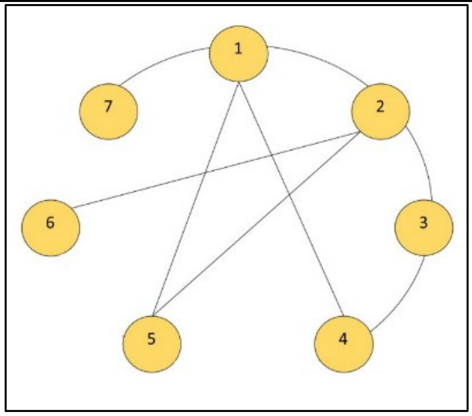


Fig 14. The architecture of I-Part of Matrix B

3.2.2.II-PART

Here, the first part of matrix L which is shown in Fig.1 is shown in Fig. 15. Let's name it L_2. This part represents partial connections of fourth, fifth, and sixth with all seven nodes of PDN architecture shown in Fig. 1. Assume this part as a matrix with 7 rows and 4 columns. Now, we have derived architecture using this part which is shown in Fig.16. On comparing Fig 1 of PDN and the second part of architecture in Fig 16, we conclude that Fig. 16 is a sub-part of Fig 1. That means a part of PDN is represented in the Fig. 16.

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 15. II-Part of Matrix B

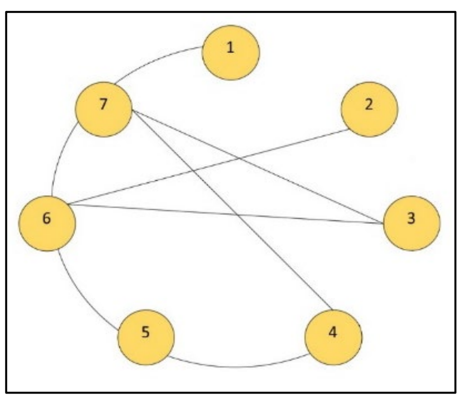


Fig 16. The architecture of II-Part of Matrix B

3.2.3.III-PART

Here, the first part of matrix L referred to as Fig. 1 is shown in Fig. 17. Let's name it L_3. This part represents partial connections of all seven nodes with the fourth, fifth, sixth, and seventh nodes of PDN architecture shown in Fig. 1. Assume this part is a matrix with 4 rows and 7 columns. Now, we have derived architecture using this part which is shown in

Fig.18. On comparing Fig 1 of PDN and the third part architecture in Fig 18, we conclude that Fig. 18 is a sub-part of Fig 1. That means a part of PDN is represented in the Fig. 18.

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 17. III-Part of Matrix B

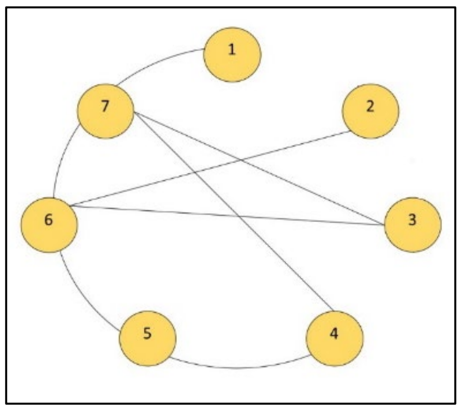


Fig 18. The architecture of III-Part of Matrix B

3.2.4.IV-PART

Here, the first part of matrix L referred to as Fig.1 is shown in Fig. 19. Let's name it L_4. This part represents partial connections of the first, second, third, and fourth nodes with all seven nodes of PDN architecture shown in Fig. 1. Assume this part is a matrix with 7 rows and 4 columns. Now, we have derived architecture using this part which is shown in Fig.20. On comparing Fig 1 of PDN and the fourth part architecture in Fig 20, we conclude that Fig. 20 is a sub-part of Fig 1. That means a part of PDN is represented in the Fig. 20.

