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MODIFIED ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS USING BALANCED TERNARY DESIGNS

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ABSTRACT

In this article, following the methods constructions of Kanna et al. (2018) Varalakshmi and Rajyalakshmi (2020, 22), a new method of modified second order response surface designs using balanced ternary designs (BTD) is suggested. A few explanatory illustrations are also presented.

Keywords: Response Surface Designs, Modified Rotatable Designs, Balanced Ternary Designs

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1. INTRODUCTION

A set of mathematical and statistical methods known as response surface methodology are helpful, when assessing the situations in which multiple independent variables affect a dependent variable. "An important step in development of response surface designs was the introduction of rotatable designs by Box and Hunter (1957). Das as well as Narasimha (1962) constructed rotatable designs dealing with balanced incomplete block designs (BIBD). Billington (1983) provided a file of balanced ternary designs (BTD) with some necessary conditions.

Billington (1984) suggested balanced n- array designs. Donovan (1985) introduced balanced ternary designs from 1-designs. Kunkle and Sarvate (1996) studied on further development of balanced ternary designs. Das et al. (1999) introduced modified second order response surface designs. Kaski and Ostergard (2004) suggested enumeration of balanced ternary designs Victor Babu along with Vasundhara devi (2005) suggested modified second order response surface designs dealing with BIBD. Victor babu et al. (2008) suggested modified rotatable central composite designs. Kanna et al. (2018) constructed a new class of SORD using balanced ternary designs. Varalakshmi and Rajyalakshmi (2020, 22) studied the optimum response for SORD using balanced ternary designs and measure of rotatability for a class of balanced ternary designs.

In this article, following the methods constructions of Kanna et al. (2018) Varalakshmi and Rajyalakshmi (2020, 22), a new method of modified rotatability for second order response surface designs using balanced ternary designs is suggested. A few exploratory illustrations are also presented".

2. MODEL AND DESIGN SPECIFICATIONS

"For example, we want to utilize the quadratic response surface model D= to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i<j}\sum b_{ij} x_{iu} x_{ju} + e_u \quad (1)$$

here indicate the level of the itch factor (i =1,2,3,4,…,v) in the uth run (u=1,2,3,4,…,N) of the experiment, are uncorrelated random errors with mean zero and variance is said to be second order rotatable design (SORD), if the variance of the estimate of first order partial derivative of with respect to each of independent variables is only a function of the distance ( ) of the point (x1,x2, …,xv) from the origin (centre) of the design. Such a spherical variance function for evaluation of second order response surface is reached if the design must satisfy the following restrictions" [cf. Box and Hunter (1957), Das and Narasimham (1962)].

$$\sum x_{iu} = 0,\ \sum x_{iu}x_{ju} = 0,\ \sum x_{iu}x_{ju}^2 = 0,\ \sum x_{iu}x_{ju}x_{ku} = 0,\ \sum x_{iu}^3 = 0,\ \sum x_{iu}x_{ju}^3 = 0,$$
$$\sum x_{iu}x_{ju}x_{ku}^2 = 0,\ \sum x_{iu}x_{ju}x_{ku}x_{lu} = 0;\ \text{for } i \neq j \neq k \neq l; \quad (2)$$

$$\sum x_{iu}^2 = \text{constant} = N\lambda_2;$$

$$\sum x_{iu}^4 = \text{constant} = cN\lambda_4;\ \text{for all } i \quad (3)$$

$$\sum x_{iu}^2 x_{ju}^2 = \text{constant} = N\lambda_4;\ \text{for } i \neq j \quad (4)$$

$$\sum x_{iu}^4 = c\sum x_{iu}^2 x_{ju}^2 \quad (5)$$

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)} \quad (6)$$

here c, λ_2 and λ_4 are treated as constants and the summation is over the design points.

When the afore mentioned criteria are met, the calculated parameters variances and covariance's become,

$$V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1)-v\lambda_2^2\right]},$$

$$V(\hat{b}_i)=\frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij})=\frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii})=\frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right],$$

$$Cov(\hat{b}_0,\hat{b}_{ii})=\frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]}$$

$$Cov(\hat{b}_{ii},\hat{b}_{jj})=\frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]} \quad (7)$$

and reaming covariance's are vanish.