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Fig 19. IV-Part of Matrix B

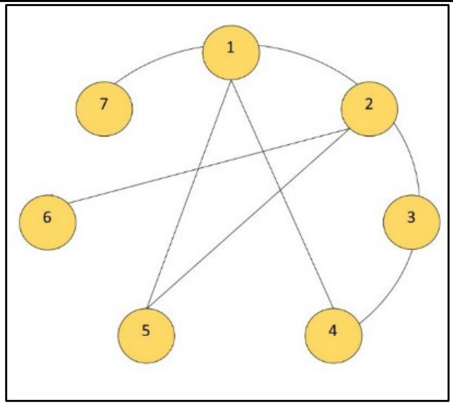


Fig 20. The architecture of IV-Part of Matrix B

From the above statements, we conclude that all four parts are subparts of PDN architecture. In the following Fig. 21 and Fig. 22 we have shown all parts together form both the cases of matrix division.

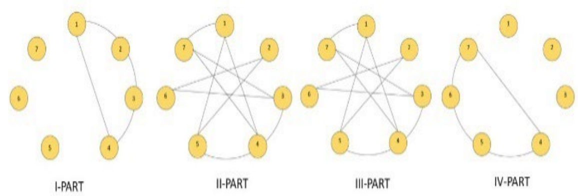


Fig 21. All Four architecture of four subparts of Matrix A

From Fig. 21, we presume that:

a) I-part L_A. and IV-part L_D are similar but in opposite directions to each other i.e., $L_A \sim L_D$.

b) 2. II-part L_B and III-part L_C are identical or commensurate to each other i.e., $L_B \cong L_C$.

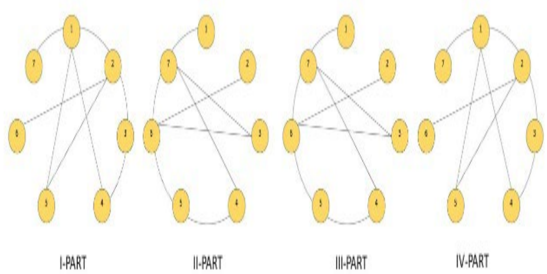


Fig 22. All Four architectures of four subparts of Matrix B

From Fig. 22, we presume that:

a) I-part and IV-part are identical or commensurate to each other i.e., $L_1 \cong L_4$.

b) II-part and III-part are identical or commensurate to each other i.e., $L_2 \cong L_3$.

5. ANALYSIS OF COMMUNICATION COMPLEXITY IN PDN

Communication Complexity of network in Parallel and Distributed Computing means nodes communicating to how many other nodes at a time i.e., amount or degree of communication for nodes in a network.[10]

Based on the above postulates, we have studied and derived several connections shown by each part of both Matrix A and Matrix B separately for every node to find the Communication Complexity of each node. We have tabulated this for Matrix A in the following Table 2 and for Matrix B in the following Table 3.

Table 2. Tabular representation of Communication Complexity of all four parts of Matrix-A

Node Number	Communication Complexity for each node			
	I-Part L_A	II-Part L_B	III-Part L_C	IV-Part L_D
1	3	4	0	3
2	3	3	0	4
3	3	4	0	5
4	3	5	3	4
5	0	4	3	4
6	0	3	3	3
7	0	4	3	4
Total	12	27	12	27

From Table 2. We conclude that,

a) Communication Complexity of I-Part L_A is equal to III-Part L_C i.e., $L_A(CC) = L_C(CC) = 12$, where CC is Communication Complexity.

b) Communication Complexity of II-Part L_A is equal to IV-Part L_C i.e., $L_B(CC) = L_D(CC) = 27$, where CC is Communication Complexity.

Table 3. Tabular representation of Communication Complexity of all four parts of Matrix-B

Node Number	Communication Complexity for each node			
	I-Part L_1	II-Part L_2	III-Part L_3	IV-Part L_4
1	5	2	2	5
2	5	2	2	5
3	3	3	3	3
4	3	3	3	3
5	3	3	3	3
6	2	5	5	2
7	2	5	5	2
Total	23	23	23	23

From Table 3. We conclude that,

c) The communication Complexity of all four parts is equal i.e., $L_1(CC) = L_2(CC) = L_3(CC) = L_4(CC) = 23$ where CC is Communication Complexity.

6. ANALYSIS OF NETWORK DENSITY IN PDN

Here in this section, we will derive the Network Density of each subpart of PDN architecture for both Matrix A and Matrix B. For that let's see what is Network Density first.