3. CONDITIONS FOR SECOND ORDER ROTATABLE DESIGNS

The most widely used design for fitting a second order model is the central composite design. Central composite designs are constructed by adding suitable "factorial combinations to those obtained from $\frac{1}{2^p}\times 2^v$ fractional factorial design (here $2^{t(v)}=\frac{1}{2^p}\times 2^v$ denotes a suitable fractional replicate of 2^v, in which no muddled interactions with fewer than five components exist. In coded form the points of $2^v(2^{t(v)})$ factorial have coordinates $(\pm a,\pm a,\ldots,\pm a)$ and $2v$ axial points have coordinates of the model $((\pm b, 0,\ldots 0), (0, \pm b,\ldots 0), \ldots, (0, 0, \ldots, \pm b))$ etc., and central points. Typically, combinations with unknown constants are used to generate SORD's, associate a factorial combination or a suitable fraction of it with factors each at levels to make the level codes equidistant. All such combinations form a design. Generally, SORD need at least five levels (suitably coded) at $0,\pm a,\pm b$ for all factors ($(0,0,\ldots 0)$)- chosen centre of the design, unknown level 'a' and 'b' are to be chosen suitably to satisfy the conditions of the rotatability) generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknown levels.

Alternatively, by imposing certain limitations that suggest a relation among $\sum x_{iu}^2, \sum x_{iu}^4$ and $\sum x_{iu}^2, \sum x_{iu}^4$ some equations involving the unknowns are obtained and their solution gives the unknown levels. In SORD the restriction used is $\sum x_{iu}^4 = 3\sum x_{iu}^2 x_{ju}^2$, i.e., $c=3$ other restrictions are also possible through, it seems, not exploited well. Das et al (1999) proposed the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ i.e., $\lambda_2^2 = \lambda_4$ to get another series of symmetrical quadratic response surface designs, which provide more exlicit estimates of response at specific points of interest than what is available from the corresponding existing designs. Further, the estimated parameters of variances and covariance's are,"

$$V(\hat{b}_0) = \frac{(c+v-1)\sigma^2}{N(c-1)}$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\sqrt{\lambda_4}}$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4}$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}$$

$$Cov(\hat{b}_0, \hat{b}_{ii}) = \frac{-\sigma^2}{N\sqrt{\lambda_4}(c-1)} \qquad (8)$$

and the reaming covariance's are vanish. "The variance of estimated response at particular moments is significantly impacted by these changes to the variances and covariance's, variance of estimated response at any point can be obtained. Let denote the evaluated response at the point. Then,

$$V(\hat{Y}_u) = V(\hat{b}_0) + d^2[V(\hat{b}_i) + 2cov(\hat{b}_0, \hat{b}_{ii})] + d^4 V(\hat{b}_{ii}) + (\sum x_{iu}^2 x_{ju}^2)[(c-3)\sigma^2 / (c-1)N\lambda_4]$$

Construction of transform response surface designs is the same as for SORD except that instead of taking the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ is to be used and this condition will provide different values of the unknowns involved. (cf. Das et al. 1999)".

4. MODIFIED ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS USING BTD

An effort is made to fit quadratic rotatable designs using balanced ternary designs, in accordance with Kanna et al. (2018), Varalakshmi and Rajyalakshmi (2020), and Varalakshmi and Rajyalakshmi (2022) techniques of building of second order rotatable designs using balanced ternary designs.

5. BALANCED TERNARY DESIGNS

The notation for BTD is (v, b, ρ1, ρ2, r, k, λ), where 'v' represents rows and 'b' represents columns. The number of times the element i appears in block j is shown by the (i, j) the cell. The incidence matrix form for a balanced ternary design, denoted by n=(nij), should meet the following requirements:

1) Every treatment may show up or not; if it does, it shows up 0–1 or 2 times in a block.
2) There are exactly the same numbers of replications, or r, for every treatment.
3) According to Billington (1984), each element appears in BTD twice in ρ2 blocks and once in ρ1 blocks.

The parametric relations of BTD are

1) vr = bk
2) $\lambda(v-1)=\rho_1(k-1)+2\rho_1(k-2)$.
3) $r=\rho_1+2\rho_2$.

The result of modified rotatability for second order response surface designs using BIBD is suggested here (cf. Victor Babu and Vasundhara devi 2005). Let (v, b, ρ1, ρ2, r, k, λ) be a BTD, denotes a fractional replicate of with levels in which no muddle interaction with fewer than five components exist. represents the design points produced from the transpose of the incidence matrix of BTD. are the design points generated from BTD by "multiplication" (cf. Raghavarao,1971). Repeat design points times. Let represents the design points generated from point set. Repeat this set of additional design points, say times. Let reflect the combination of the design points created from several sets of points and is the number of central points in the transform SORD.

6. THEOREM

The design points, $y_1[1\text{-}(v, b, \rho_1, \rho_2, r, k, \lambda)]2^{t(k)}$ U $y_2(\pm a,0,0,\ldots.0)2^1$ U (n_0) will give a - dimensional modified SORD in $N=\frac{(y_1\rho_1 2^{t(k)}+2\rho_2 2^{t(k)}+2y_2a^2)^2}{y_1\lambda 2^{t(k)}}$ design points if,

$$a^4=\frac{(3y_1\lambda\text{-}y_1\rho_1\text{-}2\rho_2)2^{t(k)-1}}{y_2}$$

$$n_0=\frac{(y_1\rho_1 2^{t(k)}+2\rho_2 2^{t(k)}+2y_2a^2)^2}{y_1\lambda 2^{t(k)}}\text{-}[y_1b2^{t(k)}+2y_2v]$$ and must be an integer.

Proof:

For the simple symmetric conditions to produced from the BTD we have

$$\sum x_{iu}^2 = y_1\rho_1 2^{t(k)}+2\rho_2 2^{t(k)}+y_2 2a^2 = N\lambda_2 \quad (9)$$

$$\sum x_{iu}^4 = y_1\rho_1 2^{t(k)}+2\rho_2 2^{t(k)}+y_2 2a^4 = cN\lambda_4 \quad (10)$$

$$\sum x_{iu}^2 x_{ju}^2 = y_1\lambda 2^{t(k)} = N\lambda_4 \quad (11)$$

To make the design rotatable, we take. From equations (10) and (11), we have

$$a^4 = \frac{(3y_1\lambda - y_1\rho_1 - 2\rho_2)2^{t(k)-1}}{y_2}$$

The modified condition $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ leads to N which is given by

$N = \frac{(y_1\rho_1 2^{t(k)} + 2\rho_2 2^{t(k)} + 2y_2 a^2)^2}{y_1\lambda 2^{t(k)}}$ alternatively, N may be obtained directly as, $y_1 b 2^{t(k)} + 2vy_2 + n_0$, where is given by $n_0 = \frac{(y_1\rho_1 2^{t(k)} + 2\rho_2 2^{t(k)} + 2y_2 a^2)^2}{y_1\lambda 2^{t(k)}} - [y_1 b 2^{t(k)} + 2y_2 v]$ and must be an integer. From equations (13) and (15) and on shortening we get.

$$\lambda_2 = \frac{y_1 r 2^{t(k)} + 2y_2 a^2}{N} \text{ and } \lambda_4 = \frac{y_1\lambda 2^{t(k)}}{N}$$

Example: With the aid of a BTD, we demonstrate the transform rotatability for quadratic response surface designs for factors $(v=3, b=3, \rho_1=1, \rho_2=1, r=3, k=3, \lambda=2)$. The design points,

$$y_1[1-(v=3,b=3,\rho_1=1,\rho_2=1, r=3,k=3,\lambda=2)]2^3 \text{ U } y_2(\pm a,0,0,....0)2^1 \text{ U } (n_0)$$

will give transform rotatability for quadratic response surface designs in N= design points. From (9), (10) and (11), we have

$$\sum x_{iu}^2 = 24+6a^2 = N\lambda_2 \quad (12)$$

$$\sum x_{iu}^4 = 24+6a^4 = cN\lambda_4 \quad (13)$$

$$\sum x_{iu}^2 x_{ju}^2 = 16 = N\lambda_4 \quad (14)$$

From equations (13) and (14) with rotatability value c=3 we get $a^4 = 4 \Rightarrow a^2 = 2 \Rightarrow a = 1.414214$ from equations (12) and (13) using the transform condition with $(\lambda_2^2 = \lambda_4)$ with $a^2 = 2$, $y_1 = 1$ and $y_2 = 3$, $N = 81$ we get, $n_0 = 39$

Table 1

Table 1 Results of Modified Rotatability for Second Order Response Surface Designs Using BTD

(v, b, ρ1, ρ2, r,k,λ)	t(k)	y1	y2	a	n0	N
(3,3,1,1,3,3,2)	3	1	3	1.4142	39	81
(5,5,1,2,5,5,4)	4	1	14	1.4142	69	289
(6,6,2,1,4,4,2)	4	1	4	1.4142	56	200
(8,8,4,1,6,6,4)	5	1	6	2	98	450

CONFLICT OF INTERESTS

None.

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None.

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