Network Density of Architecture:

Network density is defined as the ratio between the number of actual connections of a network to its number of potential connections [14]. A *"potential connection"* is a connection that could potentially exist between two "nodes" – regardless of whether or not it does. We can calculate the Network Density if the number of nodes is n as,

$$Network\ Density = \frac{Actual\ Connections}{Potential\ Connections} \ldots \ldots \ldots 1$$

$$where, Potential\ Connections = \frac{n*(n-1)}{2}$$

and here Actual Connection is equal to the total number of 1s in the subpart of a matrix of a given architecture.

We have tabulated the Network Density of each subpart of PDN architecture in Table 4 for Matrix A and in Table 5 for Matrix B.

Table 4. Tabular representation of Network Density of all four parts of Matrix-A

Sub-Parts of Matrix A	Network Density
I-Part	$\left(\frac{12}{35}\right) \times 100 = 34.28\%$
II-Part	$\left(\frac{27}{35}\right) \times 100 = 77.14\%$
III-Part	$\left(\frac{12}{35}\right) \times 100 = 34.28\%$
IV-Part	$\left(\frac{27}{35}\right) \times 100 = 77.14\%$

From above Table 4, we presume that the Network Density of I-Part and III-Part are equal since their architecture is also identical to each other concerning the degree of connections each node has. Similarly, we can see that the Network Density of II-Part and IV-Part is equal since both subparts are identical to each other concerning the degree of connections each node has.

i) Network Density of I-Part L_A is equal to III-Part L_C i.e., $L_A(ND) = L_C(ND) = 34.28$, where ND is Network Density. Communication Complexity of II-Part L_A is equal to IV-Part L_C i.e., $L_B(ND) = L_D(ND) = 77.14$, where ND is Network Density.

Table 5. Tabular representation of Network Density of all four parts of Matrix-B

Sub-Parts of Matrix B	Network Density
I-Part	$\left(\frac{23}{35}\right) \times 100 = 65.71\%$
II-Part	
III-Part	
IV-Part	

From above Table 5, we presume that the Network Density of all subparts of Matrix B is equal since their architecture also has an equal number of connections i.e., the degree of connections for each node is somehow equal.

i. Network Density of all four parts is equal i.e., $L_1(ND) = L_2(ND) = L_3(ND) = L_4(ND) = 65.71$, where ND is Network Density.

7. COMPARING THE NETWORK DENSITY OF PDN WITH ITS SUBPARTS

Network Density of PDN from equation (1) is,

$$\text{Network Density of PDN} = \frac{7 * (7-1)}{2} = \frac{7 \times 6}{2} = \frac{42}{2} = 21$$
$$= \frac{14}{21} \times 100 = 66.66\%$$

From Table 7.4. we conclude that

$L_A(ND) = L_C(ND) < L$ and $L_B(ND) = L_D(ND) > L$
From Table 7.5. we conclude that

$L_1(ND) = L_2(ND) = L_3(ND) = L_4(ND) \sim L$

8. COMPARING COMMUNICATION COMPLEXITY OF PDN WITH ITS SUBPARTS

Here, we will compare the communication complexity of PDN with other parts of it from Matrix A and Matrix B.

Table 6. Communication Complexity of Matrix-L of PDN

Node Number	Communication Complexity of PDN L
1	4
2	4
3	4
4	4
5	4
6	4
7	4
Total	28

From Table 6 and Table 2, we conclude that,

$L(CC) > L_A(CC) = L_C(CC)$ and

$$L(CC) > L_B(CC) = L_D(CC)$$

i.e., the Communication Complexity of PDN is always greater than its subparts in Matrix-A.
From Table 7.6 and Table 7.3, we conclude that,
$L(CC) > L_1(CC) = L_2(CC) = L_3(CC) = L_4(CC)$

i.e., the Communication Complexity of PDN is always greater than its subparts in Matrix-B.

9. CONCLUSION

From the above calculations and theory, we conclude that adjacency matrices also show properties of interconnection networks. We have also seen that the communication complexity and network density of parts of matrices is similar to the main complexity and density of the interconnection network. We can use this method for other properties and other interconnection networks. Also, the assumptions so far observed can further be proved using appropriate methods.

CONFLICT OF INTERESTS

None

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None

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